Small and Calculable Dirac Neutrino Mass

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We present an extension of the left-right-symmetric model by including extra singlet quarks and leptons, which leads to calculable W_L-W_R mixing and an ultralight Dirac mass for the neutrino. For a W_R mass in the teraelectronvolt range, the neutrino masses can be in the right range to explain the solar neutrino puzzle via the Mikheyev-Smirnov-Wolfenstein oscillation amplification mechanism. Crucial for our results is a "see-saw"-type mechanism for the smallness of the d-quark masses.

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Neutrino mass is a sensitive probe of new physics beyond the standard electroweak model, which predicts that $m_\nu=0$ to all orders in perturbation theory. If the neutrino has a nonvanishing mass, laboratory experiments require that its value must be much smaller than the mass of the charged fermions of the corresponding generation. It has, therefore, been a challenge to theorists to produce models that provide natural mechanisms for small neutrino masses. The see-saw mechanism[1] and its many variants[2,3] are based on the assumption that the neutrino is a Majorana particle, where its smallness is related to the existence of a new symmetry-breaking scale, M_{B-L}. Present laboratory bounds on neutrino masses require M_{B-L} to be bigger than 1 TeV, making the new physics imminently testable. It has, however, been argued recently that, if the solar neutrino puzzle owes its resolution to the Mikhaev-Smirnov-Wolfenstein (MSW) mechanism,[4] then neutrino masses must be extremely tiny[5]; for instance, one typical set of values, for $\sin^2 2\theta_{\text{mix}} > 10^{-4}$, is $(m_{\nu_i}-m_{\nu_e})^2 \simeq 5\times 10^{-5}$. If we disregard the "unnatural" possibility of degenerate neutrino masses, we may conclude that $m_{\nu_e} \ll m_{\nu_\mu} \simeq 0.7\times 10^{-2}$ eV. In terms of the conventional see-saw mechanism,[1] this would require $M_{B-L}\simeq 10^{14}$–10^{15} GeV, whereas in terms of an "improved" see-saw mechanism suggested by us recently,[6] $M_{B-L}\simeq 10^9$ GeV or so. In either case, the associated new physics is beyond the reach of experiments in the near future. The basic reason for this result is that, the Dirac mass of the neutrino ($m_{\nu\text{D}}$) in left-right or SO(10)-type models is of the order of the charged-fermion masses m_f. One way to have ultralight neutrinos coexist with a low $(B-L)$-breaking scale would be to decouple the $m_{\nu\text{D}}$ from m_f and have $m_{\nu\text{D}}=0$ naturally at the tree level. In such a theory, we would expect $m_{\nu\text{D}} \ll m_f$. The left-right-symmetric theories[7] provide a natural framework for discussing this question. In several earlier papers, it has been noted that if m_ν is set equal to zero at the tree level, it receives infinite corrections at the two-loop level.[8]

In this Letter, we present two models in which, by including singlet quarks and leptons in the conventional left-right-symmetric models,[7] we obtain a small calculable $m_{\nu\text{D}}$. In the first model we obtain $m_{\nu_i} \simeq 10^{-5} m_{l_i}\epsilon$ ($\epsilon \ll 1$). In the second model, by decoupling the leptonic sector from the quark sector, we get

$$m_{\nu_i\text{D}} \sim 7\times 10^{-6}(m_b m_t m_{l_i}/m_{W_R}^2). \tag{1}$$

In the above formula, l_i denotes the charged lepton of the ith generation. The formula in Eq. (1) implies, for instance, that if $m_{W_R}\simeq 10$ TeV, then $m_{\nu_e}\simeq 1.7\times 10^{-5}$ eV, $m_{\nu_\mu}\simeq 3.5\times 10^{-3}$ eV, and $m_{\nu_\tau}\simeq 0.6\times 10^{-1}$ eV. These values are in the right range to explain the solar-neutrino puzzle, and yet the W_R scale is low enough to be accessible to the superconducting-supercollider experiments as well as other low-energy experiments. The model has also the interesting property that the smallness of the d-quark masses arises from a see-saw-type mechanism, rather than from an arbitrary adjustment of the Yukawa couplings.

In left-right-symmetric theories based on the gauge group $SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L}$, quark and lepton masses arise from the Higgs multiplet $\phi\equiv(2,2,0)$ which has vacuum expectation value (VEV)

$$\langle\phi\rangle = \begin{bmatrix} \kappa & 0 \\ 0 & \kappa' \end{bmatrix}.$$

Denoting the quarks (Q) and leptons (ψ) by $Q_L(2,1,\frac{1}{3})$, $Q_R(1,2,\frac{1}{3})$, $\psi_L(2,1,-1)$, and $\psi_R(1,2,-1)$, respectively, and using an appropriate discrete symmetry (see later), we can have Yukawa couplings of type $\bar{Q}_L\phi Q_R$ and $\bar{\psi}_L\tilde{\phi}\psi_R$ (where $\tilde{\phi}=\tau_2\phi^*\tau_2$). We see, then, that the neutrino Dirac mass $m_{\nu\text{D}}$ and the down-quark masses are proportional to the same arbitrary parameter κ' present in $\langle\phi\rangle$. If we want to predict $m_{\nu\text{D}}$, we must set $\kappa'=0$ at the tree level and have it arise out of a finite higher-order loop graph. Since neutrino mass is in the electronvolt range, κ' arising from loops must be very tiny and would therefore fail to explain the magnitude of down quark masses. The challenge is, therefore, to construct a model with small and calculable Dirac neutrino masses, without making the down-quark masses too small. In this Letter, we report on two models which answer this challenge. Note also that these models lead to finite W_L-W_R mixing.

In addition to the already existing fermions, we introduce two new quarks $g_a(1,1,-\frac{2}{3})$ $(a=1,2)$ and a new lepton $E(1,1,-2)$. We choose the following set of Higgs bosons to break the gauge symmetry down to $\mathrm{U}(1)_{\mathrm{em}}$ and give fermion masses: ϕ_q (2,2,0), χ_L (2,1,1), χ_R (1,2,1), n_a (1,1,0), $a=1,2$. We will suppress the generation index for simplicity and restrict our discussions to one generation.

To obtain a finite neutrino mass, we will impose the following two symmetries on the Lagrangean: (1) Discrete chiral symmetry: $Q\to\exp[i\gamma_5(\pi/3)]Q$, $\psi\to\exp[-i\gamma_5(\pi/3)]\psi$, $g_1\to\exp[-(2\pi i/3)\gamma_5]g_1$, $g_2\to\exp[(2\pi i/3)\gamma_5]g_2$, $\chi_L\to e^{-i\pi/3}\chi_L$, $\chi_R\to e^{i\pi/3}\chi_R$, $\phi_q\to e^{2\pi i/3}\phi_q$, $n_2\to e^{-2\pi i/3}n_2$. All other fields are invariant. (2) Continuous U(1) symmetry : Under this symmetry the fields g_{1R}, g_{2R}, E_{1R} have charge $+1$ and χ_L, n_1, n_2 have charge -1. The Yukawa coupling invariant under these two symmetries is given by

$$\mathcal{L}_Y=h_1\bar{Q}_L\phi_qQ_R+h_2\bar{\psi}_L\tilde{\phi}_q\psi_R+f_1(\bar{Q}_L\chi_Lg_{1R}+\bar{Q}_R\chi_Rg_{1L})+f_2(\bar{g}_{1L}g_{2R}+\bar{g}_{2L}g_{1R})n_1+f_3\bar{g}_{2L}g_{2R}n_2+f_4(\bar{\psi}_L\chi_LE_{1R}+\bar{\psi}_R\chi_RE_{1L})+f_5E_{1L}E_{1R}n_1+\mathrm{H.c.} \quad (2)$$

Symmetry breaking.—The above symmetries restrict the structure of the potential to be such that the minimum corresponds to the following fields acquiring nonzero VEV:

$$\langle\phi_q\rangle=\begin{pmatrix}\kappa_1 & 0\\ 0 & 0\end{pmatrix},\quad \langle\chi_L\rangle=\begin{pmatrix}0\\ v_L\end{pmatrix},\quad \langle\chi_R\rangle=\begin{pmatrix}0\\ v_R\end{pmatrix},\quad \langle n_1\rangle\simeq\langle n_2\rangle\simeq\sigma. \quad (3)$$

We assume v_R to be in the range of several teraelectronvolts, which corresponds to right-handed W and Z bosons being in the range of several teraelectronvolts and $v_R\ll\sigma$. Furthermore, we assume $\kappa\approx v_L$. Two things are worth emphasizing about Eq. (3): First, we have chosen $\langle\phi_2^0\rangle\equiv\kappa'=0$. One may ask whether this pattern of VEV's is stable under radiative corrections. The answer is in the affirmative since terms of type $\mathrm{Tr}\phi_q\tilde{\phi}_q^\dagger$ and $\bar{\chi}_L\phi_q\chi_R$, $\bar{\chi}_L\phi_q\chi_Rn_a$, etc., which would destabilize this pattern, are forbidden by the above symmetries. The only renormalizable "mixed" term allowed by the discrete symmetries is $\bar{\chi}_L\tilde{\phi}_q\chi_Rn_2$ which only stabilizes the above minimum. It is of course clear that Eq. (3) breaks the symmetry down to $\mathrm{U}(1)_{\mathrm{em}}$.

Finite W_L-W_R mixing and neutrino mass.—Turning now to the fermion masses, we see that, at the tree level, $m_\nu=0$. The up-type quarks (u,c,t) have masses but no mixings since h_1 can be diagonalized by rotation of the quark doublets $Q_{L,R}$. All the quark mixings then arise from the matrix structure in generation space of the couplings f_1, f_2, and f_3. Again, suppressing the generation index, we get

$$M_{dg}=\begin{pmatrix}0 & f_1v_R & 0\\ f_1v_R & 0 & f_2\sigma\\ 0 & f_2\sigma & f_3\sigma\end{pmatrix}. \quad (4)$$

This leads to a down-quark mass $m_d\simeq f_3f_1^2v_Lv_R/f_2^2\sigma$ which can be much smaller than the m_W if $\sigma\gg v_R$. The two heavy quarks have masses of order $f\sigma$.

It follows from Eq. (3) that at the tree level, W_L-W_R mixing vanishes. The one-loop graph that induces W_L-W_R mixing is given in Fig. 1 and is finite by power counting: An evaluation of this graph shows that the dominant contribution to W_L-W_R mixing comes from the t,b intermediate state, leading to

$$\delta m_{L,R}^2\simeq\frac{\alpha m_bm_t}{4\pi\sin^2\theta_W}. \quad (5)$$

This leads to the W_L-W_R mixing angle $\leq10^{-7}$ for $m_t\simeq100$ GeV and $m_{W_R}\geq2.5$ TeV.[9]

Turning now to neutrino mass, we find that the one-loop graph contributing to neutrino Dirac mass is given in Fig. 2 and can be estimated to be

$$m_{\nu_l}=\left(\frac{gm_{l_i}}{2m_W}\right)\left(\frac{gm_t}{2m_W}\right)\frac{m_b}{16\pi^2}\left(\frac{v_R}{\sigma}\right)^3. \quad (6)$$

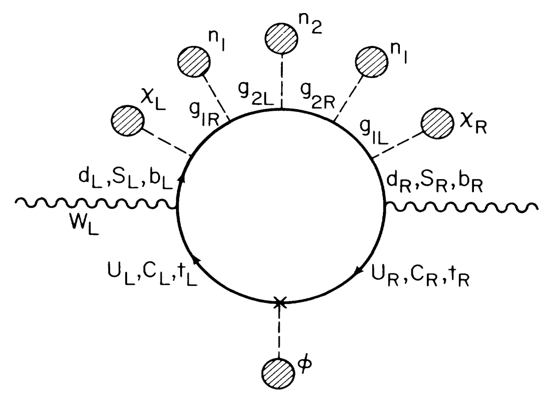

FIG. 1. One-loop graph that leads to calculable W_L-W_R mixing.

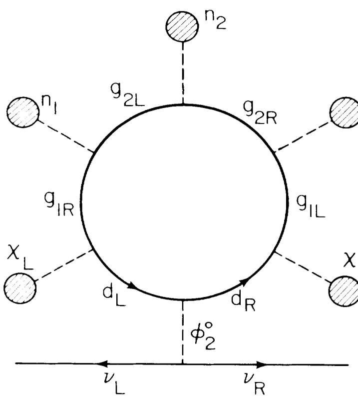

FIG. 2. The one-loop graph contributing to finite neutrino mass in model I.

For $m_t \simeq m_W$, we find $m_{\nu_i} \simeq 10^{-5} m_{l_i} \epsilon$, where $\epsilon \simeq (v_R/\sigma)^3$ is expected to be much less than 1. Even if we assume $\epsilon \simeq 1$, we find $m_{\nu_e} \simeq 5$ eV, $m_{\nu_\mu} \simeq 1$ keV, and $m_{\nu_\tau} \simeq 18$ keV. On the other hand if $v_R/\sigma \simeq 10^{-1}$, then we obtain

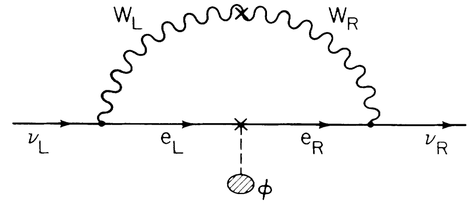

FIG. 3. One-loop graph that gives rise to infinite $m_{\nu D}$ when W_L-W_R mixing is infinite and to finite $m_{\nu D}$ when W_L-W_R mixing is finite.

$m_{\nu_e} \simeq 0.005$ eV, $m_{\nu_\mu} \simeq 1$ eV, and $m_{\nu_\tau} \simeq 18$ eV. The second set of values are cosmologically allowed even if the neutrinos are stable.

Let us consider a second model by introducing additional Higgs multiplets ϕ_l (2,2,0) and n_3 (1,1,0) to give mass to the leptons. To make the model consistent, we modify the transformation properties of ψ under the discrete symmetry as follows: $\psi \to \exp(i\gamma_5\pi/4)\psi$; $\phi_l \to i\phi_l$; $E_1 \to \exp[-i\gamma_5(7\pi/12)]E_1$; $n_3 \to \exp[-i(7\pi/6)]n_3$. The leptonic part of the $\mathcal{L}_Y$ gets modified to

$$\mathcal{L}_Y = h_l \bar{\psi}_l \phi_l \psi_R + f_4'(\bar{\psi}_L \chi_L E_{1R} + \bar{\psi}_R \chi_R E_{1L}) + f_5' \bar{E}_{1L} E_R n_3 + \text{H.c.} \tag{7}$$

Leptonic mass matrices arise from the VEV's

$$\langle \phi_l \rangle = \begin{pmatrix} 0 & 0 \\ 0 & \kappa_l \end{pmatrix}$$

and $\langle n_3 \rangle = \sigma$. Note that the above pattern of $\langle \phi_l \rangle$ is stable under radiative corrections because of the discrete symmetry. The discussion of W_L-W_R mixing is the same as in model I. However, the finite neutrino mass arises from Fig. 3 since W_L-W_R mixing is finite and we find

$$m_{\nu_i \mathrm{D}} \simeq \left(\frac{\alpha}{4\pi \sin^2\theta_W} \right)^2 \frac{m_{l_i} m_b m_t}{m_{W_R}^2}, \tag{8}$$

where m_{l_i} is the mass of the charged lepton of the ith generation. To obtain an idea about the order of magnitudes of the neutrino masses we choose $m_{W_R} \simeq 10$ TeV, and find $m_{\nu_e} \simeq 1.7 \times 10^{-5}$ eV, $m_{\nu_\mu} \simeq 3.5 \times 10^{-3}$ eV, and $m_{\nu_\tau} \simeq 6 \times 10^{-2}$ eV (for $m_t = 100$ GeV), as stated in the introduction. These values fall in the range of values required to solve the solar-neutrino puzzle via the Mikhaev-Smirnov-Wolfenstein mechanism.

As far as the neutrino mixings are concerned, they owe their origin to the couplings f_4' and f_5' and cannot be predicted by this model.

Crucial to the success of our model is the existence of new singlet heavy quarks g and heavy lepton E. These new particles have masses in the range of 1–10 TeV. The mixing between the d and g quarks produce flavor-changing neutral currents but its magnitude is of order $\sim G_F(v_L/v_R)^2$ as in the E_6 superstring models[10] and is therefore small, if we require $v/v_R \leq 10^{-2}$. For this choice of parameters universality constraints from μ decay and β decay are also respected at the present levels of experimental accuracy.

We also note that the continuous symmetry of Eq. (2) is spontaneously broken by $\langle n_a \rangle \neq 0$ at a scale in the teraelectronvolt range. This would lead to a Goldstone boson, which couples mainly to the heavy g quarks. This Goldstone boson will acquire mass if we add a soft breaking term $\mu_a^2 n_a^2$ (in addition to $\mu_a'^2 n_a^\dagger n_a$) to the potential since it is allowed by the local symmetry. Even without this mass term, such a Goldstone boson may not be physically unacceptable since it couples to the heavy quarks g_a and leptons E_i.

Let us comment briefly on the charged-lepton sector. For one generation, the light (e^-) and the heavy charged lepton (E^-) mix and we get (for model I)

$$\begin{array}{cc} & \begin{array}{cc} e_L^- & E_L^- \end{array} \\ \begin{array}{c} e_R^- \\ E_R^- \end{array} & \begin{pmatrix} h_2\kappa_1 & f_4 v_R \\ f_4 v_L & f_5 \sigma \end{pmatrix} \end{array} .$$

This has a heavy eigenstate with mass $\approx f_5\sigma$—teraelectronvolts—and a light eigenstate with mass $\approx h_2\kappa$. So we require $h_2 \approx 10^{-5}$ to understand the electron mass. When higher generations are included, h_2, f_4, and f_5 become matrices and the mixings in the leptonic sector arise from the off-diagonal elements of f_4. The same properties hold for model II.

In conclusion, we have presented two extensions of the left-right–symmetric models with calculable W_L-W_R mixing as well as calculable Dirac mass for the neutrino.

These models are consistent with all known weak-interaction data and predict finite neutrino masses. In one model, the values of neutrino masses are relevant for solving the solar-neutrino puzzle, without requiring the $(B-L)$-breaking scale to be superheavy.

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