Implications of a Heavy Top Quark and a Fourth Generation on the Decays $B \to Kl^+l^-$, $K\nu\bar{\nu}$

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We point out the importance of the Z and box diagram to the decays $B \to Kl^+l^-$, $K\nu\bar{\nu}$. The rate for $B \to Kl^+l^-$ grows rapidly for internal quark masses >100 GeV. With three generations and 25 GeV $\lesssim m_t \lesssim$ 200 GeV the branching ratio ranges roughly from 10^{-6} to 10^{-5}. With four generations, this rate could go up another order of magnitude. The mode $B \to K\nu\bar{\nu}$ typically has a higher branching ratio, but is harder to detect experimentally. The rare B decays combined with information from $K \to \pi\nu\bar{\nu}$ studies may provide a test of the symmetry-breaking mechanism of the standard model and/or evidence for a fourth generation.

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The kaon system has provided valuable information; so will studies of the B-meson system. Studies of B decays are just beginning; but already, experiments at the Cornell Electron Storage Ring and Deutsches Elektronen-Synchrotron (DESY) are setting upper limits on various exclusive rare B-decay branching ratios (R) at the 10^{-4} level.[1] New machines such as the Stanford Linear Collider (SLC) and CERN LEP, with more than 10^6 Z's expected per year and an estimated 14% R into B's, will add immensely to the study of rare B decays. Ultimately, perhaps, with 10^8–10^9 "useful" B mesons per year, the Superconducting Super Collider may afford us the possibility of studying rare B decays down to the 10^{-7}–10^{-8} level.[2,3]

At present, the most interesting rare B decays seem to be the ones that test the validity of the standard electroweak model at the one-loop level. For purely hadronic modes there are both theoretical and experimental difficulties. Recently, Deshpande, Eilam, Soni, and Kane[4] have emphasized that an experimentally cleaner and theoretically more tractable mode is the semileptonic decay $B \to Kl^+l^-$, where $l = e$ or μ. They report an inclusive rate of $(2-3)\times 10^{-6}$ as m_t ranges from 40 to 240 GeV, while the exclusive rate is estimated to be roughly half of this. However, their calculation was done taking into account only the photon contribution and did not include the Z and box-diagram contributions. In this Letter, we address this problem once again, taking into account the full set of photon, Z, and box-diagram contributions. For low m_t, the photon contribution does dominate and it remains substantial for m_t less than 100 GeV. But for large m_t, the Z contribution, which grows as m_t^2 in amplitude for very large m_t, takes over and the rate increases rapidly with m_t. Taking the recent constraint of $m_t \lesssim 200$ GeV coming from a detailed analysis of measured values for M_W, M_Z, and $\sin^2\theta_W$,[5] we find that in the three-generation case, for 25 GeV $\lesssim m_t \lesssim$ 200 GeV, the inclusive rate ranges from 1.5×10^{-6} to 1.5×10^{-5}. Moreover, the sensitivity to heavy internal quark masses makes this a natural place to study effects of the possible existence of a fourth generation. With four generations, the less constrained 4×4 Kobayashi-Maskawa (KM) matrix allows R to go up another order of magnitude,[6,7] and the current experimental limits[1] already put some constraints on the four-generation parameter space. Thus pushing the experimental limit down by an order of magnitude would yield very valuable information regarding the parameters of the 4×4 KM matrix and/or the fourth-generation quark mass $m_{t'}$.

The quark-level process $b \to sl^+l^-$ occurs at the one-loop level in the standard model. Since m_b^2/M_W^2 is small, it is a good approximation to neglect external masses and momenta in most parts of the calculations. An exception is the one-photon-exchange diagram in which one has a photon propagator $1/q^2$, and gauge invariance requires extraction of up to two powers of the photon momentum before the setting of external masses and momenta equal to zero in the remaining factors. Another exception is the absorptive parts of the loops involving light quarks ($4m_i^2 < q^2$) which, together with CP-nonconserving phases in the theory, may generate CP-nonconserving asymmetries. These have been estimated to be quite small[8] (<2%[5]), and we will not consider then further. In this approximation the result of the one-loop calculation has been given in the form of an effective Lagrangean by Inami and Lim,[9] and we follow their notation. The effective Lagrangean arising from Fig. 1 is

$$\mathcal{L}_{\rm eff}^{b\bar{s}\to l^+l^-} = 2\sqrt{2}G_{\rm F}\chi v_i\{\bar{C}_i(\bar{s}\gamma_\mu Lb)(\bar{l}\gamma_\mu Ll) - s_W^2(F_1^i+2\bar{C}_i^Z)(\bar{s}\gamma_\mu Lb)(\bar{l}\gamma_\mu l) - s_W^4F_2^i[\bar{s}i\sigma_{\mu\nu}(q_\nu/q^2)(m_sL+m_bR)b](\bar{l}\gamma_\mu l)\}, \qquad (1)$$

$$\mathcal{L}_{\rm eff}^{b\bar{s}\to \nu\bar{\nu}} = -2\sqrt{2}G_{\rm F}\chi v_i\bar{D}_i(\bar{s}\gamma_\mu Lb)(\bar{\nu}\gamma_\mu L\nu), \qquad (2)$$

where $\chi = g^2/16\pi^2$, $v_i \equiv V_{is}^* V_{ib}$, i is summed from 2 to n (where n is the number of generations),[10] s_W is the sine of the Weinberg angle, and we exhibit[11]

$$\bar{C}_i \equiv \bar{C}_i^Z + \bar{C}_i^{\rm box} = \tfrac{1}{4}x_i + \frac{3}{4}\left(\frac{x_i}{x_i-1}\right)^2 \ln x_i - \frac{3}{4}\frac{x_i}{x_i-1}, \qquad (3)$$

$$\bar{D}_i \equiv \bar{D}_i^Z + \bar{D}_i^{\rm box} = \tfrac{1}{4}x_i + \frac{3}{4}\frac{x_i(x_i-2)}{(x_i-1)^2} \ln x_i + \frac{3}{4}\frac{x_i}{x_i-1}, \qquad (4)$$

where $x_i = m_i^2/M_W^2$, and m_i is the internal quark mass. The important feature of Eqs. (3) and (4) is the term $x_i/4$,[8] which originates from the loop-induced effective sbZ vertex [the difference between $\bar{C}_i$ and $\bar{D}_i$ lies in the box graph, Fig. 1(e)]. This term dominates when the quark mass m_i is very large. We also note that the photonic contribution F_1^i has a term (in this approximation)

$$F_1^i \sim \tfrac{2}{3}Q\ln x_i/(x_i-1) + \cdots \qquad (5)$$

(Q is the internal quark charge) which is significant for very small x_i: The smallness of v_u makes the u-quark contribution unimportant numerically, but the c-quark contribution[12] through Eq. (5) actually dominates the full amplitude when m_t is not too large. We further note that although the magnetic form factor F_2^i itself is well behaved as x_i vanishes, the conserved structure $\sigma_{\mu\nu}q_\nu/q^2$ leads to a collinear type of singularity in the decay rate (the usual situation for a Dalitz-pair conversion process). Thus, both the photon contributions F_1 and F_2 have to be treated carefully.[13]

The calculated rate for the quark-level process is then

$$\Gamma = \frac{G_{\rm F}^2 m_b^5}{192\pi^3}\chi^2\left[|v_iA_i|^2 + |v_iB_i|^2 + 4s_W^2 v_i(A_i+B_i)(v_jF_2^j) + 16s_W^4\left(\ln\frac{m_b}{2m_l} - \frac{2}{3}\right)|v_iF_2^i|^2\right], \qquad (6)$$

where $\bar{A}_i = \bar{C}_i - s_W^2(F_1^i + 2\bar{C}_i^Z)$, $\bar{B}_i = -s_W^2(F_1^i + 2\bar{C}_i^Z)$ for $b\to sl^+l^-$. The result for $b\to s\nu\bar{\nu}$ is obtained from Eq. (6) by setting $s_W^2\to 0$ and $\bar{C}_i\to\bar{D}_i$. To get inclusive branching ratios, we assume that $\Gamma(B\to Kl^+l^-+X) = \Gamma(b\to sl^+l^-)$. Furthermore, we use the usual *Ansatz*

$$R(B\to Kl^+l^-X) = \frac{\Gamma(b\to sl^+l^-)}{\Gamma(b\to(u,c)l\bar{\nu})}R(B\to l\bar{\nu}X) = \frac{\chi^2[\cdots]}{|V_{ub}|^2 + |V_{cb}|^2 f(m_c^2/m_b^2)}\times 0.12 \qquad (7)$$

to reduce the uncertainties due to finite mass and binding effects. In Eq. (7), $[\cdots]$ denotes the quantity in square brackets in Eq. (6) and $f(x)$ is the usual phase-space correction factor equal to about $\frac{1}{2}$ here. For three generations,[14] the results are plotted in Fig. 2. For comparison, we have also calculated R for $b\to sl^+l^-$ with the F_1 part alone, in the 't Hooft–Feynman gauge. Our results are in good agreement[12] with Ref. 5. It is clear

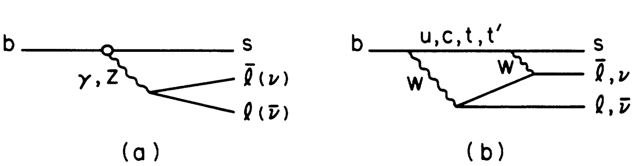

FIG. 1. The graphs that enter into the processes $b\to sl^+l^-$, $s\nu\bar{\nu}$.

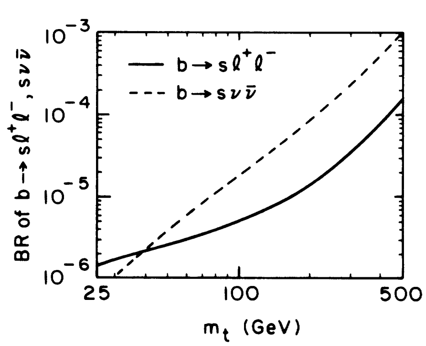

FIG. 2. Branching ratios for the processes $b\to se^+e^-$, $s\nu\bar{\nu}$ in the three-generation case. For $b\to s\nu\bar{\nu}$ the three neutrino species have been summed over. For $b\to s\mu^+\mu^-$ the rate is slightly smaller (see Refs. 15 and 16).

from Fig. 2 that the Z and box contributions are important. We note that the m_t^4 behavior only sets in for rather large m_t because of the small factor $\frac{1}{4}$ multiplying x_i and the relatively large factor of $\frac{3}{4}$ for the $\ln x_i$ term, as seen in Eqs. (3) and (4). So it will be difficult to set any useful upper bound on m_t from this decay mode. Nevertheless, it is seen, e.g., for $m_t = 240$ GeV, that $b \to s l^+ l^-$ occurs with an R of 2×10^{-5}, while $b \to s\nu\bar{\nu}$ occurs at more than 6 times this value.[15] Conversely, one can apply the limits on m_t derived from other considerations such as the ρ parameter, or the recent bound of $m_t \lesssim 200$ GeV obtained from a detailed study of M_W, M_Z, and s_W^2,[6] and deduce that $R(b \to s l^+ l^-) \lesssim 1.5 \times 10^{-5}$ in the three-generation case.

The strong dependence on internal quark mass makes these decay modes an excellent place to probe the effects of a possible fourth generation. Eilam, Hewett, and Rizzo[7] have made a detailed study of the decay mode $B \to K l^+ l^-$ in the four-generation case, but with only the photonic contribution F_1. They found that R could gain by an order of magnitude. Since the F_1 contribution has only a very mild heavy-quark-mass dependence, this indicates that from angle factors alone it is possible to increase the rate by an order of magnitude. Here we do not attempt to determine bounds on the fourth-generation parameters from currently available data; rather, we show that the contribution of a fourth generation can be determined to reasonable approximation in terms of the two parameters $m_{t'}$ and $v_{t'}$ (a comprehensive study of the role of a fourth generation in rare decays will be reported elsewhere[4]). First, we do not expect a very heavy top quark, and for $m_t \lesssim 100$ GeV, R is relatively insensitive to the precise value of m_t (see Fig. 2). Therefore in investigating the effects of a fourth generation we use $m_t = 50$ GeV. Second, with $|v_u| \ll |v_c|$, the fourth-generation unitarity constraint reduces to $v_c + v_t + v_{t'} \cong 0$. Experimentally, $v_c = V_{cs}^* V_{cb}$ is constrained[16] to lie in the range 0.024 to 0.052. For our estimates it is convenient to take $v_c \sim V_{cb} \sim 0.05$. Then v_t is determined in terms of $v_{t'}$ via the unitarity constraint. Thus there are essentially only two parameters that enter into the process under consideration. Because $v_{t'} = V_{t's}^* \times V_{t'b}$ is a product of off-diagonal KM-matrix elements, our prejudice is that it should be quite small. However, current experimental knowledge and four-generation unitarity only implies that $|v_t + v_{t'}| \sim 0.05$, and both v_t and $v_{t'}$ could be sizable. In particular, it is known[17] that the angle of rotation between the third and fourth generation is totally unconstrained and may well be large, and even $V_{t'b} \gg V_{tb}$ is possible. We only consider the range $|v_{t'}| < 0.2$, as a bigger value would make it hard to satisfy the unitarity of the 4×4 KM matrix. The results are given in Fig. 3, for various values of $m_{t'}$. For $b \to s\nu\bar{\nu}$ we have assumed that all the neutrinos are still massless, and $m_L = 50$ GeV for the fourth-generation charged lepton L.[18] It is seen that there is a range of $v_{t'}$ and $m_{t'}$ for which the inclusive R is within the range of current limits[1] on the exclusive decay $B \to K l^+ l^-$. An order-of-magnitude improvement will put considerable constrains on $m_{t'}$, $v_{t'}$. If this decay mode is found at above the 10^{-5} level, the most conservative explanation would be the existence of a fourth generation.

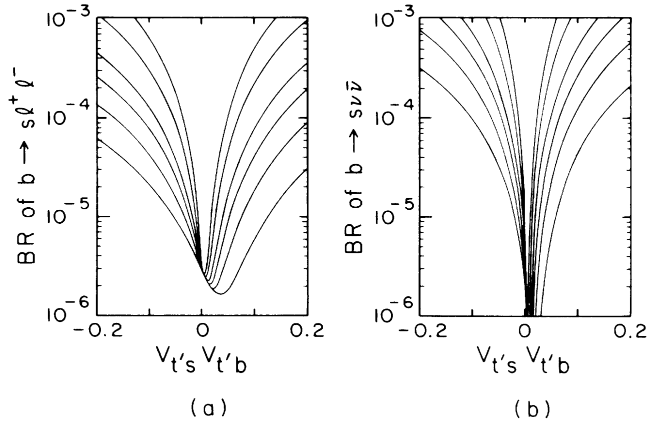

FIG. 3. The four-generation case: $v_{t'} \equiv V_{t's}^* V_{t'b}$, $V_{cb} \sim 0.05$ is assumed, $m_t = m_L = 50$ GeV, and $m_{t'} = 150$, 200, 250, 300, 400, and 500 GeV.

We close with a few remarks and comments. First of all, we have not made an effort to calculate the exclusive R's because of the usual uncertainties in evaluating hadronic matrix elements. Deshpande, Eilam, Soni, and Kane have crudely estimated that the exclusive R is roughly one half the inclusive R for $B \to K l^+ l^- + X$. If it is not a serious overestimate, then the current CLEO detector limits[1] (e.g., $B \to K e^+ e^-$ is less than 2×10^{-4}) are already putting some constraints on the parameters of a possible fourth generation. However, a reliable calculation for the exclusive rate is clearly desirable because it is experimentally very clean. The inclusive $B \to K l^+ l^- + X$ may suffer from background coming from ordinary tree-level $b \to c \to s$ decays with a virtual photon being emitted in every possible way. In Υ facilities it will be hard to separate the inclusive process $b \to s l^+ l^-$ from this background, since the B's are essentially at rest. But perhaps in the SLC or CERN LEP or other facilities which produce high-energy b jets, vertex detectors may allow us to separate $B \to K l^+ l^- X$ (where K is "prompt") from the virtual bremsstrahlung processes involving a charmed final state. Finally, we should emphasize that the counterintuitive m_t^4 (or $m_{t'}^4$) dependence of the rates is a direct consequence of the fact that in the minimal standard model, quark masses arise from the Yukawa couplings times the vacuum expectation value of the Higgs field. Measurement of these processes therefore constitute a sensitive test of the least well-founded part of the standard electroweak theory, viz. the symmetry-breaking and fermion-mass-generation mechanism. Similar m_t^4 terms occur in the process

$K \to \pi\nu\bar{\nu}$ (the decay $K^- \to \pi^- l^+ l^-$ is dominated by long-distance effects); but the rate is severely constrained by known properties of $K_L \to \mu\bar{\mu}$ and ΔM_K. Furthermore, R is suppressed by a factor

$$(|V_{td}^* V_{ts}|^2/|V_{us}|^2)(|V_{ts}^* V_{tb}|^2/|V_{cb}|^2)^{-1} \sim 10^{-5}$$

relative to B decays. Nevertheless, current experiments[19] planned to reach the 10^{-10} R level could provide complementary information.

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[8] The CP-nonconserving asymmetry $a = (\Gamma_b - \Gamma_{\bar{b}})/(\Gamma_b + \Gamma_{\bar{b}})$ is nonzero if there are CP-nonconserving phases in the theory, and if there are absorptive parts in the amplitude. The heavy-quark contributions may bring up the decay rate, but they do not contribute to the absorptive part, and so they will always suppress the asymmetry, and the physical observability of the asymmetry, viz. Ra^2, cannot be enhanced. Thus, as R increases with m_t, we expect a to decrease as $a \propto R^{-1/2}$. For the four-generation case, the asymmetry may get larger, but Ra^2 should never exceed the bounds found in Ref. 7.

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[10] The u-quark contribution has been absorbed as a subtraction to the others, through unitarity of the KM matrix, viz. $v_1 = -\sum_2^n v_i$. See Ref. 8.

[11] For F_1^i and F_2^i, see Ref. 8; they grow at most as $\ln x_i$ for large x_i. F_2^i and the combinations $F_1^i + 2\bar{C}_i^Z$, $\bar{C}_i = \bar{C}_i^Z + \bar{C}_i^{\text{box}}$, etc., are gauge invariant.

[12] We note that the other terms in F_1^i, and all the terms in the other form factors (Ref. 8), are suppressed by powers of x_i as $x_i \to 0$. Thus the sensitivity to the external momenta is isolated here [and in the kinematical structure which goes with F_2 in (1)]. One can go back to the Feynman diagram and keep the photon momentum nonzero in this piece of the amplitude, and do the phase-space integral exactly over this q^2 dependence (as was done in Ref. 4 where only the F_1 term was treated). We have done this and find that the error introduced by ignoring the q^2 dependence is no more than 20% (in rate) for small m_t and is negligible for larger m_t for which the contribution of (5) is only a small part of the total.

[13] In Ref. 4 it was argued that $F_1 \gg F_2$ and therefore the F_2 contribution was dropped. However, $F_1 \gg F_2$ holds only for small x_i. Thus, together with the collinear singularity associated with the magnetic F_2 term, its contribution is not negligible compared to F_1 for large m_t. But for the full amplitude it is relatively unimportant.

[14] As pointed out in Ref. 7, for three generations the uncertainty from KM angles is quite small. The results suffers little change in variations of KM angles within the allowed range, because $|V_{cb}|^2 \gg |V_{ub}|^2$ and $|V_{cs}^* V_{cb}|$, $|V_{ts}^* V_{tb}| \sim |V_{cb}|$.

[15] Unfortunately, $B \to K\nu\bar{\nu}$ would be extremely hard to study at Υ facilities such as the CLEO detector. It remains to be seen whether flavor and vertex identification techniques would allow this decay mode to be studied at the SLC, CERN LEP or other future facilities.

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