Gap Solitons and the Nonlinear Optical Response of Superlattices

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We study theoretically the transmission of radiation through superlattices of finite length, where the dielectric constant of one film in each unit cell contains a term linear in the local field intensity. When such a system is illuminated with radiation with frequency in a stop gap, increasing power can switch it from a state with low transmissivity to a state with transmissivity of unity. Gap solitons play a key role in the phenomenon.

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Superlattices are fascinating because the structures exhibit collective properties not shared by either constituent, and these characteristics can be controlled through variation of the structural parameters. One example is provided by the propagation of electromagnetic radiation through superlattices, normal to the interfaces. The dielectric constant varies in a stepwise manner as one moves down the structure, and thus stop gaps appear, in a manner familiar from the theory of wave propagation in periodic structures. If a superlattice is illuminated with radiation within a stop gap, the envelope of the field amplitude decays exponentially with distance down the structure, and the transmissivity of a structure of finite length is exponentially small.

We have studied theoretically transmission of stop-gap radiation (plane polarized) in a finite superlattice, with one film in each unit cell endowed with a (real) dielectric constant that depends on the local field intensity, $\varepsilon = \varepsilon^{(0)}[1+\lambda|E(z)|^2]$. The motivation is the following. With parameters arranged suitably, increasing power should close the gap partially, and the system may switch to a transmitting state at powers where the gap closes sufficiently to allow a band edge to move past the frequency of illumination. We expect bistability with origin in this mechanism.

In Fig. 1(a), we show the transmissivity of a model superlattice, calculated as described below, as a function of incident power. Indeed, we see instabilities, as evidenced by the multivalued nature of the transmissivity, but these striking results cannot be explained by the mechanism just described. The incident power is far too low to close the gap sufficiently, in the regime where the transmission coefficient is multivalued. At points P_1 and P_2 to within the accuracy of our calculation, the transmissivity is identically unity, while from the mechanism just outlined one expects an impedance mismatch to remain. The results remind one of the phenomenon of self-induced transparency,[1] though here the stop gap at low power has origin in a geometrical resonance between the radiation wavelength and structural geometry, rather than resonant response to its frequency.

The frequency used in the calculations in Fig. 1(a) lies

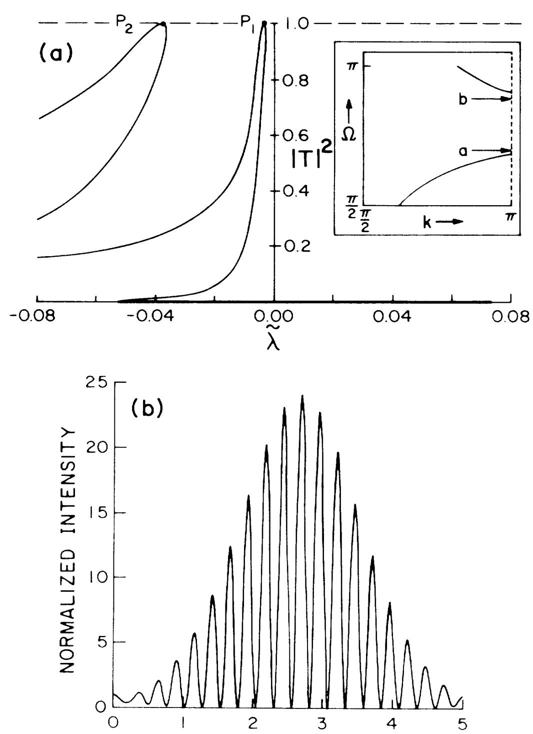

FIG. 1. (a) The transmissivity as a function of laser power, for a model superlattice with twenty unit cells. The parameter is $\tilde{\lambda} = \lambda E_0^2$, with E_0 the strength of the incident field. We have $d_1 = d_2 = 0.125$ in units of the vacuum wavelength, and $\varepsilon_1^{(0)} = 2.25$, $\varepsilon_2^{(0)} = 4.50$. The heavy line is the transmissivity in the highly reflecting state. This is in the range 10^{-4} for the example considered, and thus cannot be shown as finite on the graph. The calculation was not carried beyond $\lambda > 0.075$. Inset: The linear dispersion relation in the vicinity of the stop band considered; the dimensionless frequency is $\Omega = \omega \varepsilon_1^{(0)\,1/2}(d_1+d_2)/c$, with ω the frequency. The lower band edge is located at $\Omega = 0.74\pi$, and the transmissivity is calculated for $\Omega = 0.75\pi$. The arrow marked a in the inset indicates this frequency. (b) The field intensity, $|E(z)|^2$, in the superlattice at point P_1, measured in units of E_0^2. The value of $\tilde{\lambda}$ here is -0.004. The incident wave propagates from left to right, and distance is measured in vacuum wavelengths.

near the bottom of the stop gap [point a in the inset], and here we find a low-power transmission resonance only for $\lambda < 0$. For frequencies near the top of the gap, as at point b in the inset, we require $\lambda > 0$ to realize such behavior.

Insight into the origin of the transmission resonance is provided by Fig. 1(b), where we plot the square of the electric field, $|E(z)|^2$, in the structure at point P_1. The electric field is measured in units of E_0, the incident field, and $\tilde{\lambda} = \lambda |E_0|^2$. At the transmission resonance, the incident electric field couples to a solitonlike object which resides as a static entity in the center of the structure. We have explored the transmissivity for various frequencies within the gap, near either gap edge; we always find a transmission resonance, though the value of $\tilde{\lambda}$ where the transmissivity becomes unity varies with frequency. The soliton is always excited when the transmissivity equals unity; the width and shape of this pattern are insensitive to the number of unit cells N in this structure, provided N is sufficiently large, and the value of $\tilde{\lambda}$ at point P_1 decreases with N. So far as we can tell, the critical value of the field required to excite the transmission resonance roughly equals the field in the appropriate gap soliton at the surface of the structure; the gap soliton is then viewed as intrinsic to the superlattice of infinite extent, and it is only weakly perturbed by the surfaces of a superlattice of finite length. It acquires the character of a resonance level with finite lifetime by virtue of radiative decay through the outermost surfaces. An incident photon may then couple to the resonance mode, and excite it to produce a transmission resonance where the transmissivity reaches unity, in a nonlinear analog to the barrier transmission resonances familiar from elementary quantum mehcanics.[2]

We have established, again by numerical studies, that for frequencies near the lower gap edge, the nonlinear wave equation applied to the infinitely extended superlattice admits soliton solutions for $\lambda < 0$, while for $\lambda > 0$ such solutions exist near the upper gap edge. (In our calculations, the low-power index of refraction of the nonlinear film has been chosen larger than that of the linear film.) An example of such a gap soliton is given in Fig. 2, for an infinitely extended version of the superlattice used to generate the results in Fig. 1. For this structure, and the frequency used in Fig. 1, we have explored the properties of the solitons for values of λ in the range $10^{-4} \le \lambda \le 0.8$. The maximum field $E^{(M)}$ in the soliton obeys $\lambda E^{(M)2} = \text{const.}$, as λ is varied with frequency fixed. The envelope of the soliton is fitted accurately by the function $f(x) = E^{(M)}(\cosh\beta x)^{-1}$ with β independent of λ, suggesting that to good approximation we indeed have true solitons. At the time of this writing, we have not succeeded in deriving a simple equation for the envelope function, from the full nonlinear field equations used in the numerical work. As the frequency is moved away from either gap edge, the spatial size of the soliton shrinks. We have encountered stability problems integrating the nonlinear equations near the middle of the gap, and so here we confine our attention to frequencies near the gap edges where the solitons are many lattice constants in spatial width.

If the laser power is increased to higher values, a second transmission resonance occurs [point P_2 of Fig. 1(a)]. When the transmissivity equals unity, the field pattern inside the structure shows that two solitons have been excited within it.

We find these results most intriguing. Note that the transmissivity becomes multivalued at rather low laser powers. The parameter $\tilde{\lambda} = \lambda |E_0|^2$ measures the percentage change in the dielectric constant of the nonlinear film when the field in the film has magnitude E_0 equal to that in the incident beam. For the example shown, we have bistability when $\tilde{\lambda} < -0.0035$.

Our model, discussed next, is idealized in one regard. Dielectric constants for each film in a superlattice unit cell are presumed real, and so absorption is ignored. One supposes that in practice, the absorption lengths associated with materials incorporated into the structures must be longer than the total length of the superlattice, to ensure that the behavior found here is realized in actual samples.

The model superlattice is a stack of normal bilayers; one film in each bilayer has thickness d_1 and dielectric constant $\varepsilon_1^{(0)}$ independent of field, while the thickness of the second film is d_2, and its dielectric constant $\varepsilon_2 = \varepsilon_2^{(0)}[1 + \lambda |E(z)|^2]$, with $|E(z)|$ the amplitude of the field at point z; the z axis is normal to the interface, and we have confined our attention to plane-polarized radia-

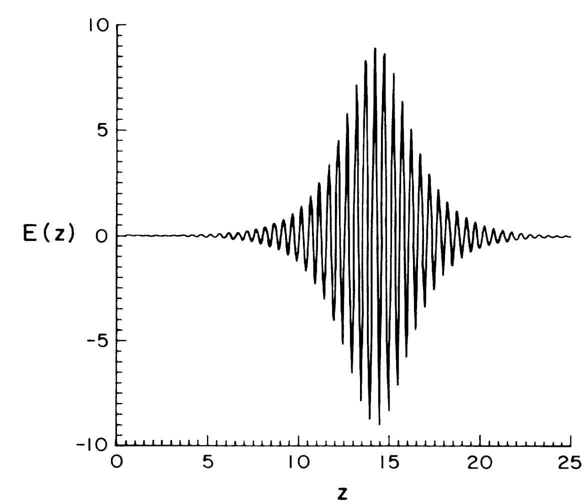

FIG. 2. The field amplitude $E(z)$ associated with a soliton in an infinitely extended superlattice, with parameters chosen as in Fig. (1). The calculations are for $\tilde{\lambda} = -0.001$. Again, distance is measured in vacuum wavelengths.

tion which propagates in the z direction. Our numerical calculations suppose that $\varepsilon_2^{(0)} > \varepsilon_1^{(0)}$, as remarked above.

We assume that the optical wave in the medium has a frequency ω, equal to that of the incident wave. Thus, we ignore higher harmonics present by virtue of the nonlinearity, as in numerous earlier studies.[3,4] In the absence of phase matching, the amplitude of the higher harmonics will be small. Then in the nonlinear films, the electric field obeys[3,4]

$$d^2E/dz^2 + k_2^2(1+\lambda|E|^2)E = 0, \tag{1}$$

with $k_2^2 = \omega^2\varepsilon_2^{(0)}/c^2$, and the linearized version of Eq. (1) with $\varepsilon_2^{(0)}$ replaced by $\varepsilon_1^{(0)}$ replaced by $\varepsilon_1^{(0)}$ applies to the linear films.

We let $E(z) = E_0\mathcal{E}(z)\exp[i\phi(z)]$, to obtain

$$d^2\mathcal{E}/dz^2 - W^2/\mathcal{E}^3 + k_2^2(1+\tilde{\lambda}\mathcal{E}^2)\mathcal{E} = 0, \tag{2a}$$

and

$$\phi(z) = \phi(z_0) + W\int_{z_0}^{z} dz'/\mathcal{E}^2(z'). \tag{2b}$$

The time-averaged Poynting vector is $S = c^2E_0^2W/8\pi\omega$ and is conserved (independent of z), and so the parameter W, in appropriate units, is the transmissivity of the structure. Our studies of the transmissivity of the finite superlattice consist of a search for those values of W for which we may achieve a solution of the nonlinear equations just described, with boundary conditions at each interface obeyed, and the appropriate incident wave illuminating the structure. The soliton solutions intrinsic to the infinitely extended superlattice have $W \equiv 0$; the envelope function vanishes exponentially as one moves far from the center of the object in either direction, so that there is no energy flow within this excitation.

The general solution of Eq. (2a) may be expressed in terms of Jacobi elliptic functions. The task one must address is to submit the general solution in each film to the apropriate boundary conditions. In linear theory, the general solution for the fields within a film is expressed in terms of four parameters. One has a wave running from right to left, superimposed with one from left to right, each described by a complex amplitude. The general solution of the nonlinear equations also involves four constants. These are W, $\phi(z_0)$, the value of $\mathcal{E}(z)$ at a reference point z_0, and a constant A which emerges from the first integral of Eq. (2a), which reads

$$A = k_2^2\mathcal{E}^2 + k_2^2\lambda\mathcal{E}^4/2 + W^2/2\mathcal{E}^2 + (d\mathcal{E}/dz)^2.$$

Even for an isolated, single nonlinear film, the task of solving for the transmissivity is formidable, as one may appreciate from a recent paper by Band.[5]

We have recently addressed the problem of calculating the transmissivity of an isolated, nonlinear film[6] and devised an efficient means of obtaining solutions. One selects a trial value of the (real) parameter W, and identities derived in our paper allow the remaining three parameters to be uniquely determined once W is chosen. Identities which link $\phi(z_0)$, $\mathcal{E}(z_0)$, and A to W follow from the boundary conditions at the film surfaces; we chose z_0 to be the right-hand (output) edge of the film. With these parameters in hand, one may use the Jacobi elliptic functions to determine $\mathcal{E}(z)$ and $d\mathcal{E}/dz$ at the left-hand (input) surface. One searches for values of W that generate values of $\mathcal{E}$ and $d\mathcal{E}/dz$ on the left which satisfy the boundary conditions. We thus reduce the problem to a one-parameter search.

It has proven possible to extend our earlier scheme to an arbitrary multilayer structure,[7] each film of which is nonlinear in the manner described by Eq. (1). We are able to retain the feature that one chooses a single parameter W and a sequence of identities may be used to express all remaining parameters in the solution of the multilayer problem in terms of W. We do this by moving from the right (output) end to the left through the structure, to find $\mathcal{E}(z)$ and $d\mathcal{E}/dz$ at the left-hand side. We search for values of W that lead to fulfillment of the boundary conditions on the left. Accuracy can be checked by our explicitly calculating the reflectivity R from $\mathcal{E}$ and $d\mathcal{E}/dz$ obtained in this manner, then testing whether $R+T=1$, with T the transmissivity.

As remarked earlier, the soliton in the infinitely extended superlattice is a solution with $W\equiv 0$. The solitons such as that in Fig. 2 are generated by a different procedure. Note that with $W=0$, the phase $\phi(z)$ is constant everywhere [Eq. (2b)], and thus may be chosen to be zero. In the "tail" of the soliton, the field has small amplitude, the nonlinear terms are unimportant, and linear theory relates $\mathcal{E}(z)$ and $d\mathcal{E}/dz$. We choose a small value for $\mathcal{E}(z)$ at a selected point and use linear theory to determine $d\mathcal{E}/dz$ there, and the solution is matched to the elliptic-integral description provided by the full theory. We move from right to left, requiring in the linear regime that the solution match to the exponentially increasing solution, and use the full elliptic-integral representation to move out of the linear regime, into and through that where the nonlinearity asserts itself.

The calculations suggest one should study experimentally, as a function of laser power, the transmissivity of finite superlattices illuminated with radiation of frequency within a stop gap, near a band edge. Gap-soliton–mediated bistability should be observed, if the intrinsic absorption is sufficiently small. From the theoretical point of view, a derivation of the form of the nonlinear equation obeyed by the envelope function would be welcome.

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