Low-Energy Antikaon-Nucleon Interaction

J. Schnick and R. H. Landau
Physics Department, Oregon State University, Corvallis, Oregon 97331
(Received 12 January 1987)

The $\bar{K}N$ S-wave interaction is investigated with a coupled-channels potential model. The importance of using relativistic kinematics and of requiring a subthreshold resonance while fitting to low-energy scattering data is explored. A model reproducing the above-threshold data, and agreeing with the kaonic-hydrogen energy shift, is found.

PACS numbers: 13.75.Jz, 11.80.Gw, 12.40.Qq, 14.20.Jn

X rays given off in transitions to the $1S$ level of kaonic hydrogen show that the nuclear interaction of the kaon shifts the atomic level to a more bound energy.[1-3] This is a puzzle in that the shift is in a direction opposite to that observed in heavier atoms,[4] and to that calculated[5-7] with use of the analyses of low-energy $\bar{K}N$ scattering experiments.[8-16] Unraveling this puzzle is important for an understanding of the $\bar{K}N$ interaction and the structure of the subthreshold $\Lambda(1405)$ resonance.

The $\Lambda(1405)$ is a strangeness -1, isosinglet resonance which decays only to $\Sigma\pi$. This resonance also couples strongly to the $\bar{K}N$ system, but does not decay to it since the $\Lambda(1405)$ is some 30 MeV below the $\bar{K}N$ threshold. Whether the $\Lambda(1405)$ is composite ($\bar{K}N$ bound state or $\Sigma\pi$ resonance) or is elementary (three-quark state) is controversial.[17-20] In a recent investigation into the nature of the $\Lambda(1405)$, Refs. 19 and 20 include a bare three-quark state in their cloudy-bag model. They find the $\Lambda(1405)$ to consist of a $\bar{K}N$ bound state with only a small admixture of elementary three-quark state. This result suggests that while quark degrees of freedom may be needed to describe some details of the $\bar{K}N$ interaction (see also Maltman and Isgur[21]), a reasonable model is possible with the $\bar{K}$, N, Σ, and π as elementary particles interacting via potentials or exchanged mesons.

Results of the three different x-ray experiments[1-3] are displayed in Fig. 1, which clearly shows the large statistical errors. In addition, since many elements produce x rays with energies similar to the $1S$ transition in kaonic hydrogen, and since a full cascade down to the $1S$ level has not been observed, it is difficult to be certain of the identity of the observed x rays, and questions regarding systematic errors have been raised.[4,23] In light of these experimental difficulties, a possible reconciliation of the scattering and x-ray experiments is obtained by our disregarding the x-ray results. However, the x-ray results represent the *only* direct experimental evidence on the near-zero-energy $\bar{K}N$ interaction and we choose to search for an interpretation of the low-energy scattering data which is not at odds with them.

In the simplest analysis, the atomic-level shift and width are related to scattering data via the proportionality

$$(\Delta E_n)/E_n \approx -4f(E_n)/nR_B, \tag{1}$$

where $f(E_n)$ is the complex K^-p scattering amplitude evaluated at the Coulomb bound-state energy E_n, and R_B is the Bohr radius. The amplitude is "known" from the many years of work on M- and K-matrix analyses,[24] and its use in (1) predicts a shift to the less bound —which disagrees with experiment. Since corrections to (1) found in exact calculations[6,7,23] are large, but not large enough to reverse the sign of ΔE_n, there must be a problem with either the experiments or $f(E)$.

The M- and K-matrix analyses extrapolate scattering data (cross sections and amplitudes determined by Coulomb-nuclear interference) measured at higher energies down to threshold and below. The extent of the extrapolation is illustrated in Fig. 2 where it can be seen that no data are available below kaon laboratory momenta of 100 MeV/c, yet use of (1) requires the amplitude "below" 0 MeV/c. The hydrogen experiment calls this extrapolation into question. If we assume the measured shift to be correct, the puzzle centers on the energy dependence of the scattering amplitude $f(E)$ in the region from the Coulomb bound-state energy (8 keV below

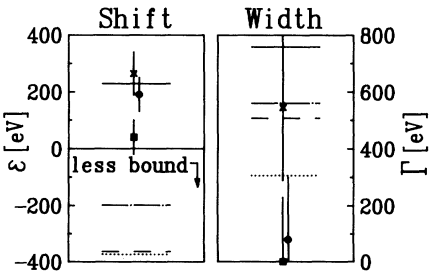

FIG. 1. The shifts and widths of the $1S$ level in kaonic hydrogen as measured by Ref. 1 (squares), Ref. 2 (crosses), and Ref. 3 (circles). An exact bound-state calculation with several K^-p potentials yields the dotted lines (nonrelativistic update of Ref. 22), dot-dashed lines (relativistic update), solid lines (new relativistic), and dashed lines (Ref. 22, set C).

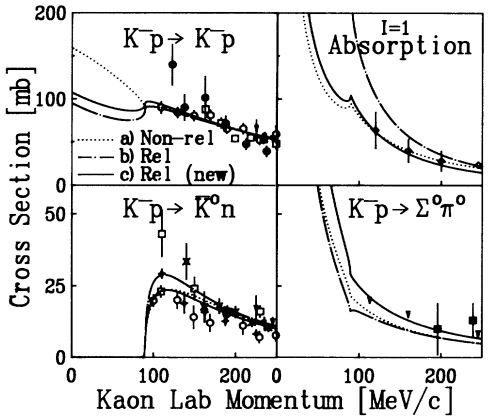

FIG. 2. Calculated cross sections for the three sets of potential parameters. The data are as follows: plusses, Ref. 8; open circles, Ref. 9; filled circles, Ref. 10; filled triangles, Ref. 11; filled squares, Ref. 12; open squares, Ref. 13; crosses, Ref. 14. The $I=1$ absorption data are derived from the following: filled lozenges, Refs. 12 and 13; open lozenges, Refs. 15 and 16.

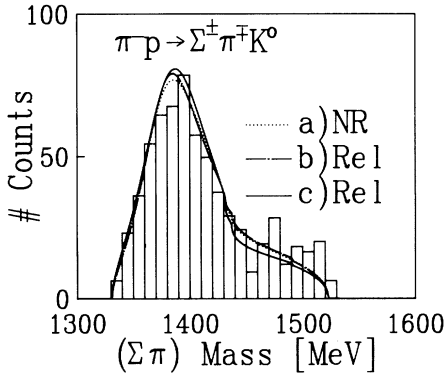

FIG. 3. The $\Sigma\pi$ mass spectrum of Ref. 28 compared to predictions from three fits.

K^-p threshold) to the low end of the scattering data (5 MeV above threshold). Either a second sign change in $f(E)$ is needed (i.e., in addition to the one produced by the subthreshold resonance), or—contrary to K- and M-matrix analyses—there is no sign change at all in $f(E)$. Indeed, models have been proposed[25] with a second sign change in this region. While they may be correct, they are *ad hoc,* and have drawn theoretical criticism.[26]

In this Letter we describe our attempt to find a coupled-channels potential model of the low-energy K^-p interaction consistent with both the scattering and atomic data. We work with a potential since it builds analyticity and unitarity into our scattering amplitudes and allows us to solve the Coulomb plus nuclear bound-state problem exactly; we suspect that similar results can be obtained with more microscopic models. Our model is designed to reveal the physics of the interaction while keeping the computations manageable. No Coulomb corrections to the scattering data were made, and only S waves are considered (P waves may affect differential cross sections at laboratory momenta as low as 150 MeV/c [13,14]). Our approach is similar to that given by Alberg, Henley, and Wilets,[22,27] and we refer the reader there for details on notation.

At low energies the K^-p system (our main channel) couples to the $\bar{K}^0n$, $\Sigma^+\pi^-$, $\Sigma^-\pi^+$, $\Sigma^0\pi^0$, and $\Lambda^0\pi^0$ systems. Since resonance formation in the isospin-0, $\Sigma\pi$ channel is the main mechanism for annihilation from the K^-p channel, we use a second channel for it. Likewise, since the $\bar{K}^0n$ channel lies only 5 MeV above K^-p threshold, it strongly affects the value of the K^-p scattering amplitude near threshold, and it is our third isospin-0 channel:

$$|1\rangle=|K^-p\rangle,\quad |2\rangle=|\Sigma\pi\rangle,\quad |3\rangle=|\bar{K}^0n\rangle.$$

For isospin 1 we employ only channels $|1\rangle$ and $|3\rangle$ but add an imaginary part to the potential to account for nonresonant absorption into the $\Sigma\pi$ and $\lambda\pi$ channels. This permits a direct comparison of our potential parameters with those of Ref. 27, and reduces the number of parameters. We further reduce that number by placing the three charged $\Sigma\pi$ states into the isospin-0 channel 2.

We solve the S-wave Lippman-Schwinger equation, $T=V+VGT$, with potential, propagator, and transition matrices, V, G, and T, 3 by 3 matrices. We use separable potentials with Yamaguchi form factors,

$$V^{ij}(k'|k)=\sum_{I=0}^{1} g_I(k')\lambda_I^{ij}g_I(k),$$

$$g_I(k)=1/(k^2+\beta_I^2),$$

where the subscripts indicate isospin and the superscripts channels. The inverse range, β_I, and the isospin-0 coupling-strength parameters, λ_0^{ij}, are real. The isospin-1 coupling parameter, λ_1^{11}, is given an imaginary part to provide the absorption discussed above. We invoke isospin symmetry insofar as $\lambda_I^{33}=\lambda_I^{13}=\lambda_I^{31}=\lambda_I^{11}$, but use actual masses in the propagator. The propagator,

$$G_{ij}=\delta_{ij}/[E^+-(p^2+m_{1i}^2)^{1/2}+(p^2+m_{2i}^2)^{1/2}],$$

incorporates relativistic kinematics for the two particles in channel i. For the nonrelativistic calculation, the channel masses are chosen to make the energy agree with the relativistic expression at K^-p threshold.[27]

The reactions fitted are shown in Figs. 2 and 3, and the parameters determined are given in Table I. Our work differs from Ref. 27 in our (1) using additional, more recent data, (2) fitting the individual K^-p elastic and charge-exchange cross sections (Ref. 27's good fit to the sum did not produce equally good fits for the individual reactions), (3) including the $\Sigma\pi$ mass spectrum as a further constraint on the potentials, (4) fitting directly to data—as opposed to K-matrix parameters derived from data, (5) treating $\bar{K}^0n$ as a separate channel with a mass

TABLE I. Potential parameters for fits a (nonrelativistic update of Ref. 22), b (relativistic update), c (new relativistic), and d (Ref. 22, set C).

Fit	β_0^{-1} (fm)	λ_0^{I1} (MeV^2)	λ_0^{I2} (MeV^2)	λ_0^{22} (MeV^2)	β_1^{-1} (fm)	λ_1^{I1} (MeV^2)
a	0.412	-2.62×10^4	4.91×10^4	3.49×10^4	0.301	$-3.51\times10^4-3.93\times10^4i$
b	0.261	-2.76×10^7	2.87×10^7	-3.00×10^7	0.226	$-1.20\times10^5-1.99\times10^4i$
c	0.0962	-1.05×10^6	1.71×10^5	1.29×10^6	0.0678	$-2.75\times10^6-3.77\times10^4i$
d	0.180	-3.58×10^5	-1.23×10^5	-4.52×10^5	0.500	$-6.08\times10^4-4.42\times10^4i$

different from that of K^-p, (6) using relativistic kinematics (the pion *is* relativistic), and (7) computing all integrals numerically.

We find that, while the elastic and absorption data fix the square modulus of the scattering amplitudes at energies above the $\bar{K}^0n$ threshold (right of the rightmost arrow in Fig. 4), large differences in the amplitudes occur below that threshold, when no subthreshold data are fitted. In particular, fitting above-threshold data alone does not produce a subthreshold "resonance" [peak in Im$f(E)$ and sign change in Re$f(E)$], but imposing the additional constraint of reproducing the zero-energy K-matrix elements does lead to a resonance near the $\Lambda(1405)$. Apparently, since the K-matrix elements are derived from the low-energy data *plus* a number of theoretical assumptions,[24] such as a linear energy dependence for K^{-1}, forward dispersion relations, and crossing symmetry, these assumptions are important in producing the resonance.

To constrain our potential further, and to build in more experimental information, we used our $\Sigma\pi$ elastic-scattering amplitude in a simple model for the reaction $\pi^-p\rightarrow\Sigma\pi K^0$. If we assume that the experimental mass spectrum[28] is dominated by the final-state $\Sigma\pi$ interaction, and use relativistic three-body phase space, we obtain the spectrum shown in Fig. 3. Being required to fit this spectrum tends to produce a resonance, and is a strong constraint—most of the potential parameter sets fitted to above-threshold data alone failed. Yet even the mass-spectrum constraint leaves many parameter sets —each a local χ^2 minimum—from which to choose. For example, in Figs. 2–4, the dotted curves, fit a, were obtained by our using a nonrelativistic propagator and starting the search with the parameters of Ref. 27; it is thus an update of their work and yields similar scattering amplitudes. The dot-dashed and solid curves, fits b and c, were obtained with use of the relativistic propagator and different starting points for the seven-dimensional searches.

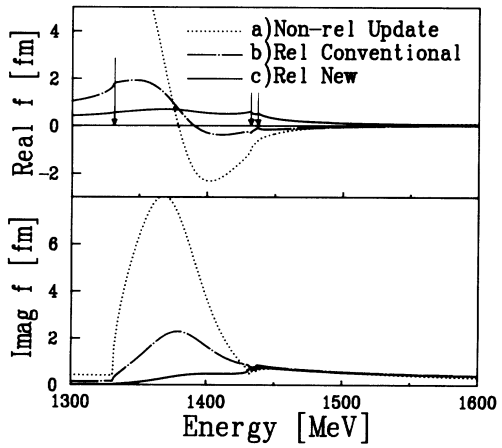

FIG. 4. Real and imaginary parts of the K^-p elastic-scattering amplitude. Arrows indicate threshold energies for the $\Sigma\pi$, K^-p, and $\bar{K}^0n$ channels in order from left to right.

If we look at the curves of K^-p scattering amplitude versus energy in Fig. 4, we see below-threshold resonances for fits a and b, but not for fit c. This is a consequence of the relativistic kinematics, and of a somewhat weaker $I=0$ amplitude in the K^-p channel; the fit-c $\Sigma\pi$ amplitude (not shown) does have a nice resonance!

The importance of fit c arises from its having the sign and magnitude needed to provide agreement with the kaonic-hydrogen experiments. To actually test that assertion, we used all three coupled-channels potentials to calculate *exactly* the kaonic-hydrogen energy shifts and widths. For this purpose we used the code of Ref. 6, which searches for $\det(1-VG)=0$, with V containing Coulomb plus coupled-channels nuclear potentials. The results obtained are shown in Fig. 1. Fit c (solid line) agrees with the present state of the kaonic-hydrogen data. Even if one chooses to disregard the hydrogen experiments, however, fit c is the best fit in that it reproduces the elastic and reaction data with the smallest χ^2. In that both the scattering data and the hydrogen data are well described by a model which does not include a three-quark state, our results support the conclusion of the cloudy-bag-model work,[19] namely, that the $\Lambda(1405)$ consists predominantly of a composite state. At present we are testing our potential in optical potentials for heavier kaonic atoms.

This work was supported by U.S. Department of Energy Grant No. DE-FG06-86ER40283,A001. We wish to acknowledge the assistance of B. Cheng, P. Fink, M. Sagen, and E. Viet and thank B. Jennings and M. Alberg for their helpful conversations.

[1]J. D. Davies *et al.*, Phys. Lett. **83B**, 55 (1979).
[2]M. Izycki *et al.*, Z. Phys. A **297**, 11 (1980).
[3]P. M. Bird *et al.*, Nucl. Phys. **A404**, 482 (1983).
[4]C. J. Batty, Nukleonika **25**, 545 (1980).
[5]R. C. Barrett, J. Phys. G **9**, 355 (1983).
[6]R. H. Landau, Phys. Rev. C **28**, 1324 (1983); R. H. Landau and B. Cheng, Phys. Rev. C **33**, 734 (1986).
[7]J. Thaler, J. Phys. G **10**, 1037 (1984).
[8]W. Kittel, G. Otter, and I. Wacek, Phys. Lett. **21**, 349 (1966).
[9]J. K. Kim, Phys. Rev. Lett. **19**, 1074 (1967).
[10]W. E. Humphrey and R. R. Ross, Phys. Rev. **127**, 1305 (1962).
[11]T. S. Mast *et al.*, Phys. Rev. D **14**, 13 (1976).
[12]A. D. Martin, Nucl. Phys. **B179**, 33 (1981).
[13]J. Ciborowski *et al.*, J. Phys. G **8**, 13 (1982).
[14]D. Evans *et al.*, J. Phys. G **9**, 885 (1983).
[15]R. O. Bangerter *et al.*, Phys. Rev. D **23**, 1484 (1981).
[16]T. S. Mast *et al.*, Phys. Rev. D **11**, 3078 (1975).
[17]R. H. Dalitz and J. G. McGinley, in *Low and Intermediate Energy Kaon-Nucleon Physics,* edited by E. Ferrari and G. Violini (Reidel, Boston, 1981), p. 97.
[18]S. Capstick and N. Isgur, in *Hadron Spectroscopy—1985,* edited by S. Oneda, AIP Conference Proceedings No. 132 (American Institute of Physics, New York, 1985), p. 267.
[19]E. A. Veit, B. K. Jennings, A. W. Thomas, and R. C. Barrett, Phys. Rev. D **31**, 1033 (1985).
[20]B. K. Jennings, Phys. Lett. **176B**, 229 (1986).
[21]K. Maltman and N. Isgur, Phys. Rev. D **34**, 1372 (1986).
[22]E. M. Henley, M. A. Alberg, and L. Wilets, Nukleonika **25**, 567 (1980).
[23]H. H. Brouwer, J. W. de Maag, and L. P. Kok, Z. Phys. A **318**, 1 (1984).
[24]A. D. Martin, in Ref. 17, p. 97.
[25]J. G. McGinley, D. Phil. dissertation, University of Oxford, 1979 (unpublished); K. S. Kumar and Y. Nogami, Phys. Rev. D **21**, 1834 (1980).
[26]R. H. Dalitz, J. McGinley, C. Belyea, and S. Anthony, in *Proceedings of the International Conference on Hypernuclear and Kaon Physics, Heidelberg, Germany 1982,* edited by B. Povh (Max-Planck-Institute für Kernphysik, Federal Republic of Germany, 1982), p. 201.
[27]M. Alberg, E. M. Henley, and L. Wilets, Ann. Phys. **96**, 43 (1976).
[28]D. W. Thomas, A. Engler, H. E. Fisk, and R. W. Kraemer, Nucl. Phys. **B56**, 15 (1973).