Geometric Interpretation of Hadron-Proton Total Cross Sections and a Determination of Hadronic Radii

B. Povh and J. Hüfner

Max-Planck-Institut für Kernphysik, Heidelberg, Federal Republic of Germany, and Institut für Theoretische Physik der Universität Heidelberg, Heidelberg, Federal Republic of Germany

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Hadron-proton total cross sections σ_{hp}^t and slope parameters b_{hp} from elastic scattering are found to be closely related. If effective radii R_i are defined by $b_{hp}=R_h^2+R_p^2$, we find from the data $\sigma_{hp}^t=gR_h^2R_p^2$. This geometric factorization property, which holds for c.m. energies $\sqrt{s}\geq 15$ GeV, is understood within the eikonal approach. Total cross sections σ_{hp}^t can be used to deduce hadronic radii of mesons and baryons.

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The total cross section $\sigma_{pp}^t(s)$ for proton-proton (pp) collisions is about 40 mb at the c.m. energy of $\sqrt{s}=15$ GeV. May this value be understood geometrically, for instance as $\sigma_{hp}^t=\pi R^2$, where the radius parameter $R\simeq 1$ fm is related to the size of the colliding hadrons? *Elastic* hadron-hadron scattering is indeed related to the shape of the colliding hadrons. The ideas of Wu and Yang[1] and Chou and Yang[2] have stimulated a long series of papers, the most recent ones being those by Dias de Deus and Kroll,[3] Bourrely, Soffer, and Wu,[4] Donnachie and Landshoff,[5] and those given at the First International Workshop on Elastic and Diffractive Scattering.[6] For example the differential cross section for the elastic scattering of hadrons 1 and 2 is written as[5]

$$\frac{d\sigma_{12}^e}{dt}=\frac{[n_1\beta_1F_1(t)]^2[n_2\beta_2F_2(t)]^2}{4\pi}\times(s/s_0)^{2[\alpha_P(t)-1]}, \qquad (1)$$

where $F_i(t)$ is the Dirac form factor of hadron i, n_i the number of its constituent quarks, β_i a coupling constant, and $\alpha_P(t)$ the Pomeron trajectory. Equation (1) has been tested for πp and pd, but best for pp collisions. Apart from the last factor in Eq. (1), the elastic cross section is proportional to the form factors $F_i(t)$, which reflect the shape of the two colliding objects.

Historically, *total* hadron-hadron cross sections have not been interpreted along geometric lines but within the additive quark mode; cf. Levin and Frankfurt,[7] Lipkin,[8] Kokkedee and van Hove,[9] and later papers. The empirical fact that the πp total cross section is smaller than σ_{pp}^t by a factor of about $\frac{2}{3}$ is related to the number of valence quarks, two for the pion and three for the proton. A total quark-proton cross section σ_{qp} is defined for u and d quarks by $\sigma_{qp}=\sigma_{\pi p}^t/2=\sigma_{pp}^t/3\simeq 12$ mb. The fact that σ_{Kp}^t is smaller than $\sigma_{\pi p}^t$ by 4 mb is interpreted as a consequence of a smaller elementary cross section for the strange quark. This reasoning then leads, for example, to the prediction that the Ξp total cross section is smaller than σ_{pp}^t by $2(\sigma_{\pi p}^t-\sigma_{Kp}^t)=8$ mb, which is borne out experimentally since $\sigma_{pp}^t-\sigma_{\Xi p}^t=9.2\pm 0.4$ mb.

Despite the clear success of the additive quark model to explain systematics in the total cross sections one does not see how its assumptions can be reconciled with the shape dependence of the elastic cross sections. We propose an alternative explanation for this systematics in σ_{hp}^t which may bridge this conceptual gap. We claim and prove that the variation of σ_{hp}^t in its dependence on the projectile h is a geometrical effect and not primarily related to the number of valence quarks nor to their flavor. For instance, the cross section $\sigma_{\pi p}^t$ is smaller than σ_{pp}^t because the pion has a smaller radius than the proton. How can we prove our claim? Unfortunately electromagnetic radii $\langle r^2\rangle$ are known only for the pion and proton. But at least for these two systems we have empirically $\sigma_{\pi p}^t/\sigma_{pp}^t=\langle r_\pi^2\rangle/\langle r_p^2\rangle=\frac{2}{3}$. Furthermore, for the systems pp and πp the success of Eq. (1) proves experimentally that the slope parameter b_{hp} for elastic scattering,

$$b_{hp}=\frac{d}{dt}\ln\left(\frac{d\sigma_{hp}^e}{dt}\right)=R_h^2+R_p^2, \qquad (2)$$

is related to the sum of the squares of the two radii. Except for the contribution in the t dependence from the Pomeron trajectory, the radii defined by Eq. (2) are related to the mean square electromagnetic radii by $\langle r^2\rangle=3R^2$. Including the contribution from $s^{2\alpha_P(t)}$ in Eq. (1), we use Eq. (2) to *define* effective, possibly energy-dependent radii $R_i^2(s)$. The data which we will present suggest a relation between the total cross sections and these effective radii. To our knowledge, this relation has not been observed before. However, the suggestion that total cross sections may depend on the size of the particles has already been put forward by Gunion and Soper.[10]

We have collected data on total cross sections and slope parameters from various hadron-proton experiments in the energy regime 50 GeV $\leq E_{\mathrm{lab}}\leq$ 200 GeV. The measured values for σ_{hp}^t and b_{hp} including the relevant references[11-15] are listed in Table I. The cross sections and the slope parameters have been deduced

TABLE I. Experimental values for slope parameters b_{hp} and total cross sections σ_{hp}^{t} together with the references. The published value for the slope parameter b in the Ξp experiment (Ref. 13) is just the average value for $t < 0.6$ GeV^{-2}. We use the value at $t=0$ which has been obtained by reanalysis of the data in accord with the authors of Ref. 13.

hp	$\sqrt{s}$ (GeV)	b_{hp} (GeV^{-2})	σ_{hp}^{t} (mb)	$\langle r_h^2 \rangle$ (fm)
pp	16[a]	11.3 ± 0.1	38.57 ± 0.19	0.82 ± 0.01
$\pi^+ p$	16[a]	9.0 ± 0.1	23.43 ± 0.12	0.64 ± 0.01
$\pi^- p$	16[a]	9.2 ± 0.2	24.00 ± 0.12	0.65 ± 0.01
$K^+ p$	16[a]	8.3 ± 0.2	19.23 ± 0.10	0.57 ± 0.01
$K^- p$	16[a]	9.1 ± 0.3	20.50 ± 0.10	0.60 ± 0.01
ϕp	4.2[b]	6.7 ± 0.7	10.2 ± 0.6	0.42 ± 0.03
$J/\psi p$	16[c]	5.2 ± 0.6	2.2 ± 0.7	0.20 ± 0.06
Λp	5[d]	...	34.6 ± 0.40	0.77 ± 0.01
Σp	16[e]	...	34.14 ± 0.30	0.77 ± 0.01
$\Xi^- p$	16[e]	9.0 ± 0.5	29.35 ± 0.3	0.72 ± 0.01
$\bar{p}p$	16[a]	12.6 ± 0.4	41.80 ± 0.21	0.84 ± 0.01

[a]Reference 10.
[b]Reference 13.
[c]Reference 14.
[d]Reference 11.
[e]Reference 12.

from direct measurements of elastic hadron-proton scattering with the exception of the ϕ (Ref. 14) and J/ψ (Ref. 15) data which are obtained from electroproduction experiments under the assumption of the validity of the vector dominance. The value for the p interaction is known only at low energies, $\sqrt{s} \simeq 4$ GeV, and is plotted just as an indication for the general trend of σ_{hp}^{t} and b_{hp}. The experimental data on J/ψ are taken at the energy of comparison. The only reservation concerning these data is whether the vector-dominance model still works at these high energies. It should be pointed out, however, that even though the following considerations have been triggered by the new result on the J/ψ hadronic cross sections, the model developed below is mainly supported by the experimental results on hadron-proton interactions in the direct experiments.

In Fig. 1 we have plotted σ_{hp}^{t} against b_{hp}. There is a clear correlation between the various data points which may be parametrized in the form of a straight line,

$$\sigma_{hp}^{t} = g(b_{hp} - \tfrac{1}{2} b_{pp}) \tfrac{1}{2} b_{pp} = g R_h^2 R_p^2. \qquad (3)$$

While the dependence on the factor $b_{hp} - \frac{1}{2} b_{pp}$ is dictated by the data and describes the dependence on the incident hadron, the factor $\frac{1}{2} b_{pp}$ is introduced for symmetry. It is constant and therefore not detectable as long as one holds the projectile energy fixed. The slope of the straight line in Fig. 1 determines the value of the constant g. Above $\sqrt{s} \simeq 30$ GeV, total pp and $\bar{p}p$ cross sections are approximately equal and have been measured up to $\sqrt{s} = 630$ GeV. Figure 2 shows these data[16] plotted against b^2. One observes a straight line which is pre-

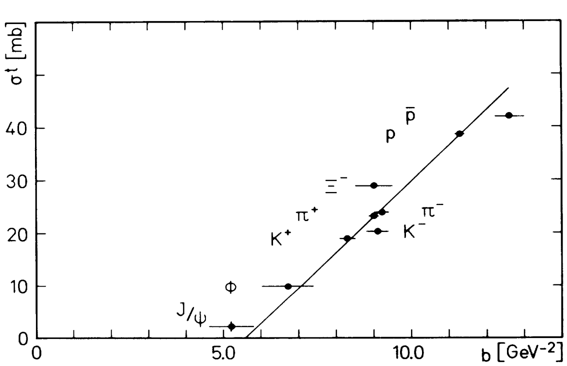

FIG. 1. Data for total cross sections σ^t plotted against the experimental slope parameters b for various reactions at a c.m. energy $\sqrt{s} = 16$ GeV. The experimental points are labeled by the projectile hadron.

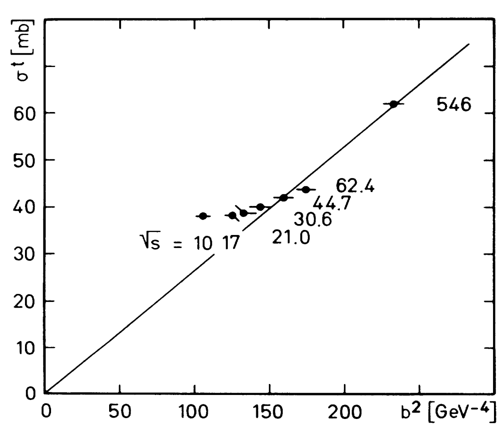

FIG. 2. The total cross section σ^t for pp (points for $\sqrt{s} \leq 62.4$ GeV) and $\bar{p}p$ (points for $\sqrt{s} \geq 21.0$ GeV) collisions plotted against b^2, the square of the slope parameter. The various experimental points are labeled by the c.m. energy.

dicted from Eq. (3) for $h=p$ provided g is independent of $\sqrt{s}$. How accurately is the shape given by Eq. (3) supported by the data? We have tested a more general expression for σ_{hp}^t by replacing R_i^2 in Eq. (3) by $R_i^{2+\delta}$ but leaving Eq. (2) unchanged. We find fits to the data in Figs. 1 and 2 with $|\delta| \leq \frac{1}{2}$ acceptable, but stay with $\delta=0$ for the subsequent discussion. Figure 2 also indicates deviations for this systematics for energies $\sqrt{s} \lesssim 15$ GeV. Our geometrical model corresponds to Pomeron exchange in the Regge approach. Also there, for smaller energies deviations are observed and attributed to other trajectories.

A consistent mathematical framework which relates total cross sections σ_{12}^t, slope parameters b_{12}, and elastic cross sections $d\sigma_{12}^3/dt$ can be formulated with use of the eikonal approach. The amplitude $f_{12}^e(\mathbf{q})$ for elastic scattering ($\mathbf{q}^2=-t$) is expressed as a Fourier transform

$$f_{12}^e(\mathbf{q})=\frac{ik}{2\pi}\int d^2B\, e^{-i\mathbf{q}\cdot\mathbf{B}}(1-e^{-\chi_{12}(B)}), \tag{4}$$

where $\chi_{12}(B)$ is called the phase-shift function. We assume it to be generated by the transverse density profiles or opacities $T_i(\mathbf{s})$ for the colliding hadrons, $\mathbf{s}$ being a vector in the impact-parameter plane:

$$\chi_{12}(\mathbf{B})=g'\int d^2s\, T_1(\mathbf{B}-\mathbf{s})T_2(\mathbf{s}), \tag{5}$$

with a complex coupling strength g'. We *assume g' and* the normalization $T_i(0)$ to be independent of the colliding hadrons and of the energy. Since both the total cross section and the slope parameter are obtained from the elastic amplitude at *small* momentum transfers, the detailed shape of $\chi_{12}(B)$ and therefore of the $T_i(\mathbf{s})$ is not needed. For this reason and for convenience we choose Gaussians,

$$T_i(b)=\exp(-b^2/2R_i^2)/\sqrt{8}\pi. \tag{6}$$

We assume that $\chi_{12}(B)$ is sufficiently small so that the exponential in Eq. (4) can be expanded. Then to first order in $\chi_{12}(B)$ in Eq. (4), one derives from Eqs. (4)-(6) using the optical theorem

$$b_{12}=R_1^2+R_2^2, \tag{7}$$

$$\sigma_{12}^t=\mathrm{Re}\,g'R_1^2R_2^2, \tag{8}$$

and

$$\frac{d\sigma_{12}^e}{dt}=\frac{[R_1^2\exp(\frac{1}{2}R_1^2t)]^2[R_2^2\exp(\frac{1}{2}R_2^2t)]^2}{4\pi}\times|2\pi g'|^2. \tag{9}$$

Equations (7) and (8) contain the empirically observed relation between σ_{12}^t and the slope parameter as displayed in Figs. 1 and 2 with $g=\mathrm{Re}\,g'=75$ fm^{-2}. Equation (9) for elastic scattering is of the shape given by Eq. (1), where the form factors $F_i(t)$ are approximated by exponentials in t and one has for the coupling parameters $3\beta_i=2\pi|g'|R_i^2$.

Equation (1) which is well tested empirically contains an energy- and momentum-dependent factor $s^{2[\alpha_P(t)-1]}$ arising from the Regge trajectory $\alpha_P(t)=1+\epsilon+\alpha' t$ with two adjustable parameters ϵ and α'. For pp scattering our Eq. (9) contains only one energy-dependent parameter, the effective radius $R_p^2(s)$. The factor from the Pomeron exchange can therefore be translated into an energy dependence of the effective radii $R_i(s)$ in two ways. Comparing Eqs. (1) and (3) we find

$$R_i^2(s)/R_i^2(s_0)=(s/s_0)^{\epsilon/2}, \tag{10a}$$

$$R_i^2(s)-R_i^2(s_0)=\alpha'\ln(s/s_0), \tag{10b}$$

where s_0 is a normalization point. If our approach is correct, Eqs. (10a) and (10b) must give the same numerical values for the two fit parameters ϵ and α' from the Regge approach, at least within the experimentally tested range $15\leq\sqrt{s}\leq 630$ GeV. Donnachie and Landshoff[5] report values $\epsilon=0.08$ and $\alpha'=0.25$ (GeV/c)$^{-2}$ which correspond to an increase of the effective proton radius R_p by about 15% when one goes from $\sqrt{s}=15$ to $\sqrt{s}=630$ GeV.

The expressions Eqs. (7) to (9) for the total and elastic cross sections have been derived by our expanding the exponential in Eq. (4) and calculating the first order in the phase-shift function $\chi_{12}(\mathbf{B})$. How good is this approximation? We have evaluated the correction $\Delta\sigma_{12}^t$ from the next-order term in χ_{12} and find

$$\Delta\sigma_{12}^t/\sigma_{12}^t\simeq-(g/16\pi)R_1^2R_2^2/(R_1^2+R_2^2), \tag{11}$$

where σ_{12}^t is given by Eq. (8). For pp collisions at $\sqrt{s}=15$ GeV this correction amounts to 15% and is smaller for all other hadrons but rises to 25% for pp scattering at $\sqrt{s}=630$ GeV. We note that the higher-order corrections destroy the factorization property in σ_{12}^t and $d\sigma_{12}^e/dt$ for the dependence on projectile and target. We have tested the validity of the systematics of Eq. (3) also for pd[17] and $p\,^4$He[18] between 100 and 300 GeV and observed sizable deviations. We attribute them to the fact that higher-order corrections like Eq. (11) have been neglected.

Our empirical relation, Eq. (3), between the total cross section and the effective radii R_i^2 has an important consequence. *Hadronic* radii—not necessarily equal to the electromagnetic ones—can be deduced from the values of the total cross sections by

$$\langle r_h^2\rangle/\langle r_p^2\rangle=\sigma_{hp}^t/\sigma_{pp}^t \tag{12}$$

or, if the slope parameter is measured, from

$$b_{hp}=\tfrac{1}{3}(\langle r_h^2\rangle+\langle r_p^2\rangle). \tag{13}$$

Table I contains radii for many known hadrons calculated from Eqs. (12) and (13). The values for proton and pion hadronic rms radii agree with the electromagnetic ones, a rather surprising observation. Equation (12) predicts for the J/ψ meson a radius $\langle r_\psi^2 \rangle^{1/2} = 0.20 \pm 0.02$ fm. Calculations within the nonrelativistic quark model[19] support this value. One observes a clear systematics in the radii of Table I: (i) Systems with three valence quarks are larger than those with two, provided they have a similar flavor content. (ii) The heavier the valence quarks, the smaller is the system, e.g., $\langle r_p^2 \rangle > \langle r_\Lambda^2 \rangle > \langle r_\Xi^2 \rangle$ or $\langle r_\pi^2 \rangle > \langle r_K^2 \rangle > \langle r_{J/\psi}^2 \rangle$.

This systematics may find its explanation within the additive quark model. We decompose the mean square radius $\langle r_h^2 \rangle$ of a hadron in a sum

$$\langle r_h^2 \rangle = \sum_i a_{q_i} \langle r_{q_i}^2 \rangle, \quad (14)$$

where the r_{q_i} are the radii of the valence quarks contained in the h and the a_{q_i} their coupling strength to the probe which measures the radius (e.g., for electron scattering $a_{q_i} = e_{q_i}/e$, where e_q is the charge of the quark). If we assume flavor independence for hadronic probes, the a_{q_i} are independent of the q_i and one recovers the rules of the additive quark model for the total cross sections. In our interpretation the flavor dependence of the elementary quark-proton cross sections σ_{qp} defined in the introduction does not imply a flavor dependence of the strong interaction but is rather a consequence of the quark q being *bound* in the hadron h. Its density distribution inside the hadron depends on its mass and therefore its flavor.

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