Constraints on Anomalous Scattering of Neutrinos from Crystals

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We present a general sum rule in the Born approximation governing neutrino scattering from local potentials in a crystal. This relation facilitates the placing of stringent bounds on the force exerted on a crystal by a flux of neutrinos. The force is found to depend linearly on the number of scattering centers and no exotic coherent effects are predicted.

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The recent suggestion by Weber[1] that the cross section for finite-angle scattering of neutrinos from crystals can be proportional to N^2, the square of the number of scattering centers, even for neutrinos with energies on the order of 1 MeV, is indeed intriguing. In fact, the results of several experiments have been presented which are claimed to support this hypothesis.[1,2] There have, however, been several arguments given against the existence of such phenomena.[3,4]

Casella[4] has made a detailed calculation which resulted in the corresponding force on the crystal being proportional to N rather than N^2 except for neutrinos with h/p of order of the crystal dimensions. The purpose of this paper is to present a general sum rule for the scattering of neutrinos from an arbitrary aggregate of scattering centers via local potentials.[5] This sum rule implies a rigorous inequality which can be used to place an upper bound on the force of a "ν wind" on a crystal. We also comment on other possible consequences of such anomalous interactions should they exist.

Let us consider the scattering of a weakly interacting particle from a system of N local, nonoverlapping stationary potentials at coordinates $\mathbf{r}_n$. We shall now derive the following sum rule:

$$\int dE\, d\Omega (1-\cos\theta)\frac{d\Sigma(E,\Omega)}{d\Omega} = \sum_{n=1}^{N}\int dE\, d\Omega (1-\cos\theta)\frac{d\sigma_n(E,\Omega)}{d\Omega}, \quad (1)$$

which relates the differential cross section of the composite system, $d\Sigma/d\Omega$, to that of the individual scattering centers, $d\sigma/d\Omega$, when both are calculated in the Born approximation.

The nonrelativistic amplitude for scattering from the composite system is given by

$$F(\mathbf{p}_i,\mathbf{p}_f) = -\frac{\mu}{2\pi}\int d^3r\, V(\mathbf{r})e^{i\mathbf{q}\cdot\mathbf{r}} \equiv -\frac{\mu}{2\pi}\tilde{V}(\mathbf{q}), \quad (2)$$

where μ and $\mathbf{q}$ are the mass and momentum transfer of the incident particle, respectively, and the tilde indicates the Fourier transform. Parseval's theorem relating the scattering potential of the system, $V(r)$, and its transform $\tilde{V}(\mathbf{q})$ is expressed as follows:

$$\int \frac{d^3q}{(2\pi)^3}|\tilde{V}(\mathbf{q})|^2 = \int d^3r\, |V(\mathbf{r})|^2, \quad (3)$$

where

$$V(\mathbf{r}) = \sum_{n=1}^{N} v_n(\mathbf{r}-\mathbf{r}_n). \quad (4)$$

The fact that the potentials at different sites do not overlap implies that $v_n(\mathbf{r}-\mathbf{r}_n)v_m(\mathbf{r}-\mathbf{r}_m) = \delta_{mn}v_n^2$, which, when substituted into Eq. (3), yields

$$\int \frac{d^3q}{(2\pi)^3}|\tilde{V}(\mathbf{q})|^2 = \sum_{n=1}^{N}\int d^3r\, |v_n(\mathbf{r}-\mathbf{r}_n)|^2. \quad (5)$$

Parseval's relation can also be applied to the individual potentials yielding an expression analogous to Eq. (3),

$$(2\pi)^3\int d^3r\, |v_n(\mathbf{r})|^2 = \int d^3q\, |\tilde{v}_n(\mathbf{q})|^2, \quad (6)$$

which, when substituted into Eq. (5), gives

$$\int d^3q\, |\tilde{V}(\mathbf{q})|^2 = \sum_{n=1}^{N}\int d^3q\, |\tilde{v}_n(\mathbf{q})|^2. \quad (7)$$

The scattering cross section of the composite system and of the individual potentials are proportional to $|\tilde{V}(\mathbf{q})|^2$ and $|\tilde{v}(\mathbf{q})|^2$, respectively, so that $|\tilde{V}(\mathbf{q})|^2 \propto d\Sigma/d\Omega$ and $|\tilde{v}_n(q)|^2 \propto d\sigma_n/d\Omega$. We also find it convenient to change variables $q_x = p\sin\theta\cos\phi$, $q_y = p\sin\theta\sin\phi$, and $q_z = -p(1-\cos\theta)$ so that $d^3q = p^2 dp\, d\Omega(1-\cos\theta)$.

Equation (7) can then be written as

$$\int_0^\infty p^2dp\int d\Omega(1-\cos\theta)\frac{d\Sigma}{d\Omega}=\sum_{n=1}^{N}\int_0^\infty p^2dp\int d\Omega(1-\cos\theta)\frac{d\sigma_n}{d\Omega}, \quad (8)$$

$$\int_0^\infty dE\,d\Omega(1-\cos\theta)\frac{d\Sigma(E,\Omega)}{d\Omega}=N\int_0^\infty dE\,d\Omega(1-\cos\theta)\frac{d\sigma(E,\Omega)}{d\Omega}. \quad (9)$$

At sufficiently high incident ν energy, Λ, the integrands in Eq. (9) become identical because the fraction of atoms recoiling coherently becomes negligible. The contributions to both integrals for $E>\Lambda$ are identical; hence, we obtain the following useful relation:

$$\int_0^\Lambda dE\,d\Omega(1-\cos\theta)\frac{d\Sigma(E,\Omega)}{d\Omega}=N\int_0^\Lambda dE\,d\Omega(1-\cos\theta)\frac{d\sigma}{d\Omega}(E,\Omega). \quad (10)$$

An interesting feature of Eq. (10) is that the integrated quantity $(1-\cos\theta)d\Sigma/d\Omega$ is proportional to N and not N^2. It is true, nevertheless, that $d\Sigma/d\Omega\propto N^2$ for very small scattering angles; however, the small integration measure, as well as the extra factor $(1-\cos\theta)\cong\theta^2/2$, vitiate the N^2 contributions. Also, we will discover below that the magnitude of the total z component of the force exerted on the crystal by neutrinos contains the same factor $(1-\cos\theta)$.

To obtain an expression for the force, we consider a ν flux, $\phi(E)$, incident on the crystal in the z direction. We can express the force as follows:

$$F_z=\int_0^{E_0}dE\,\Phi(E)\int d\Omega\left[\frac{E}{c}(1-\cos\theta)\right]\frac{d\Sigma(E,\Omega)}{d\Omega}, \quad (11)$$

where $(E/c)(1-\cos\theta)$ is the momentum transfer to the crystal by a neutrino of energy E when it elastically scatters through an angle θ, and E_0 is the end-point energy of the neutrino spectrum. In the case of reactor neutrinos, the quantity $E\phi(E)$ has a maximum value for $E=2$ MeV; hence, we evaluate it at this energy.[6] This yields the following useful inequality:

$$F_z\lesssim\left\{\frac{E}{c}\Phi(E)\right\}_{\max}\int_0^{E_0}dE\,d\Omega(1-\cos\theta)\frac{d\Sigma(E,\Omega)}{d\Omega}. \quad (12)$$

In Eq. (10), we can choose Λ as high as we please; hence, if we simply choose $\Lambda>E_0$, for the moment, we can use Eq. (10) with Eq. (12) to obtain

$$F_z\le\left\{\frac{E}{c}\Phi(E)\right\}_{\max}N\int_0^{\Lambda}dE\,d\Omega(1-\cos\theta)\frac{d\sigma(E,\theta)}{d\Omega}. \quad (13)$$

Let us now evaluate this expression for a 100-g crystal of Al_2O_3 ($N=3.3\times10^{24}$) in a flux of reactor neutrinos of 5×10^{11} ν/cm^2 sec. Using a calculated reactor spectrum,[6] we find $N\{(E/c)\Phi(E)\}_{\max}=1.8\times10^{25}$ ν cm^{-3}. For $d\sigma_n(E,\theta)/d\Omega$, we use the expression given by Drukier and Stodolsky[7] which can be reexpressed as

$$\frac{d\sigma}{d\Omega}\cong3.34\times10^{-46}(A-Z)^2E^2(1+\cos\theta)\ \mathrm{cm}^2. \quad (14)$$

We obtain an effective value $(A-Z)^2\simeq117$ and, performing the trivial integrations in Eq. (13), we find $F_z<3\times10^{-22}\Lambda^3$ dyn when Λ is in megaelectronvolts.

Since we are dealing with a sum rule, Λ must be chosen high enough to ensure that for $E>\Lambda$ an insignificant fraction of the scattering events result in coherent scattering. We require that the momentum transfer $\mathbf{q}$ correspond to a length scale not much smaller than the dimensions of the crystal. A straightforward calculation, using Eq. (14) and $q_z=(E/c)(1-\cos\theta)$, yields a simple expression for the fraction of scattering events which will be coherent, namely, $f=\theta_0^2$ when E is large and hence θ is small, and where $\theta_0\equiv\arccos\{1-cq/E\}$. We see then that $\Lambda=10$ MeV is a very conservative cutoff energy. In this case, $F_z<3\times10^{-19}$ dyn, which is a factor of 10^{-14} smaller than the experimental result given in Ref. 1. We note as an aside that even if we chose $\Lambda=1$ GeV, we find $F_z<3\times10^{-13}$ dyn. At such high energies, even coherent scattering from nucleons is lost other than in the forward direction.

It is worth emphasizing that the present discussion differs completely from previous investigations[8,9] of the pressure due to the 3-K (wavelength of 1 cm) background neutrinos which *do* scatter coherently from macroscopic grains. The speculation refuted in Refs. 8 and 9 was that the pressure is $O(G_F)$ rather than $O(G_F^2)$.

In the above discussion, the quantum character of the scatterers was completely neglected. We now address the question of possible enhancement effects due to this aspect. Let us consider an extreme case in which such effects are maximal, but one which is very unrealistic for neutrino scattering. We will see that under the most optimistic conditions with slow, heavy "neutrinos," no unexpected enhancements occur; hence, *a fortiori,* none should occur for ordinary neutrinos.

Let us consider an analogy in which slow massive neutrinos ($10^{-4}<\beta<10^{-3}$), interact via vector Z^0 exchange, impinging on a macroscopic superconductor. The electron pairs are all in the same macroscopic quantum state and they move rapidly compared to the neutrino transit time. In this case, an adiabatic Born-Oppenheimer approximation applies, and the incident neutrino experiences an average effective potential V_{eff} extending over the entire lattice, rather than experiencing the potentials of individual electrons. We illustrate this point by representing the individual potentials by square wells of width $a \equiv 10^{-16}$ cm $= M_Z^{-1}$, and depth $V_0 = M g_w^2 \simeq 10$ GeV. The average effective potential has a depth $\bar{V} = V_0 a^3/d^3 \cong V_0 \times 10^{-24} \cong 10^{-13}$ eV, where $d \cong 10^{-8}$ cm is the average interatomic distance. While all neutrinos will experience $\bar{V}$, its effect is completely negligible compared with the individual kinetic energies, $\frac{1}{2} mc^2\beta^2 \cong 10^{-4}$–$10^{-3}$ eV, for $m \cong 1$ keV. No significant scattering is expected.

All of the above arguments notwithstanding, it is still interesting to ask if there are not some dramatic effects which should have already been observed if Weber's results were indeed correct. In this case, the macroscopic cross section $\Sigma \propto N^2$ and $\Sigma \cong 1$ cm^2 for a crystal with dimensions $L \cong 1$ cm. In smaller crystals, the macroscopic cross section is much smaller. For a crystal of dimension L, $\Sigma_L \propto L^6$ and the mean free path for a neutrino is $(n\Sigma_L)^{-1} \cong L^{-3}$. In polycrystalline materials comprised of single crystals of $L = 1.0$, 0.1, 0.01, and 0.001 cm, for example, the mean free paths are 1 cm, 10 m, 10 km, and 10^4 km, respectively. This is in sharp contrast to the ordinary case in which $\Sigma \propto N$, and the mean free path is not dependent on how the macroscopic sample is subdivided. It seems then that one would have to understand at least the size distribution of crystalline materials in the Earth's crust and mantle in order to interpret solar-neutrino experiments. The N^2 nature of the cross sections could possibly lead to strong diurnal variations in the detection rates, an effect which could only be observed in a new generation of direct-counting solar-neutrino experiments.[10] It might then be remotely possible that the distribution of crystal sizes conspires to reduce the expected count rate in Davis's ^{37}Cl experiment by a factor of 3.

If this were the case, however, the total solar energy absorbed by the Earth would increase by $\sim 20\%$ over that due to photon absorption alone. One is tempted to wonder whether the present understanding of the Earth's temperature due to solar photons and terrestrial radioactivity is complete enough to exclude such a revision; the change in the overall temperature due to solar neutrinos would be $\cong 15\,°$C.

One final amusing thought concerns the possible effect of the pressure exerted by the solar ν wind on experiments of the Dicke-Braginski type.[11,12] These measurements verified that various objects experience the same solar gravitational acceleration to within one part in 10^{12}. The ν-wind forces in such experiments, on a 100-g single crystal and on other amorphous mixture of equal mass, would differ by approximately 10^{-6} dyn. The measured solar gravitational attraction would be 60 dyn, but with an uncertainty of only 6×10^{-11} dyn. It seems that such experiments would be best suited for testing Weber's hypothesis as well as his experimental results.

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[5]It is straightforward to show that when calculating first-order elastic scattering amplitudes the scatterers can be replaced by potentials. We demonstrate this with N scatterers in harmonic-oscillator ground states $\exp[-(\mathbf{r}_j - \mathbf{R}_j)^2/2\Delta^2]$, a neutrino plane wave $e^{i\mathbf{k}\cdot\mathbf{r}_0}$, via the interaction $\lambda\delta(\mathbf{r}_0 - \mathbf{r}_j)$. The corresponding first-order scattering amplitude is proportional to the expression

$$\int d\mathbf{r}_0 e^{i(\mathbf{k}_i - \mathbf{k}_f)\cdot\mathbf{r}_0} \int d\mathbf{r}_1 \cdots d\mathbf{r}_N \left[\prod_{j=1}^{N} e^{-(\mathbf{r}_j - \mathbf{R}_j)^2/2\Delta^2} \right] \sum_k \lambda\delta(\mathbf{r}_0 - \mathbf{r}_k) \prod_{m=1}^{N} e^{-(\mathbf{r}_m - \mathbf{R}_m)^2/2\Delta^2}.$$

The integrals in the square brackets are trivial, and the above expression becomes

$$\int d\mathbf{r}_0 e^{i(\mathbf{k}_i - \mathbf{k}_f)\cdot\mathbf{r}_0} \sum_{j=1}^{N} \lambda e^{-(\mathbf{r}_0 - \mathbf{R}_j)^2/\Delta^2}.$$

This is tantamount to our replacing the particles with Gaussian potentials. Obviously, this general result does not depend on the fact that we approximated the quantum states of the scatterers by harmonic-oscillator states nor on the form of the interaction used.

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