Testing Nucleon Distribution Amplitudes: Relations between Neutron and N-Δ Form Factors

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The proton and neutron magnetic form factors, the axial-vector form factor, and the leading N-Δ transition form factor are connected by the nucleon distribution amplitude. If the distribution-amplitude moments obey the QCD sum-rule results, a correlation exists between the neutron magnetic form factor G_{Mn} and the leading N-Δ transition form factor. For existing proposed distribution amplitudes, one or the other of them will be small and the other not. We comment on what available data say.

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Calculations of high-momentum-transfer exclusive processes[1,2] involve a process-independent "distribution amplitude," the light-cone quark wave function integrated over transverse momentum, and process-dependent but calculable[3-7] hard-scattering amplitudes. In this work, we point out a correlation between the nucleon magnetic form factor and the leading N-Δ transition form factor that can help determine the nucleon distribution amplitude and test the validity of perturbative QCD at a given Q^2; we consider QCD sum-rule constraints on distribution-amplitude moments and comment on their consequences; and we examine data on N-Δ electromagnetic transitions to check consistency with the different possibilities[8] for G_{Mn}/G_{Mp}.

Independently of the distribution amplitude, we can identify the leading amplitude for each process and state each amplitude's high-Q^2 scaling. For example, nucleon electromagnetic form factors F_1 (F_2) asymptotically fall like[3] $1/Q^4$ ($1/Q^6$) up to $\ln Q^2$ corrections, F_1 equals G_M at high Q^2, and N-Δ transition form factors behave like elastic nucleon form factors given parallel definitions.[5] The leading helicity amplitude preserves the hadrons' helicity.[9] Rewritten in terms of multipole amplitudes for N-Δ transitions, this means[5] that $F_{E2}/F_{M1}=\sqrt{3}$ at high Q^2.

Normalized calculations depend on the distribution amplitude. The latter are incalculable *ab initio*, although there exist the QCD-sum-rule constraints, and we must make plausible or flexible guesses.

The nucleon magnetic form factor G_{Mn} and the asymptotically leading N-Δ transition amplitude $G_{MN\Delta}$ are together sensitive adjudicators among the several proposed distribution amplitudes. ($G_{Mp\Delta^+}$ is a linear combination of the $E2$ and $M1$ multipole amplitudes; the M in $G_{Mp\Delta^+}$ emphasizes that the definition parallels G_{Mp} and is used although $G_{Mp\Delta^+}$ is not purely magnetic.) In particular, for asymmetric wave functions, the symmetric-antisymmetric interference term contributes to $G_{MN\Delta}$ with relative sign opposite to its contribution to G_{Mn} so that one of those two form factors tends to have a large magnitude when the other is small.

Explicitly, including the expressions for G_{Mp} and g_A, we have

$$\begin{aligned}
\tfrac{1}{2}\sqrt{2}Q^4G_{Mp\Delta^+}&=f\int[dx\,dy]\{\tfrac{2}{3}(T_1-T_2)\phi_\Delta(x)\phi_S(y)+\tfrac{1}{3}\sqrt{3}T_1[\phi_\Delta(x)\phi_A(y)+(x\rightleftarrows y)]\},\\
Q^4G_{Mn}&=f\int[dx\,dy]\{-\tfrac{2}{3}(T_1-T_2)\phi_S(x)\phi_S(y)+\tfrac{2}{3}\sqrt{3}T_1[\phi_S(x)\phi_A(y)+(x\rightleftarrows y)]+\tfrac{2}{3}(T_1-T_2)\phi_A(x)\phi_A(y)\},\\
Q^4G_{Mp}&=f\int[dx\,dy]\{2T_1\phi_S(x)\phi_S(y)-\tfrac{2}{3}\sqrt{3}T_1[\phi_S(x)\phi_A(y)+(x\rightleftarrows y)]+\tfrac{2}{3}(T_1+2T_2)\phi_A(x)\phi_A(y)\},\\
Q^4g_A&=f\int[dx\,dy]\{\tfrac{2}{3}(4T_1+T_2)\phi_S(x)\phi_S(y)-\tfrac{4}{3}\sqrt{3}T_1[\phi_S(x)\phi_A(y)+(x\rightleftarrows y)]-2T_2\phi_A(x)\phi_A(y)\},
\end{aligned}\tag{1}$$

where[1] ϕ_S and ϕ_A are the parts of the nucleon distribution amplitude that are respectively symmetric and antisymmetric under $x_1\rightleftarrows x_3$, with quarks 1 and 3 having parallel helicity, ϕ_Δ is the ($x_1\rightleftarrows x_3$ symmetric) distribution amplitude for the helicity-$\frac{1}{2}$ delta, and $f=(16\pi\alpha_s/9)^2$. We let ϕ_Δ and ϕ_S be the same. T_1 and T_2 come from the hard-scattering amplitude and are known.[1,3]

Two statements lead to a simple result connecting $G_{Mp\Delta}$ and G_{Mn}. First, the observed size of G_{Mp} implies a broad distribution amplitude.[10] In such a case, the T_1 terms dominate the T_2 terms. Also, the $\phi_A\phi_A$ terms are numerically small.[10] Hence the $T_1\phi_S\phi_S$ and $T_1\phi_S\phi_A$ terms dominate and

$$2\sqrt{2}G_{Mp\Delta^+}=3G_{Mp}+5G_{Mn}. \qquad (2)$$

Since distribution amplitudes which give a good G_{Mp} are of greatest interest, the above is effectively a relation between $G_{Mp\Delta^+}$ and G_{Mn}.

Table I gives form factors calculated for several distribution amplitudes, which, except the one labeled $\eta=0.6$, are given by their expansion in Appell polynomials,[1,3,11]

$$\begin{aligned}\phi_N&=120x_1x_2x_3\sum_{i=0}^{5}B_i\tilde{\phi}_i(x_1,x_2,x_3)\\&=\text{const}[\phi_S(x_1,x_2,x_3)-\sqrt{3}\phi_A(x_1,x_2,x_3)].\end{aligned} \qquad (3)$$

The notation[2] ϕ_N is normalized by $B_0=1$ or equivalently

$$\int[dx]\phi_N(x_1,x_2,x_3)=1. \qquad (4)$$

The ϕ_S and ϕ_A are normalized from the wave function,

$$\int[dx][d^2k_T][\,|\psi_S(x,k_T)|^2+|\psi_A(x,k_T)|^2]=P_{3q}, \qquad (5)$$

where P_{3q} is the probability for a three-quark proton Fock component and

$$\phi_{S,A}(x)=\int[d^2k_T]\psi_{S,A}(x,k_T). \qquad (6)$$

This defines the constant in Eq. (3). The coefficients B_i change with $\ln Q^2$ to some calculable power[1] (which is below $\frac{1}{2}$ for all polynomials we keep) but the polynomials do not mix; we take the B_i at some fixed Q^2.

Table I includes the Chernyak-Zhitnitsky amplitude,[2] four others that satisfy the QCD sum rules,[12] and the distribution amplitude (denoted GS) proposed by Gari and Stefanis[13] (GS). The tradeoff between G_{Mn} and $G_{MN\Delta}$ is visible in all these examples. Included for comparison is a totally symmetric amplitude

$$\phi_N(x)=K(x_1x_2x_3)^\eta \qquad (7)$$

with the power $\eta=0.6$. (This distribution amplitude does not satisfy the moment constraints and was normalized to give the G_{Mp} shown in Table I.)

The constraints already mentioned come from calculations of moments

$$M_{n_1n_2n_3}=\int[dx]x_1^{n_1}x_2^{n_2}x_3^{n_3}\phi_N(x_1,x_2,x_3), \qquad (8)$$

with use of QCD sum rules.[2,12,13] Some qualitative features are of interest here. One striking fact emerges from the well-determined $N=n_1+n_2+n_3=1$ moments. The (momentum-)space wave function of the quarks in the nucleon is asymmetric, and, at least for the integral that gives the distribution amplitude, the asymmetry is large. For the quadratic moments, the quoted[2] uncertainties mean that the coefficients of the quadratic Appell polynomials are not strongly constrained.[12] (The moments up to a given N determine the Appell-polynomial–expansion coefficients up to the same order.) The consequent range of G_{Mp} is large, as may be seen from examples 1–4 in Table I. (The larger G_{Mp}'s in this table match the data.) Remarkably, G_{Mn}/G_{Mp} is always about $-\frac{1}{2}$ if the moment constraints are satisfied. This result is marginally consistent with the data.

As regards data on G_{Mn}, the electron-neutron differential cross section σ_n is measured[14] at $Q^2=2.5$, 4, 6, 8, and 10 GeV^2 but only at one scattering angle, and so a separation of G_{Mn} and G_{En} is impossible. The data show

TABLE I. Form factors for some nucleon-distribution amplitudes. The Chernyak-Zhitnitsky (CZ) distribution amplitude and examples 1–4 satisfy the moment constraints from the QCD sum rules. The Gari-Stefanis (GS) distribution amplitude gives a small asymptotic G_{Mn}/G_{Mp}. The foregoing distribution amplitudes give a large $|G_{Mn}|$ (relative to G_{Mp}) for a small $G_{MN\Delta}$ and vice versa. B_i are coefficients for the Appell-polynomial expansion. Results from a totally symmetric distribution amplitude with power $\eta=0.6$ are given for comparison.

		Examples					
Distribution amplitude	CZ	1	2	3	4	GS	$\eta=0.6$
		Form factor (GeV^4)					
Q^4G_{Mp}	0.89	0.29	0.37	0.68	0.38	0.88	1.00
Q^4G_{MN}	−0.43	−0.13	−0.18	−0.34	−0.18	−0.09	−0.31
Q^4g_A	1.36	0.45	0.58	1.06	0.59	1.00	1.36
$Q^4G_{Mp\Delta^+}$	0.01	0.04	−0.02	−0.03	−0.02	0.72	0.43
		Coefficients B_i					
B_0	1	1	1	1	1	1	
B_1	4.3	3.9	3.9	4.0	3.9	4.1	
B_2	1.9	1.9	1.9	2.0	1.9	2.1	
B_3	2.3	1.3	1.2	1.9	1.2	−4.7	
B_4	−3.5	9.0	0.5	−5.0	0	5.0	
B_5	0	1.8	1.7	2.0	1.7	9.3	

σ_n/σ_p falling roughly like $1/Q^2$ between 5 and 10 $(\text{GeV})^2$ and the ratio is roughly $\frac{1}{4}$ at the higher Q^2 value. Recall that at small angles

$$\sigma_N \sim (4m_N^2/Q^2)G_{EN}^2 + G_{MN}^2 \tag{9}$$

for $N=n$ or p, and that G_{Ep} contributes little to σ_p at high Q^2. Then

$$|G_{Mn}/G_{Mp}| \le \tfrac{1}{2} \tag{10}$$

at Q^2 of 10 GeV^2. Possibly $Q^4|G_{Mn}|$ flattens out at 10 GeV^2 and is about $\frac{1}{2}$ of Q^4G_{Mp} thereafter. However, the falling of σ_n/σ_p suggests the possibility that G_{Mn} is asymptotically small. Then σ_n is dominated by G_{En} and a $1/Q^2$ falloff of σ_n/σ_p is naturally explained. This behavior was found in Ref. 8 by our combining the perturbative QCD predictions at high Q^2 with meson dynamics at low Q^2 to fit the electromagnetic form factors of the proton and neutron. A further feature of Ref. 8 is the near vanishing of the form factor F_{1n} at all Q^2 values.[15]

The GS amplitude, constructed by two of us,[12,13] gives a small asymptotic G_{Mn}/G_{Mp} and good agreement with the new high-Q^2 G_{Mp} data.[16]

Experimental information on $G_{MN\Delta}$ comes from[17] $e+N \rightarrow e+N+\pi$. There are data on the total cross section out to Q^2 of 6 Gev.[2] At 6 GeV^2, data for the resonance part of the cross section[17] give

$$|Q^4G_{Mp\Delta^+}| \le 0.6 \text{ GeV}^4; \tag{11}$$

the limit is less than but comparable to Q^4G_{MP} at the same Q^2. Data on the ratio F_{E2}/F_{M1} go out to 3 GeV^2. The ratio is compatible with 0 $(15\% \pm 15\%)$ at its highest measured Q^2. However, model studies for photoproduction[18] suggest that the background to the $E2$ amplitude is not small, and so the experimental result should be viewed carefully.

Consider two possibilities for $G_{MN\Delta}$.

(a) The asymptotic value of $Q^4G_{MN\Delta}$ is small and other form factors or soft contributions still dominate at $Q^2=6$ GeV^2. This favors the Chernyak-Zhitnitsky distribution amplitude and the "accidental zero" it gives for the leading N-Δ transition amplitude. (The zero is due to cancellations between symmetric and antisymmetric nucleon terms and is little affected by changes in the delta distribution amplitude.) This possibility also fits in with a small F_{E2}/F_{M1} since now the bulk of the result need not conserve hadron helicity. Incidentally, for the $D_{13}(1520)$, $F_{15}(1588)$, and $D_{33}(1670)$ resonances at Q^2 of a few square gigaelectronvolts there is evidence that the hadron-helicity–conserving amplitudes are larger and falling more slowly with Q^2 than the hadron-helicity–nonconserving ones.

(b) Asymptotically, $Q^4G_{MN\Delta}$ is close to 0.6 GeV^4. After all, its elastic-scattering analog G_{Mp} shows asymptotic behavior at 6 GeV^2 though not at 3 GeV^2, and one could expect that F_{E2}/F_{M1} will rise. This possibility disfavors distribution amplitudes that give $|G_{Mn}/G_{Mp}| \approx \frac{1}{2}$, but encourages use of distribution amplitudes that give smaller G_{Mn} and in particular it fits in with the possibility that $F_{1n} \approx 0$ at all Q^2.

To conclude:

(1) Assuming the applicability of perturbative QCD and of the QCD sum-rule results, we predict a correlation between G_{Mn} and $G_{Mp\Delta^+}$. If one of them is small, the other is not. If the correlation fails, at least one of the assumptions should be questioned.[19]

(2) We have not focused on g_A. It does not drop dramatically to 0 for some distribution amplitude as does G_{Mn} or $G_{Mp\Delta^+}$. Data[20] for g_A extrapolate to $Q^4g_A \approx 1.5$ GeV^4 at high Q^2, but with 50% uncertainty if we use only the highest-energy experiment. This fits with any distribution amplitude that gives a good G_{Mp}. A more accurate determination of g_A at high Q^2 would have consequences for our judging of distribution amplitudes.

(3) Data relevant to both G_{Mn} and $G_{Mp\Delta^+}$ exist and the means to improve them can be available. An experimental G_{Mn}-G_{En} separation appears feasible if G_{Mn} is small and G_{En} is big.[21] Better isolation of the $e+N \rightarrow e+\Delta$ part of $e+N \rightarrow e+N+\pi$ could be gotten from a manageable extension of existing theoretical modeling, and measurements of individual multipole amplitudes at higher Q^2 would be most interesting.

(4) We have a means of distinguishing among proposed nucleon distribution amplitudes because they differ in their predictions for G_{Mn} and $G_{Mp\Delta^+}$. A small $G_{MN\Delta}$ comes as an accidental zero if one uses the Chernyak-Zhitnitsky nucleon distribution amplitude and explains the up-until-now failure of the F_{E2}/F_{M1} prediction. On the other hand, the data can be seen as favoring distribution amplitudes that give a larger $G_{Mp\Delta^+}$ and small G_{Mn}. This is consistent with a vanishing neutron form factor F_{1n} and inconsistent with the idea that $G_{Mn} \approx -\frac{1}{2}G_{Mp}$ at high Q^2.

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