# New Value for the Rydberg Constant from the Hydrogen Balmer-$\beta$ Transition

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The Rydberg constant $R_\infty$ was determined to 3 parts in $10^{10}$ by direct comparison of the four H,D Balmer-$\beta$ transitions with a National Bureau of Standards standard laser. This is the most precise value: $R_\infty = 109\,737.315\,73(3)\ \mathrm{cm}^{-1}$; it approaches the limits of accuracy for optical measurements. The fine-structure splittings and isotope shift are in excellent agreement with theory. The result agrees with less precise experiments by Zhao *et al.* and Biraben and Julien, but disagrees with the result of Hildum *et al.*

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The Rydberg is now the most precisely measured fundamental physical constant. $R_\infty$ is basic to all calculations of atomic and molecular energy levels. It affects other fundamental constants via the least-squares adjustment, which it is taken as exact.

The meter is now defined exactly in terms of the speed of light $c \equiv 299\,792\,458$ m/sec[1]; wavelength and frequency are formally identical. The atomic second is defined in terms of the Cs clock: 1 sec $= 9\,192\,631\,770$ periods of the $^{133}$Cs ground-state hfs, to a precision in the radio-frequency region of several parts in $10^{14}$. In the optical region the standard of frequency/wavelength, the $^{127}I_2$ lines at 633 nm, is given to 1.6 parts in $10^{10}$.[2] Since hydrogen spectra occur in both regions, the Rydberg constant has the potential of tying together the frequency spectrum.

Quantum electrodynamics (QED) is one of the most important and precise of our modern theories. Balmer's formula is modified by the Dirac relativistic equation and by QED, in the Lamb shift $S$. Recent rf measurements of $S$ have been quoted to a few kilohertz.[3,4] Balmer's formula (plus Dirac and QED) should be tested with lasers to the same absolute precision, which amounts to measuring $R_\infty$ to a few parts in $10^{12}$. Such a measurement probes theoretical terms untestable by rf techniques, such as the Lamb shift in the ground state of H.

The present experiment is a step in that direction. It is an improvement on that of Amin, Caldwell, and Lichten[5] and Zhao *et al.*,[6] which are Balmer-$\alpha$ analogs of the classical Lamb-Retherford measurement of the Lamb shift.

A piezoelectrically scanned etalon had $\lambda/200$ curved ($R = 284.3 \pm 0.1$ cm) and flat mirrors at opposite ends. These mirrors were identical to those used by one of us in a previous experiment,[7] but were recoated to 94% reflectivity at both standard (633 nm) and unknown (486 nm) wavelengths. Unlike the previous case,[7] the Fresnel phase-shift errors were negligible because of the much shorter wavelength of Balmer $\beta$ (486 nm) compared to the He-Ne, $CH_3$ laser (3390 nm).[7]

The wavelength measurements were made by the method of virtual mirrors, which eliminated corrections due to optical phase shifts in the mirror coatings. The spacings were 12.34890, 82.03979 cm, which gave a virtual spacing of 69.69089 cm.

A tungsten oven was heated to 2850 K to dissociate molecular hydrogen. A beam of H (or D) atoms emerged from a slit of the oven to be bombarded by electrons and excited to the metastable $2^2S_{1/2}$, $F = I + \frac{1}{2}$ state, which lived long enough to reach a secondary-electron-emission detector. A chopped, retroflected, cw dye-laser beam at $\lambda = 486$ nm crossed the atomic beam at right angles and diminished the signal by exciting the $2^2S_{1/2}$ state to the $4P_{1/2}$ and $4P_{3/2}$ states. A cw He-Ne laser was offset from the standard laser with a variable frequency depending on an rf oscillator. Both offset-

TABLE I. Corrections and estimated errors, Rydberg constant (parts in $10^{10}$; average of all four H and D transitions).

| Effect | Correction | Error |
|---|---|---|
| 2$S$ hfs | 470.6 | ... |
| 4$P$ hfs | 0.0 | 0.5 |
| 2nd order Doppler | 4.3 | 0.9 |
| Photon recoil | −10.2 | ... |
| Refractive index | −0.2 | ... |
| Statistical | ... | 1.5 |
| Energy-level calculations | ... | 0.2 |
| Standard laser | ... | 1.6 |
| Stark effect, light shifts, and other minor effects | ... | 1.0 |
| Root mean square error | | 2.6 |
| Rounded to | | 3 |

TABLE II. Calculations for the Rydberg constant.

| Transition to | Measured λ $\lambda_m - 4860$ Å | Corr. ($10^{-10}$ Å) | Corrected λ $\lambda_c - 4860$ Å | Calculated[a] λ $\lambda_t - 4860$ Å | Rydberg constant $R - 109\,737$ cm$^{-1}$ |
|---|---|---|---|---|---|
| H, $4P_{1/2}$ | 2.656 050 9(5) | 721.0 | 2.655 700 3(5) | 2.655 714 94 | 0.315 770(12) |
| H, $4P_{3/2}$ | 2.645 229 8(13) | 702.8 | 2.644 888 1 (13) | 2.644 900 51 | 0.315 720(29) |
| H combined | | | | | 0.315 745(25) |
| D, $4P_{1/2}$ | 1.333 060 1(11) | 219.8 | 1.332 953 3(11) | 1.332 964 42 | 0.315 692(25) |
| D, $4P_{3/2}$ | 1.322 437(8) | 214.2 | 1.322 139 6(8) | 1.322 152 92 | 0.315 741(14) |
| D, combined | | | | | 0.315 717(25) |
| Grand average | | | | | 0.315 731(16) |

[a]References 3–6, 16, with $R_\infty = 109\,737.315\,44$ cm$^{-1}$, $1/\alpha = 137.035\,989\,5$, Lamb shift for $H = 1\,057.851\,4(19)$ MHz, for $D = 1\,059.235(27)$ MHz, $m_p/m_e = 1\,836.152\,701(37)$.

laser and dye-laser outputs were mode matched into the piezoelectrically modulated plano-concave etalon for wavelength comparison. A feedback loop locked the dye laser to an external cavity by means of the technique of Hänsch and Couillaud.[8] Another loop locked both to the measuring etalon and a further feedback loop locked the measuring etalon to the offset laser.

Thus the frequency locking chain was formed as follows: dye laser–external cavity–measuring etalon–offset laser–standard–$^{127}I_2$ peak. Data were taken by scanning of the oscillator frequency so that every link on the chain followed and the dye laser swept across the Balmer-$\beta$ transition line.

The Balmer-$\beta$ frequency $f$ is $6.17\times10^8$ MHz, and the experimental linewidth $\Delta f$ was 23 MHz (D) and 28 MHz (H), which gave a resolution of $f/\Delta f \approx 2.5\times10^7$, a threefold improvement over the previous value for Balmer $\alpha$.[5,6] Although two-photon measurements in positronium[9] and H($1s$-$2s$)[10–13] may have better resolution, the present experiment had higher precision because of lower systematic errors and direct use of the primary standard of length. This standard was identical to that used by Jennings *et al.* to establish the current definition of the meter.[2] As a further check on the possibility of a foreign-gas shift in the iodine cell, its Hanle effect at 502 nm was measured by Z. Zhou after the technique of Marx and Rowley.[14]

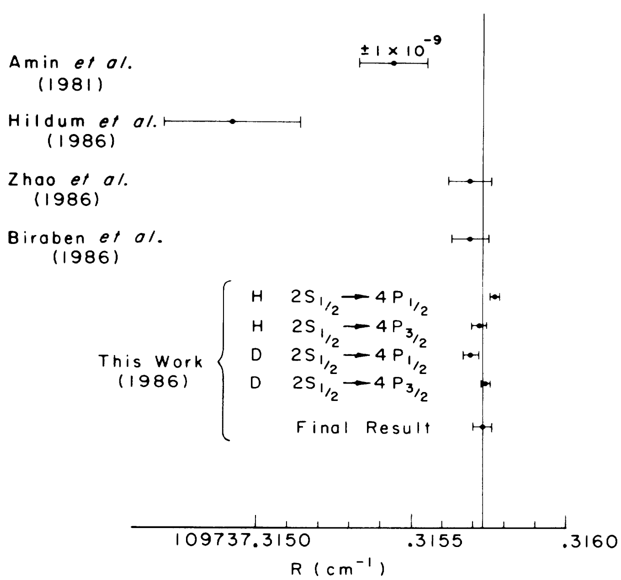


FIG. 1. Comparison of recent measurements of the Rydberg constant.

The atomic-beam collimation ratio was 1400 for most of the data taken. Typical operating conditions were 50% quenching of the metastables, signal-to-noise ratio of 1000/1 with a 3-sec time constant, detector current of 0.14 pA, noise 0.12 fA, $6\times10^7$ metastable atoms/sec, and one-way laser power 30 mW/cm$^2$. The transition region was shielded magnetically (∼20 mG or less) and electrostatically (∼1 mV/cm or less). No corrections were believed necessary for Zeeman, dc or ac Stark, or polarization effects.

The theoretical isotope shifts and fine-structure separations for the H,D Balmer-$\beta$ lines were used to find the correct integral order of the mirrors by the method of exact fractions.[15] During a scan across the line, a simultaneous record of offset frequency and signal strength was taken digitally. The observed lines were symmetric and had a width and line shape that agreed with theoretical calculations.

TABLE III. Comparison with recent measurements for $R$.

| Reference | $R_\infty - 109\,737$ cm$^{-1}$ |
|---|---|
| Amin, Caldwell, and Lichten[a] | 0.315 44 ± 0.000 11 |
| Hildum *et al.*[b] | 0.314 92 ± 0.000 22 |
| Zhao *et al.*[c] | 0.315 69 ± 0.000 07 |
| Biraben and co-workers[d] | 0.315 69 ± 0.000 06 |
| Present result | 0.315 73 ± 0.000 03 |

[a]Reference 5. [b]Reference 13. [c]Reference 6. [d]Reference 11.

TABLE IV. Fine structure splittings and isotope shift.

| | Theory [a] | This work |
|---|---|---|
| | Fine structure $4P_{1/2} \leftrightarrow 4P_{3/2}$ | |
| H | 1371.1294(1) | 1370.9(3) |
| D | 1371.5036(1) | 1371.8(3) |
| | Isotope shift H $\leftrightarrow$ D | |
| $2S_{1/2} \leftrightarrow 4P_{1/2}$ | 167752.791(2) | 167752.4(3) |
| $2S_{1/2} \leftrightarrow 4P_{3/2}$ | 167753.165(2) | 167753.3(3) |
| Weighted avg. | 167753.040(2) | 167753.0(3) |

[a]From Reference 16.

Table I lists the systematic corrections and sources of error for the Rydberg constant. Table II gives the measured wavelengths and the calculated Rydberg constant for infinite mass: $R_\infty = 109\,737.31573(3)$ cm$^{-1}$, with the meter defined such that $c$ is exactly 299792458 m/sec, and the "$i$" peak of $^{127}I_2$ measured to have a frequency $\nu(^3\mathrm{He}\text{-}^{20}\mathrm{Ne}, ^{127}\mathrm{I}_2, i) = 473\,612\,214.789(74)$ MHz (1.6 parts in $10^{10}$).[2]

The error is one standard deviation ($1\sigma$). By the more conservative Comité Consultatif pour la Définition du Mètre error[1] for the wavelength standard of 3.4 parts in $10^{10}$, our error changes to 4 parts in $10^{10}$ at the $1\sigma$ level.

Figure 1 and Table III compare recent measurement of $R_\infty$. Our result is in excellent agreement with our most recent and reliable Balmer-$\alpha$ measurement (Zhao *et al.*)[6] and with Biraben and co-workers,[11] but highly significantly disagrees with the result of Hildum *et al.*[13]

Table IV compares measured fine-structure and isotope shifts with theory.[16] The agreement is excellent.

A more detaile discussion is given elsewhere.[6]

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