Lower Bound on the Neutron Electric Dipole Moment in Models with Spontaneous *CP* Nonconservation in Scalar Exchanges

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We deduce a lower limit on the neutron electric dipole moment in the Weinberg *Ansatz* for *CP* nonconservation. The resulting number is comparable to the existing experimental upper limit; this *Ansatz* will therefore be critically tested by the next round of experiments using ultracold neutrons.

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A nonvanishing electric dipole moment for the neutron, d_N, would represent the first direct observation of a microscopic violation of time-reversal invariance. In the $SU(3)\otimes SU(2)\otimes U(1)$ standard model one predicts, however, very tiny values: $d_N < 10^{-30}$ e cm. Instanton effects could produce much larger numbers; yet once a Peccei-Quinn symmetry is invoked to obtain a natural solution to the strong *CP* problem, one again finds $d_N < 10^{-30}$ e cm.

Much larger predictions for d_N are obtained when using the Weinberg *Ansatz*[1] for *CP* nonconservation as noted by several authors[2]: The estimates range typically from 10^{-24} to 10^{-25} e cm. Experimentally the following bounds have been obtained:

$$d_N = \begin{cases} (-2.0\pm 1.0)\times 10^{-25}\, e\text{ cm} & \text{Ref. 3,} \\ (-1.8\pm 2.9)\times 10^{-25}\, e\text{ cm} & \text{Ref. 4.} \end{cases}$$

It is expected that the experimental sensitivity will reach the 10^{-26}-e cm level in the near future. Motivated by these exciting prospects we have reexamined as carefully as possible the prediction on d_N as it is obtained in the Weinberg *Ansatz*. Our treatment is very similar to that of Cheng[5]; yet we have analyzed the long-distance effects in considerably more detail than has been done before and have specifically included top-quark contributions as well as QCD radiative corrections. Our results based on all these considerations are presented in Fig. 1. In short, d_N indeed cannot be significantly smaller than 10^{-25} e cm; for most of the allowed range in the model parameters, d_N actually exceeds 10^{-25} e cm substantially. The next round of measurements, therefore, has to reveal a nonvanishing value for d_N if the Weinberg mechanism represents the major source of *CP* nonconservation.

There are three doublets of Higgs fields in addition to the three families of quarks and leptons and the gauge bosons. *CP* nonconservation occurs spontaneously; thus the Kobayashi-Maskava (KM) matrix is orthogonal and all *CP* asymmetries can be traced back to complex vacuum expectation values of Higgs fields that enter the Yukawa couplings after a redefinition of the Higgs fields:

$$\mathcal{L}_Y = \frac{g}{\sqrt{2}M_W}\sum_{i=1}^{3}(\alpha_i \bar{U}_R M_U K D_L + \beta_i \bar{U}_L M_D K D_R)H_i^\dagger + \text{H.c.}, \tag{1}$$

with H_i denoting the (charged) Higgs fields and U and D the three families of quarks with diagonal mass matrices M_U and M_D, respectively: $U=(u,c,t)$; $D=(d,s,b)$; K is the KM matrix.

It is the relative phase between α and β that drives *CP* asymmetries; the range of allowed values for $\text{Im}\alpha^*\beta$ is derived from data on ϵ and ϵ'. One finds the general expressions[6]

$$\epsilon = \frac{1-D}{2\sqrt{2}}e^{i\pi/4}\left\{\epsilon_m + 2\xi + \frac{D}{1-D}\chi\right\}, \tag{2}$$

$$\frac{\epsilon'}{\epsilon} = -e^{i(\pi/4+\delta_2-\delta_0)}\frac{2\xi}{20(1-D)[\epsilon_m+2\xi+D\chi/(1-D)]}, \tag{3}$$

with the following notation

$$M_{12}=(M_{12})_{\text{SD}}+(M_{12})_{\text{LD}}\equiv(M_{12})_{\text{SD}}+DM_{12},\quad \epsilon_m=\frac{\text{Im}(M_{12})_{\text{SD}}}{\text{Re}(M_{12})_{\text{SD}}},\quad \xi=\frac{\text{Im}\langle 2\pi,I=0|H|K^0\rangle}{\text{Re}\langle 2\pi,I=0|H|K^0\rangle},$$
$$\text{Im}(M_{12})_{\text{LD}}=(-2\xi+\chi)\text{Re}(M_{12})_{\text{LD}}. \tag{4}$$

SD (LD) stands for short (long) distance dynamics.

In the Weinberg *Ansatz* it was found[7] that

$$\epsilon_m \ll 2\xi. \quad (5)$$

This leads to $\epsilon'/\epsilon \sim -0.05$, a number clearly inconsistent with experimental bounds, unless chiral symmetry introduces a sufficiently strong suppression to yield[8,9]

$$|2\xi| \ll |(D\chi/1-D)|. \quad (6)$$

We assume this to happen—otherwise the model is already ruled out—and therefore read off[10]

$$|\epsilon| \approx D\chi/2\sqrt{2}. \quad (7)$$

In this model ϵ is thus produced mainly by long-distance dynamics; therefore they have to be studied very carefully.

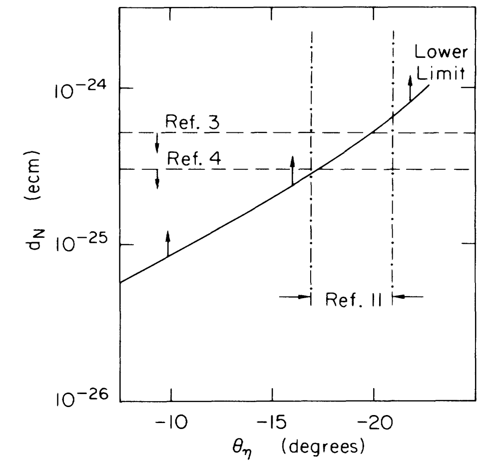

FIG. 1. Lower limit on the neutron electric dipole moment.

SU(3) symmetry together with current algebra implies the phases of $\langle 2\pi, I=0|H|K^0\rangle$, $\langle\pi^0|H|K^0\rangle$, and $\langle\eta_8|H|K^0\rangle$ to be equal; in the Wu-Yang phase convention the 2π, π^0, and η_8 contributions to $(M_{12})_{\rm LD}$ are therefore purely real. $\mathrm{Im}(M_{12})_{\rm LD}$ must then be produced mainly by the $K^0 \to \eta_0 \to \bar{K}^0$ transition, η_0 being the SU(3)-singlet component in the nonet of pseudoscalar mesons:

$$\epsilon = \frac{\mathrm{Im}\langle K^0|H|\eta_0\rangle\langle\eta_0|H|\bar{K}^0\rangle}{2\sqrt{2}\mathrm{Re}M_{12}} = \tfrac{8}{9}\rho\xi_0\frac{|\langle K^0|H|\pi_0\rangle|^2}{\sqrt{2}\Delta M^2}\sum_{P=\eta,\eta'}\frac{(1-4\rho)X_P^2-(1+2\rho)Y_P^2-(1/\sqrt{2})(1+8\rho)X_PY_P}{m_K^2-m_P^2}, \quad (8)$$

where[11]

$$\frac{\langle K^0|H|\eta_0\rangle}{\langle K^0|H|\eta_8\rangle} = -2\sqrt{2}\rho(1-i\xi_0), \quad \Delta M^2 = 2m_K\Delta m_K.$$

We have used as representation for the pseudoscalar wave functions[12]:

$$|P\rangle = X_P|(u\bar{u}+d\bar{d})/\sqrt{2}\rangle + Y_P|s\bar{s}\rangle + Z_P|G\rangle, \quad P=\eta,\eta'; \quad (9)$$

$|G\rangle$ denotes an additional SU(3)-singlet component, such as a glueball.

Equation (8) contains three types of parameters — $\{X_P, Y_P\}$; ρ; ξ_0—which will be discussed in turn.

(i) A comprehensive analysis of decays involving η and η' in the initial or final state leads to the conservative bounds[13]

$$0.6 \le X_\eta \le 0.85, \quad 0.55 \le -Y_\eta \le 0.95, \quad 0.3 \le X_{\eta'} \le 0.6, \quad 0.55 \le Y_{\eta'} \le 0.85. \quad (10)$$

A very recent reanalysis of Mark III data yields[14]

$$X_\eta = 0.81 \pm 0.04, \quad Y_\eta = -0.58 \pm 0.04, \quad X_{\eta'} = 0.58 \pm 0.04, \quad Y_{\eta'} = -0.81 \pm 0.04. \quad (11)$$

These numbers are quite consistent with an η-η' mixing angle $\theta = (-19 \pm 2)°$ as predicted by a $1/N$ treatment of QCD.[15] However, there is still room for a sizable glueball component in the η' wave function.

(ii) In the next step one obtains the parameter ρ by solving

$$A(K_L \to \gamma\gamma) = \langle K_L|H|\pi^0\rangle A(\pi^0 \to \gamma\gamma)\left\{\frac{1}{m_K^2-m_\pi^2} + \frac{1}{3}\sum_{P=\eta,\eta'}\frac{A(P\to\gamma\gamma)}{A(\pi^0\to\gamma\gamma)}\frac{1}{m_K^2-m_P^2} \times [X_P - \sqrt{2}Y_P - 2\sqrt{2}\rho(\sqrt{2}X_P + Y_P)]\right\}. \quad (12)$$

The relative sign of the amplitudes $A(\eta \to \gamma\gamma)$ and $A(\eta' \to \gamma\gamma)$ is taken to be positive as predicted by the quark model. The quark model actually predicts $\rho = 1$. We have found this to hold to within a factor of 2 for the set of parameters that give the lower limit on d_N. A note of caution is in order here: The obtaining of a value for ρ in this way and subsequent use of it in a *CP*-nonconserving matrix element represents a reasonable, yet not rigorous, procedure.

(iii) The remaining task consists of computing ξ_0 in the Weinberg *Ansatz*. One-loop diagrams involving Higgs-boson exchange yield the CP-odd transition operator[7,16]

$$\mathcal{L}_- = if\bar{d}\sigma^{\mu\nu}(1-\gamma_5)t^A s F^A_{\mu\nu} + \text{H.c.}, \tag{13}$$

where $F^A_{\mu\nu}$ denotes the gluon field-strength tensor and

$$f = \frac{G_F}{\sqrt{2}} \frac{g_2}{32\pi^2} m_s m_c^2 \frac{\alpha^*\beta}{m_H^2} \left[\eta_c K_{cs} K_{cd}^* G\left(\frac{m_c^2}{m_H^2}\right) + \frac{m_t^2}{m_c^2} \eta_t K_{ts} K_{td}^* G\left(\frac{m_t^2}{m_H^2}\right)\right], \tag{14}$$

where $G(x) = -[\frac{1}{2} + 1/(1-x) + 1/(1-x)^2 \ln x]$; η_c and η_t denote QCD radiative corrections; a leading-logarithm treatment yields

$$\eta_Q \simeq \left[\frac{\alpha_s(m_Q^2)}{\alpha_s(\mu^2)}\right]^{-1/6b} \left[\frac{\alpha_s(m_Q^2)}{\alpha_s(\mu_H^2)}\right]^{8/b}, \quad Q = c,t;\ b = 11 - \tfrac{2}{3} n_F; \tag{15}$$

μ denotes the normalization or infrared cut-off scale. Forming the matrix element will in principle lead to a compensating dependence on μ; in practice, however, an uncertainty is thus introduced since the models used to evaluate the matrix elements do not exhibit the μ dependence explicitly. In this case the μ dependence is extremely mild because of the tiny exponent $1/6b$ and we use $\eta_c \sim 3.2$, $\eta_t \sim 1.2$ for $m_t \sim 40$ GeV, $M_H \sim 100$–500 GeV.

Then one has

$$\langle K^0 | \mathcal{L}_- | \eta_0 \rangle \simeq -2(\tfrac{2}{3})^{1/2} \rho \langle K^0 | \mathcal{L}_- | \pi^0 \rangle = -2(\tfrac{2}{3})^{1/2} \rho f^* A_{K\pi}, \tag{16}$$

and therefore

$$\xi_0 = \frac{\text{Im} f A_{K\pi}}{\langle K^0 | H | \pi^0 \rangle}. \tag{17}$$

Inserting (17) into (8) and solving for Imf we find

$$\text{Im} f = \frac{9}{8\rho} \epsilon \frac{\sqrt{2}\Delta M^2}{A_{K\pi} |\langle K^0 | H | \pi^0 \rangle|} \frac{1}{F}, \quad F = \sum_P \frac{(1-4\rho)X_P^2 - (1+2\rho)Y_P^2 - (1/\sqrt{2})(1+8\rho)X_P Y_P}{m_K^2 - m_P^2}. \tag{18}$$

Equation (18) together with (14) allows us, finally, to determine Im$\alpha^*\beta$ for given values of M_H, M_t; for $A_{K\pi}$ we use the bag-model result[10] $A_{K\pi} = 0.4$ (GeV)^3.

In the nonrelativistic approximation d_N is simply expressed in terms of d_d and d_u, the electric dipole moments of down and up quarks:

$$d_N = \tfrac{1}{3}(4d_d - d_u). \tag{19}$$

The one-diagrams lead to (since $d_u \ll d_d$)

$$d_N = \frac{2\sqrt{2} G_F e}{18\pi^2} \frac{m_c^2 m_d}{m_H^2} \text{Im}(\alpha\beta^*) \left[\bar{\eta}_c |K_{dc}|^2 g\left(\frac{m_c^2}{m_H^2}\right) + \bar{\eta}_t \frac{m_t^2}{m_c^2} |K_{td}|^2 g\left(\frac{m_t^2}{m_H^2}\right)\right] \tag{20}$$

with

$$g(x) = \frac{1}{(1-x)^2}\left[\frac{5}{4}x - \frac{1-\frac{3}{2}x}{1-x}\ln x - \frac{3}{4}\right];$$

$\bar{\eta}_c$ and $\bar{\eta}_t$ are the radiative QCD corrections. In the leading-logarithmic approximation one finds

$$\bar{\eta}_Q \simeq \left[\frac{\alpha_s(m_Q^2)}{\alpha_s(\mu^2)}\right]^{4/3b} \left[\frac{\alpha_s(m_Q^2)}{\alpha_s(m_H^2)}\right]^{8/b}, \tag{21}$$

and we therefore use $\bar{\eta}_c \sim 2.5$–3, $\bar{\eta}_t \sim 0.7$–0.9 in the same spirit as expressed after Eq. (15).

It turns out that the minimum of d_N is obtained by minimizing the t-quark contribution. This occurs when the inequalities

$$m_t \gtrsim 23 \text{ GeV}, \quad K_{td} \gtrsim 0.001 \tag{22}$$

are saturated. The former follows from DESY PETRA data, the latter from the unitarity of the KM matrix (assuming there are only three families). The use of $K_{td} = 0$ will actually increase the value of d_N.

The resulting lower bound for d_N has only a weak dependence on M_H: The variation is at most 20% in the range 10 GeV $\lesssim M_H \lesssim$ 500 GeV. In our evaluation we have set $M_H = 500$ GeV.

In many computations of this type, one encounters large cancellations between π^0 and η, η' contributions, which amplify uncertainties introduced by, for example, SU(3) breaking and chiral symmetry breaking. In Eq. (8), we are spared from this possibility since only η and η' contribute. In Eq. (12), we have taken the symmetry-breaking effect into account by introducing

$$\frac{\langle K^0|H|\eta_0\rangle}{\langle K^0|H|\pi^0\rangle}=\frac{1}{\sqrt{3}}(1+0.17),$$

which was computed by Donoghue, Holstein, and Lin.[17] In principle, the same correction factor should be incorporated in Eqs. (16) and (18). Here, the uncertainty comes in as an overall multiplicative correction and can be treated together with the uncertainty in $A_{K\pi}$. The main uncertainty is due to the procedure by which we have determined ρ. We are confident that a *further* reduction of the lower bound by a factor of 3 reflects these uncertainties sufficiently. This has been done in Fig. 1 which shows our findings. However, because of the caveat stated above, we do not pretend to have deduced a rigorous lower bound. (In principle, there could also be cancellations between different scalar exchanges; yet a scan of the parameter space shows this to be a very unlikely occurrence.)

As stated in the beginning the experimental sensitivity for d_N is expected to reach the 10^{-26}-e cm level soon. These measurements will have to reveal a nonvanishing value for d_N if the Weinberg *Ansatz* describes the major source of *CP* nonconservation. Otherwise this model would clearly be ruled out as a significant contributor to ϵ. Two further notes in passing:

(i) The $\bar{\theta}$ parameter is calculable in this model. It vanishes naturally on the tree level; yet on the one-loop level one finds[18] $\bar{\theta}$(1-loop)$\sim 10^{-3}$ which is much too large thereby creating a pronounced need for a Peccei-Quinn symmetry.

(ii) The presence of scalar couplings produces a transverse polarization of muons in $K^+\to\mu^+\nu\pi$ decays. Yet we find Pol$(\mu)\sim 10^{-4}$. It appears hopeless to observe such a tiny effect, however important it would be.

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