High-Resolution Thermal-Conductivity Measurements near the Lambda Point of Helium

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We report measurements of the thermal conductivity of helium in the region from 6×10^{-8} to 6×10^{-4} K above the lambda point at the vapor pressure. Primarily because of uncertainties in the Kapitza boundary resistance, reliable results were obtained only in the region from 6×10^{-7} to 6×10^{-4} K above the transition. Within 5×10^{-6} K of the transition we find conductivity values significantly lower than expected on the basis of theoretical extrapolations from previous experiments. When the parameter R_K is calculated, we find a maximum about 2×10^{-6} K above the transition, in disagreement with the predictions of the two-loop dynamic renormalization-group theory. The uncertainties in our results in this region are discussed in detail.

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Near the lambda point of helium a unique opportunity exists to perform high-resolution tests of the theory of cooperative phase transitions. In contrast to other systems, the absence of a strongly divergent compressibility combined with the intrinsic strain-free nature of the system allows a resolution of less than 10^{-7} in the parameter $t=|1-T/T_\lambda|$ to be reached before severe difficulties are encountered. Here we report very high-resolution measurements of the thermal conductivity, K, obtained just above the transition temperature, T_λ, which can be used to obtain insight into the recently developed[1,2] dynamic renormalization-group theory (DRG). Previous measurements,[2,3] while extending to $t\sim3\times10^{-6}$, have been compared with the theory most carefully at relatively large values of t, extending to $t\sim10^{-2}$. This region represents the weak-coupling regime of the theory, and is also the region where the experimental results are most accurate. For $t<10^{-3}$ we enter the crossover region to strong coupling, and comparison with experiment is still found to be good. Our results span the range $3\times10^{-8}<t<3\times10^{-4}$, with maximum accuracy over the range $3\times10^{-7}<t<10^{-4}$. This allows us to explore the validity of the two-loop DRG deeper in the asymptotic region than has previously been possible.

To compare our results with DRG we compute the parameter R_K defined by

$$R_K=K(k_B/g^2\xi C_p)^{1/2}, \tag{1}$$

where K_B is Boltzmann's constant, g is the bare dynamic coupling constant,[3] ξ is the correlation length, and C_p is the heat capacity per unit mass. This transformation removes the asymptotic singular behavior from K and gives a dimensionless quantity that is expected to tend to a fixed-point value at the transition. We find that R_K reaches a maximum of 0.325 ± 0.01 near $t=10^{-6}$. In this region two-loop DRG predicts a smooth, monotonic increase of R_K based on fits[3,4] to results obtained further from the transition. The presence of the maximum implies that the two-loop approximation is insufficient for $t<3\times10^{-6}$, at least at the saturated vapor pressure of helium. Also, the behavior of R_K must be more complex than expected in the region very close to the transition if this quantity is ever to reach a nonzero fixed-point value.

Since a number of effects can perturb our results, we describe our experimental method and analysis in some detail. To obtain the thermal-conductivity data we used a cell of conventional design described elsewhere,[5] in which ΔT, the temperature drop across a helium sample[6] confined to a cylindrical region between two copper end plates, can be measured as a function of the power applied to the bottom. The temperature difference was measured with very high-resolution thermometers having an rms noise level of less than 3×10^{-10} K in a 0.1-Hz bandwidth. This allowed us to make accurate measurements with a maximum power input to the sample of 2.5×10^{-9} W/cm^2, about 2 orders of magnitude less than in previous experiments. This situation maximizes the linearity of the measurements close to T_λ, and minimizes the broadening of the transition. In our measurements the transition broadening due to the finite power input was about a factor of 5 less than the gravitational width due to the compression of the fluid in the 0.219-cm-high sample space. Two different techniques were used to collect data. In the primary mode, the temperature of the top of the sample was held constant and power was applied to the bottom. After steady-state conditions had been established, the resulting ΔT was recorded. For all data obtained in this mode, power inputs of (0,1,2, and 3)$\times10^{-9}$ W over the 1.2-cm^2 sample cross section were used. A background power of 5×10^{-10} W was observed, which was found to be somewhat temperature dependent. The thermal conductivity of the helium was computed from the change in ΔT with power, eliminating the effect of the background power. After this group of measurements was completed, the servomechanism stabilizing the top-plate temperature was disabled and the cell moved to a new temperature. For $t<2\times10^{-6}$ data were also taken with a continuous mode in which

the temperature of the sample was allowed to drift slowly at fixed power input to the bottom and the resulting temperature drop recorded. Because of the thermal lags in the system and the heat absorbed by the sample, results obtained while heating in this mode differed by a few percent from those obtained while cooling. We assumed that these effects were linear in the heating rate and averaged the heating and cooling data accordingly. A numerical model indicated that this was a reasonable procedure for $t > 10^{-8}$. For $10^{-7} < t < 2 \times 10^{-6}$, where overlap between the two methods exists, excellent agreement was obtained.

Before the thermal conductivity of the helium was calculated, we applied corrections to the raw data for the effect of the heat flow through the stainless-steel walls of the cell (including end effects[3]) and the boundary resistance at the surfaces of the end plates. At $t = 3 \times 10^{-4}$, where it was largest, about 10% of the heat passed through the walls. Closer to the transition its importance is diminished because of the increased conductivity of the helium. To estimate the temperature drop across the sample we assumed that the Kapitza boundary resistance at the surface of the end plates was independent of temperature over the range of the data and that its magnitude was given by the value observed 5×10^{-6} K below the transition. The effect of other assumptions about the behavior of this quantity was also investigated, as described below. Far from the transition these corrections were sufficient to allow an accurate determination of K. For $t < 10^{-5}$, it was also necessary to allow for the variation of K over the height of the cell due to gravity and the finite ΔT. To perform these corrections, we assumed the following model for the behavior of $K(T,P)$ in the region near T_λ and close to the vapor pressure: Within the cell, K was taken to be independent of pressure when measured at constant temperature interval from the lambda line, i.e., $K(T,P) = K(T - T_\lambda(P))$. In addition we assumed that $K(T - T_\lambda) = At^\delta$, where δ was determined from a fit to our data and A was determined at each datum point. To calculate A, we used an iterative procedure. The cell was divided into 100 horizontal slices and the temperature drop across each was calculated from the power input by use of an initial estimate for A. The total temperature drop was then compared with the observed ΔT, and A was adjusted until convergence was obtained. It was then a simple matter to calculate K for any element in the cell. When the variation of K across the cell was small, this procedure gave results that were insensitive to the value of δ. However, for $t < 3 \times 10^{-7}$, moderate sensitivity was found and it was necessary to use a value of δ appropriate for this region. The optimum value was determined from a plot of $K(t)$ close to the transition. For $10^{-7} < t < 2 \times 10^{-6}$ this correction procedure was checked by comparison of the computed conductivities of the surface layers on opposite sides of the cell. In the continuous mode two independent measurements give results for K at a given t that have different correction terms applied, one for each side of the cell. Agreement was obtained to within the noise level of the data. All results reported here have been obtained with no superfluid in the cell.

In Fig. 1 we show the results obtained with the two data-collection modes described above, on a logarithmic temperature scale. To facilitate the display of small effects in the data, the thermal-conductivity values are plotted as percentage deviations from a simple power-law function chosen to fit the data roughly over a broad range. Also shown for comparison are the results of Tam and Ahlers[3] (TA) for cell F, which had a vertical height of 0.203 cm. It can be seen that near $t = 10^{-4}$, where the combined uncertainty of the two experiments is lowest, our results are higher by about 2%. This discrepancy is within the uncertainties of the absolute values in the two experiments.[7] Closer to the transition larger discrepancies exist. We note that previous experiments lose accuracy in this region because of the large nonlinearities in the measurements induced by the use of power densities over 100 times larger than ours. Recent measurements by Dingus, Zhong, and Meyer[8] are also included in the figure and show somewhat better agreement in this region.

To calculate R_K we used the heat-capacity data[9] extending to $t \sim 5 \times 10^{-8}$ and the functional form for ξ given by TA. We also multiplied our data by a normalizing factor of 0.98 to allow a more direct comparison with those of TA. The results are shown in Fig. 2. The most significant feature of our measurements is the indication of a maximum in R_K near $t = 10^{-6}$. In this region the largest uncertainty in our results is due to the

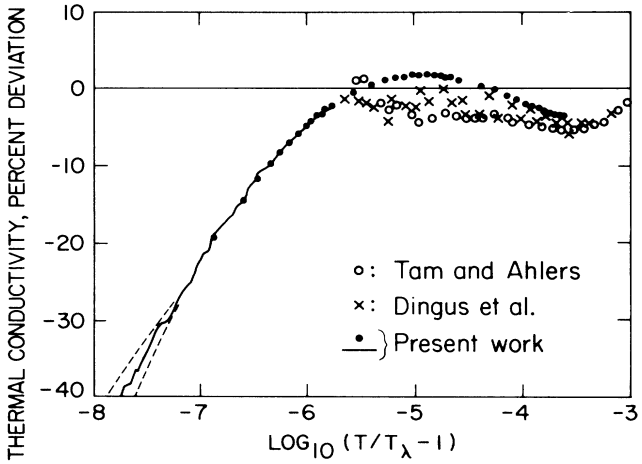

FIG. 1. Percentage deviations of thermal-conductivity data from the function $K = 1.38 \times 10^{-5} t^{-0.43}$ as a function of log$t$. Filled circles, our data from the static method; full line, our data from the continuous method; dashed line, range of uncertainties due to lambda point location; other symbols as shown.

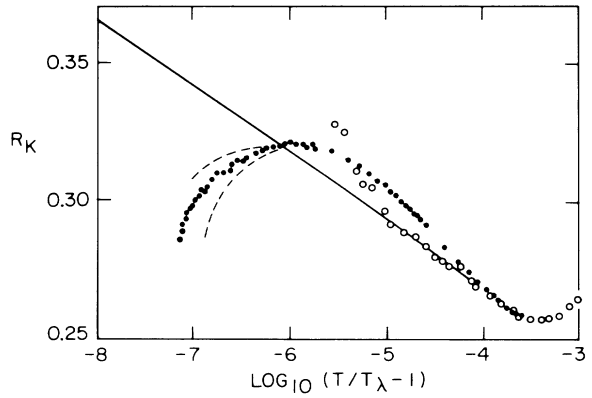

FIG. 2. Variation of the thermal-conductivity parameter R_K as a function of log$t$. Filled circles, our results; dashed line, effect of varying Kapitza boundary resistance term as described in text; open circles, results of TA; full line, fit of two-loop DRG to data of TA.

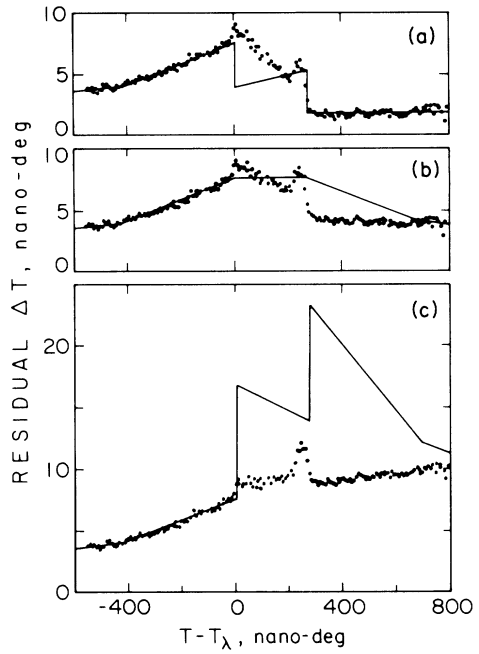

FIG. 3. Comparison of observed (circles) and calculated (lines) temperature residuals after subtraction of sample contribution. (a) With no boundary resistance anomaly above the transition; (b) with a symmetric anomaly on each side of the transition; and (c) with three times the anomaly above the transition. The temperature taken as T_λ is that where normal fluid first appears in the bottom of the cell.

possible existence of an anomaly in the Kapitza boundary resistance. We have been able to set limits on the extent of this potential anomaly as follows: First, we can easily observe the corresponding anomaly *below* the transition[10] simply by applying the measurement techniques described above. Next, we can calculate the expected temperature drop across the helium alone from our measured conductivities and a reasonable extrapolation to smaller values of t. Finally, we can compute the excess temperature drop across the cell from the ΔT data less the sample contribution. Clearly this can be done at any temperature including those in the gravitationally induced two-phase region (TPR). Since the derivation of the conductivity data involves an assumption about the boundary resistance, the above procedure should ideally lead to consistency, with the residuals just giving the input boundary resistance. Figure 3 shows the results with the use of three different assumptions about the behavior of the boundary resistance: (a) no anomaly above T_λ, (b) a symmetric anomaly, and (c) three times the anomaly above T_λ. The points show the calculated residuals and the lines the input boundary resistance. In each case a four-parameter function was fitted to the data for K to allow extraction of the residuals. Qualitatively, the results were found to be only weakly dependent on the choice of this function.[11] With model (a) we obtain excellent agreement above the TPR, indicating internal consistency, but within the TPR some deviations are seen.[12] Model (b), on the other hand, is poorly behaved above the TPR but reasonably good within the region. It can be seen that one feature that this model lacks is the ability to produce a sharp reduction of the residuals at the point where the upper plate of the cell reaches the transition temperature. Model (c) is clearly excluded. Over all, model (a) gives the best results, but the discrepancies in the TPR admit some possibility of a weaker anomaly above the transition. We conclude that (a) and (b) bracket the range of possible behavior with (a) favored. Since there is also the remote possibility that the regular term in the boundary resistance is less above the transition, the approximate extremes shown in Fig. 2 are these: upper dashed line, model (b); lower dashed line, one half of the boundary resistance above the transition with no anomaly.

In Fig. 2 we also show the expected behavior of R_K based on the fit by TA of two-loop DRG to their data extending to $t=10^{-2}$. Clearly for $t<3\times10^{-6}$ our results depart significantly from the extrapolated curve. TA found that throughout the region of our measurements an expansion parameter of the theory is close to unity. It is therefore not surprising that poor results are obtained here. However, we would expect R_K to continue its general monotonic trend as t is reduced, possibly to the fixed-point value of order unity estimated by TA. A maximum of $R_K=0.325\pm0.01$ is incompatible with this, implying that the behavior at small t is more complex than previously suspected.

In conclusion, it appears that our results for R_K show significant deviations from the predictions of two-loop DRG in the region very close to the transition, with the

unanticipated appearance of a maximum in this quantity. In this region the theory is not expected to be accurate, but a general continuation of the monotonic behavior seen further from the transition is presumed. An attempt[2] has been made to fit some previous data with a model including a phenomenological three-loop term. In some cases[4] a decrease in slope of R_K vs $\log(t)$ below $t \sim 10^{-6}$ was obtained, giving hope that the complete three-loop model will be able to explain the experimental behavior. Additional work in this area is urgently needed. To determine more clearly the temperature dependence of R_K very close to the transition, an accurate measurement of the boundary resistance above T_λ is essential.

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[6]The concentration of ^{3}He in the sample was approximately 1 part in 10^9.

[7]The absolute scale-factor uncertainty of our results is 2%, while at $t = 10^{-4}$ that of TA is 0.6 %.

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[10]R. V. Duncan, G. Ahlers, and V. Steinberg [Phys. Rev. Lett. **58**, 377 (1987)] report similar observations for a gold/helium interface.

[11]We assumed that $K = At^\delta$ where A and δ were smooth functions of $\log t$ derived from a fit to the data for $t < 2 \times 10^{-6}$. An earlier analysis with A and δ constant gave essentially the same results.

[12]There is some possibility that this effect is related to the presence of the normal-superfluid interface.