Magnetic Resonance as a Probe of Permeability in Porous Media

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Results of a theoretical study of the permeability k and proton relaxation rate T_1^{-1} in fully and partially saturated porous media are presented. Numerical calculations on two distinct geometrical models of granular materials predict a strong correlation between k and $\phi^4 T_1^2$ for fully saturated systems (with porosity ϕ) in agreement with a recent experimental study.

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Among the most important (and elusive) properties of porous media is the permeability k which measures the ease with which a viscous fluid flows through the solid matrix. k, which has the dimensions of area, is defined in terms of the Darcy equation, $\mathbf{v}=(k\eta^{-1})\nabla p$, where $\mathbf{v}$ is the flow velocity, p is the pressure drop across the sample, and η is the viscosity. Because k is difficult to measure *in situ,* NMR has been proposed as an *indirect* technique for its estimation.[1] At first glance it would appear that NMR is not a likely candidate to provide information about permeability. A porous medium can be visualized as a network of large open regions (nodes) connected by narrow channels (throats). While most of the porosity is concentrated in the nodes, the system's transport properties are clearly controlled by the throats. What about magnetic resonance? Since almost all of the fluid is concentrated in the nodes, the observed signal is dominated by these regions. Indeed, we would expect essentially the same lifetime to be measured for a system in which all the throats are blocked (as shown for the upper left node) and the permeability vanishes. Interest in this problem has been heightened by the results of a recent experimental study of the relation between NMR and permeability in a suite of 56 water-saturated sandstones.[2] For the present purposes, the results of Ref. 2 may be summarized as follows: (1) Within this suite, the decay of the proton magnetization $M(t)$ was well described by a stretched exponential form $M(t)=M_0 \times\exp[-(t/T_1)^a]$. (2) The best correlation between the measured values of k, ϕ, and T_1 was found when k was cross plotted against the combination $\phi^4 T_1^2$ (see Fig. 1).

To understand these results, we have undertaken a study of proton relaxation in the grain-consolidation (GC) and shrinking-tube (ST) models of porous media.[3,4] While these models do not exhibit all the complexity of sedimentary rocks, they retain the essential features of granular media and allow us to study NMR and permeability on an equal footing. We conclude (1) that the calculated permeabilities and lifetimes of both models show a strong correlation between k and $\phi^4 T_1^2$, (2) that this correlation is consistent with two recently proposed expressions for the permeability,[5,6] and (3) that the techniques we have developed can be applied to the study of partially saturated systems.

In the conventional picture of NMR in porous media, the fluid's bulk relaxation processes are overwhelmed by the influence of the pore-grain interface.[1,7-9] There the proton magnetization is believed to decay rapidly because of either the presence of paramagnetic impurities or hindered rotation of the H_2O molecules. This decay is usually described in terms of a phenomenological killing strength, ρ. The magnetic moment per unit volume $M(\mathbf{r},t)$ then satisfies the diffusion equation[7,8] $D\nabla^2 M - \tau_B^{-1} M = \partial M/\partial t$, with the boundary condition $[D\hat{\mathbf{n}}\cdot\nabla M = \rho M]_S = 0$. Here D is the diffusion constant for bulk H_2O, $\hat{\mathbf{n}}$ is a unit normal directed from the pore into the grain, and the term involving τ_B^{-1} provides for bulk relaxation. Given a pore of diameter a, the problem is characterized by three time scales: the diffusion time a^2/D, the killing time a/ρ, and the bulk lifetime τ_B. As-

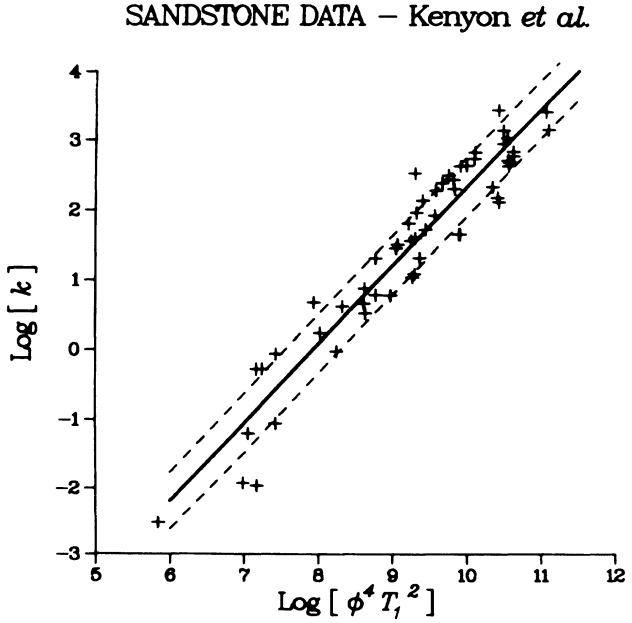

FIG. 1. Experimental scatter plot of k vs $\phi^4 T_1^2$ for a suite of water-saturated sandstones (Ref. 2). k is given in millidarcies (1 darcy $\approx$ 1 μm^2), ϕ is in percent, and T_1 is in milliseconds.

suming that τ_B is so long as to be irrelevant, we can distinguish two regimes. If $\rho a/D \ll 1$, we are in the fast diffusion limit and the lifetime is controlled by ρ. In the diffusion-limited regime, $\rho a/D \gg 1$, the lifetime is controlled by D. In general, a wide range of length scales characterizes the pore space. Protons in the smallest pores will exhibit fast diffusion, while those in the largest pores might exhibit slow diffusion; there is some coupling between the large and small pores and the entire system's behavior is quite complex.

Both the GC and ST models are designed to emulate the process of diagenesis by which granular systems evolve from high to low porosity. In the ST model the initial configuration of the pore space $[\phi \equiv \phi_0]$ is represented as a simple cubic array of cylindrical tubes whose radii are distributed uniformly within a specified range. The system is taken to lower porosities by repeated random shrinking of the tubes by a factor x $[0 < x < 1]$. In the present work, we set $\phi_0 = 0.38$ (the porosity of dense random packed spherical grains), computed the corresponding permeability by mean-field theory, and used the results of Ref. 4 to estimate k following the shrinking process. To calculate the decay of magnetization, we assume that the tubes are long enough to be independent (i.e., diffusion does not take a significant number of protons from one tube to another). The diffusion equation can then be solved for each tube[7] and the results summed to give the system's net response. The GC model begins with an ordered packing (fcc in the present case) of monodisperse spherical grains.[3] The porosity is reduced by our allowing the spheres to grow uniformly into polyhedra. In this model the pore space divides naturally into relatively large nodes connected by narrow throats and the transport properties can be either calculated by network simulation or estimated by minimum area arguments.[3] The most direct way to study proton relaxation in the GC model is by random-walk simulation of the diffusion equation. A grid (with cube edge ε) is created on the pore space and each H_2O molecule moves randomly until it encounters a grain boundary where the proton magnetization decays with probability (per unit time step τ) γ. [A discrete version of the boundary condition[10] yields a relation between the microscope and macroscopic parameters, viz., $\rho = (\varepsilon/6\tau)\gamma(1-\gamma)^{-1}$.] The ST and GC models are in many ways complementary. The first emphasizes the role of disorder and allows us to treat a system with a wide range of pore sizes. The GC model provides a somewhat more realistic picture of the pore shape and allows us to treat diffusion from node to throat to node.

The principal results of our work are shown in Fig. 2. For both models we have constructed *scatter plots* based on three choices of grain size, four (ST) or five (GC) porosities (for each grain size), and three values of ρ (for each grain size and porosity).[11] The results for the two models are strikingly similar; as in Fig. 1, both show a

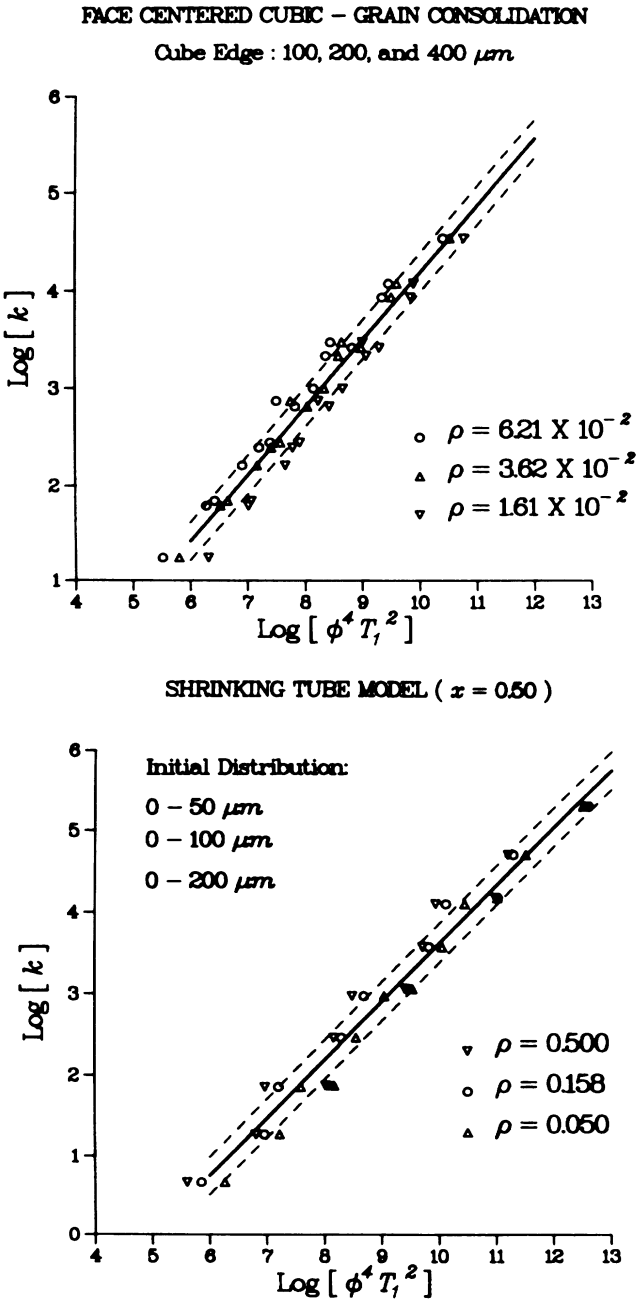

FIG. 2. Scatter plots of k vs $\phi^4 T_1^2$ for the water-saturated GC and ST models. The values of ϕ are, for GC, 38.0, 28.2, 22.1, 16.6, and 12.0; for ST, 38.0, 19.5, 9.5, and 4.75. Units are as in Fig. 1.

strong correlation, over roughly four decades in k, between permeability and $\phi^4 T_1^2$. We emphasize that the values of ρ chosen in both cases are representative of naturally occurring sedimentary rocks and correspond to the *intermediate*-diffusion regime for reasonable pore sizes ~ 10 μm. In this regime the lifetime is not very

sensitive to variations in ρ. By contrast, in the fast-diffusion regime, a good correlation would be obtained only if ρ were essentially constant for all porous media. While the qualitative features of Figs. 1 and 2 are quite similar, we note (1) that the slope of the experimental cross plot (≈ 1.13) is noticeably higher than that found in the model calculations and (2) that, for a given value of $\phi^4 T^2$, the values of k for the models are higher than those of the sandstones. Both effects arise because the geometrical models underestimate the complexity of the pore space in real reservoir rocks. Dead-end channels, isolated pockets, and rough interfaces are all expected to decrease k while having little effect on the value of T_1.

It is of interest to interpret the present results in terms of the relations proposed independently by Katz and Thompson[5] (KT) and Johnson, Koplik, and Schwartz[6] (JKS). In both Refs. 5 and 6 the permeability was shown to be well described by an equation of the form

$$k \sim L^2(\sigma/\sigma_f) = L^2\phi^m. \quad (1)$$

Here σ and σ_f are the *electrical* conductivities of the fully saturated rock and the pore fluid, respectively, and L is a length scale characteristic of the pore-space geometry. In the geophysical evolution of a family of rocks, the exponent m describes the relationship between the sizes of the throats (which govern conductivity) and the nodes (which determine the porosity). (The versions of the GC and ST models employed here yield $m \approx 1.80$ and $m \approx 1.85$, respectively.[3,4]) KT have shown theoretically and confirmed experimentally that the length scale entering Eq. (1) (say, L_{KT}) can be obtained from a capillary-pressure measurement. On the other hand, JKS argue that $L = \Lambda$ where Λ is defined in terms of the solution of the pore-space electrical conduction problem. While the KT and JKS approaches might appear to be rather different, it has recently been shown that the lengths L_{KT} and Λ are in fact proportional to one another in many cases of interest.[12] How, then, can we reconcile these results with the present NMR calculations? To be specific, we limit our attention to the GC calculations and to the JKS version of Eq. (1). In the GC model Λ is proportional to the pore volume-to-surface ratio V_p/S and we have $k \sim (V_p/S)^2\phi^{1.8}$. In order that this expression be consistent with the results presented in Fig. 2 [i.e., $k \sim (\phi^4 T_1^2)^{0.7}$], it must be true that for all grain sizes and all values of ρ we have

$$T_1 \sim (V_p/S)^{1.43}\phi^{-0.71}. \quad (2)$$

Our GC data are, in fact, in excellent accord with this equation and the exponent 1.43 in Eq. (2) indicates that we are indeed in the *intermediate*-diffusion regime. Further, our discussion of Eqs. (1) and (2) suggests that the NMR-permeability correlation is related to the electrical conductivity problem. This is not entirely surprising because, as we noted above, the exponent m provides information about the interrelationship of the pore-space nodes and throats.[13] The length L required to go from conductivity to permeability can be estimated from NMR, interfacial effects (JKS), or capillary-pressure measurements (KT).

To this point we have considered only fully saturated porous media. However, in many cases one must deal with systems in which a mixture of two fluids [e.g., oil and water] resides in the pore space. For simplicity, we have considered the case of an air-filled system being invaded by water. [Because air has such a low density of protons, its NMR response is negligible.] Water is assumed to be the wetting fluid and the displacement process is one of imbibition. As long as the flow rate is sufficiently small (so that capillary forces dominate the effects of both viscosity and buoyancy), the rules of invasion percolation[14] state that water will first enter the *smallest* accessible pore. The ST model, with its wide distribution of pore sizes is well suited for a first analysis of this problem. Calculations were carried out on a network of 6100 tubes with radii initially distributed uniformly between 0 and 200 μm. The tubes were invaded with water and the NMR response was calculated as a function of the water saturation, S. In Fig. 3 the lifetime $T(S)$ (normalized to its value at $S=1$) is shown as a function of S for two values of ρ for the initial distribution of tube radii. Within a mean-field approximation, where the tubes simply fill in order of increasing size, it is straightforward to estimate the behavior of $T(S)$ in

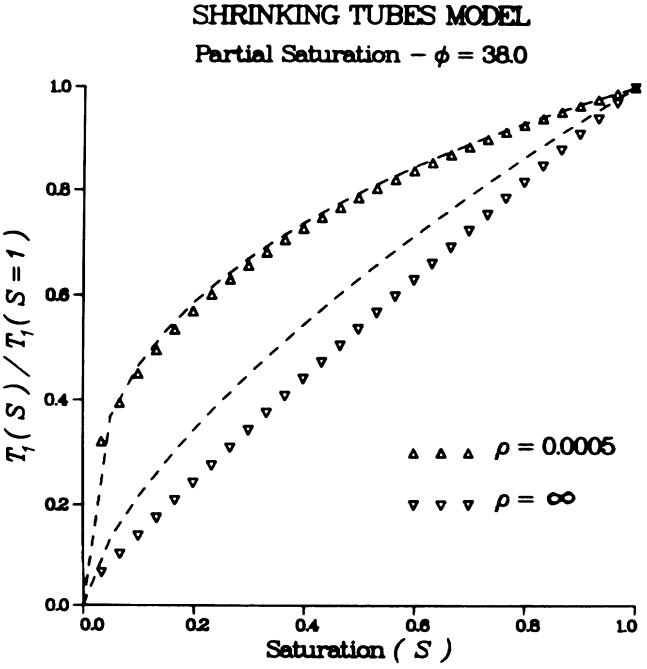

FIG. 3. Relative lifetime vs saturation for invasion percolation. The dashed curves correspond to the $S^{1/3}$ and $S^{2/3}$ estimates for the limiting behavior.

the fast- and slow-diffusion limits. We find $T(S)\sim S^{1/3}$ for fast diffusion and $T(S)\sim S^{2/3}$ for slow diffusion as indicated by the dashed curves in Fig. 3. The numerical data indicate that the mean-field approximation is quite good. (The $\rho=\infty$ results lie slightly below the $S^{2/3}$ curve because of contributions due to higher normal modes which are omitted in our mean-field estimate.) Note that for $S>0.5$, the curves shown in Fig. 3 are confined to a relatively narrow band, which would indicate that one could reasonably hope to estimate $T(S=1)$ from data at one or more partial saturations.

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[11]All the lifetimes reported in this paper were obtained by fitting the decay by a stretched exponential (see Ref. 2). Thus T_1 is the time at which the magnetization reaches e^{-1} of its initial value and the exponent α is a measure of the distribution of relaxation times governing the overall decay.

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[13]Indeed, we find that the slope of the k vs $\phi^4 T_f^2$ cross plot is controlled by the conductivity exponent m. To verify this we have carried out calculations on the ST model with $x=0.25$ ($2.55<m<2.96$) and $x=0.75$ ($m\approx 1.32$). The slopes of the corresponding k vs $\phi^4 T_f^2$ cross plots are 0.92 ($x=0.25$) and 0.48 ($x=0.75$).

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