Test for Relativistic Gravitational Effects on Charged Particles

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Experimental results are presented which provide the first measurement of the effects of a gravitational field on charged particles, equivalent to the red shift for photons. Two Josephson-effect batteries ($V \simeq 300\ \mu$V) having a vertical separation of 7.2 cm are connected in opposition by superconducting wires. A voltage difference of 2.35×10^{-21} V is maintained between these batteries by means of the gravitational red shift. The emf around this loop is, however, measured to be less than 1×10^{-22} V, consistent with the predicted invariance of the gravito-electrochemical potential along the wires.

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The strong equivalence principle, which states that locally the effects of a uniform gravitational field are indistinguishable from those due to an accelerating reference frame, is a fundamental postulate of general relativity. A consequence of this equivalence, along with special relativity, is that the rates of identical clocks running in different gravitational potentials will differ. This variation in clock rates implies the gravitational red shift of photons, which was first observed by Pound and Rebka[1] and has, during the past two decades, been verified by a number of workers[2] to a present accuracy of better than 1 part in 10^4. There have, however, previously been no experimental tests of the corresponding effects for charged particles.

One such effect occurs for the conduction electrons in a metal, such as a superconductor, in a gravitational field where, in equilibrium, the electrochemical potential μ will in general not be constant. Rather, a related quantity $\tilde{\mu} = \mu(1+\lambda)$ called the gravito-electrochemical potential will be constant along the wire.[3-6] Here $\lambda = zg/c^2$ and depends on the gravitational potential through the height z; g is the gravitational acceleration at the earth's surface and c the speed of light in a vacuum, giving $\lambda/z = 1.09 \times 10^{-16}$/m. For a resistanceless metal such as a superconductor, $\tilde{\mu}$ should also be independent of position even with a dc current flowing.[6] Thus a circuit with two identical batteries (as measured by local observers) of emf V, separated in height by z and connected in opposition by superconducting wires, would have a net emf $\Delta V = \lambda V$ around the zero-resistance loop leading to a loop current increasing linearly in time.[4]

Within a single wire a purely Newtonian gravitational field can produce a number of effects which shift the electrochemical potential. For instance, the gravitational force on the electrons must be balanced by an electrostatic force giving rise to an electric field (the Schiff-Barnhill field[7]). This latter effect is in fact contained in the potential $\tilde{\mu}$ if the electron rest mass is included, since $\lambda mc^2 = mgz$. An electric field can also be produced because of the shift in the Fermi level induced by lattice distortion due to the gravitational force on the ions.[8] These gravitational effects can be 10 to 15 orders of magnitude greater[9] than the relativistic shift in potential difference discussed above. However, they do not contribute to the net emf around a loop.

In this Letter we report the results[10] of a null experiment to compare the fractional change with height of the electrochemical potential difference between two superconducting wires to the fractional change in photon frequency over the same height, which is given by $\Delta\nu/\nu = \lambda$. The schematic of the experiment is shown in Fig. 1. The "batteries" used to provide the highly precise reference emf's are Josephson junctions separated vertically by a height $z = 7.2$ cm and phase locked to a common external microwave source. The relation between the frequency ν of the external radiation and the dc electrochemical potential difference across the junction, $V \equiv \Delta\mu$, is given by the Josephson equation as

$$V = n\nu\Phi_0, \tag{1}$$

where the integer n is the order of the radiation-induced step in the junction I-V curve. Since the radiation reaching the upper junction is red shifted with respect to

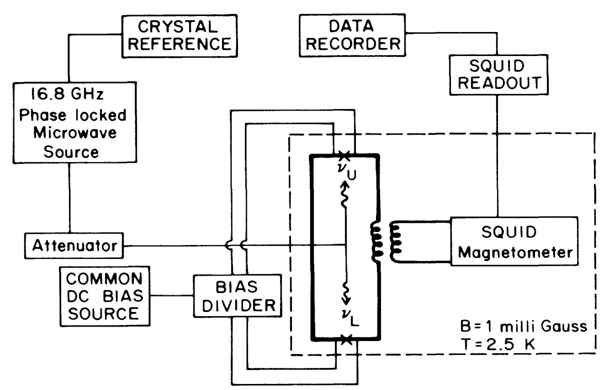

FIG. 1. Schematic of the measurement circuit. The crosses denote the Josephson batteries. The darker lines indicate superconducting portions of the circuit.

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that reaching the lower junction, the potential differences across the upper and lower junctions, V_u and V_l respectively, will differ slightly so that

$$V_u = V_l(1-\lambda). \tag{2}$$

In the absence of additional relativistic effects on the Cooper pairs, one would thus expect the net emf ΔV in the loop containing the junctions to be $\lambda V \simeq 2\times 10^{-21}$ V, since $V \simeq 300\ \mu$V and $\lambda \simeq 7\times 10^{-18}$ for this experiment. Such a nonzero value for ΔV would lead, as discussed below, to a time-varying flux through the loop. This could be detected by use of a superconducting quantum interference device (SQUID) magnetometer, as shown in Fig. 1. The relativistic prediction that $\tilde{\mu}=$const in the superconducting leads, however, implies that the potential difference V between the leads varies with height so that $V(z)=V(0)(1-\lambda_q)$. The symbol λ_q, which in theory equals λ, has been introduced to allow for the possibility of corrections to the predicted relativistic effects on charged particles. The loop emf is thus $\Delta V = V_l \times (\lambda - \lambda_q)$ and is predicted to be zero on the basis of the strong equivalence principle.

The basic technique for the detection of extremely small voltage differences between two Josephson junctions by monitoring of the flux change in a superconducting loop was developed[11] shortly after the discovery of the Josephson effect. The apparatus used for these measurements is a refined version of that previously used by us to demonstrate the universality of Eq. (1) for different types of Josephson junctions[12] to 2 parts in 10^{16}. The Josephson junctions used here are lead-alloy tunnel junctions with critical currents of $I_c \simeq 900\ \mu$A. The junctions are shunted with low-inductance 0.5-Ω resistors to ensure nonhysteretic operation and have I-V characteristics that are extremely close to those predicted for ideal resistively shunted junctions (RSJ model). The junctions and superconducting leads connecting them are placed over a niobium ground plane giving a total loop inductance of $L \simeq 1$ nH, associated primarily with the coupling inductor to the magnetometer. The microwave radiation is coupled to the junctions by our placing each junction at the end of a capacitively grounded microstrip which is in turn coupled to a coaxial cable through a small section of coplanar waveguide. In order to minimize any extraneous shifts in the relative phases of the radiation reaching the two junctions, these two coaxial cables are connected, just outside the sample cell and halfway between the junctions, to a common cable leading out of the cryostat.

In general Josephson junctions will phase lock to applied radiation over a range of bias current $\Delta I_i = J_i \sin(\delta_i - \pi/2)$, where the maximum current variation—or locking range—is dependent on microwave power. Here, $\delta_i \equiv \theta_i - n\omega_i t$ ($\omega_i = 2\pi\nu_i$) is the difference between the linear component θ_i of the phase of the Josephson oscillations in the ith junction and the phase of the applied radiation at the junction.[13] At a constant gravitational potential the time-averaged fluxoid quantization condition for the loop of inductance L as shown in Fig. 1 is then[14]

$$(\Phi_0/2\pi)(\theta_2-\theta_1)+LI_l+\Phi_a = m\Phi_0, \tag{3}$$

where Φ_a is the flux applied to the loop and $I_l = (\Delta I_2 - \Delta I_1)/2$. For simplicity, identical junctions and locking ranges for 1 and 2 have been assumed.

The emf which drives the changes in the loop current I_l is, from Eq. (3),

$$\frac{\Phi_0}{2\pi}\frac{d}{dt}(\delta_2-\delta_1)+\frac{\Phi_0}{2\pi}n\,\Delta\omega+\frac{d\Phi_a}{dt},$$

where $\Delta\omega \equiv \omega_2-\omega_1$. For the SQUID's used in these measurements $\beta_l' \equiv 2\pi LJ/\Phi_0 \gg 1$; thus the terms in δ can be neglected, giving

$$-L\frac{dI_l}{dt} \simeq \frac{d\Phi_a}{dt}+\frac{\Phi_0}{2\pi}n\,\Delta\omega.$$

The response of this phase-locked system to a small frequency difference $\Delta\omega$, as described by Eq. (3), is the same as the response to an applied flux varying linearly with time for the system biased below the junction critical current in the absence of radiation. For example, if Φ_a is changed (or $\Delta\omega \neq 0$), a loop current will be induced. As long as $I_l < J$, m will remain constant and I_l will increase (nearly) linearly with Φ_a (or time) until $I_l \simeq J$, then m will change and I_l will decay in steps of approximately Φ_0/L as flux quanta enter the loop. In the linear region, for $I_l \ll J$, the rate of change in the loop current is related to $\Delta\omega$ (for constant Φ_a) by

$$-L\frac{dI_l}{dt}l \simeq \Delta V \simeq \frac{\Delta\omega}{\omega}V \tag{4}$$

with the junction voltage $V = n\nu\Phi_0$.

It is important to note from this discussion that, within the constraint of fluxoid quantization, it is possible for the two junctions to have slightly different average voltages while in a fixed fluxoid state, phase locked to the applied radiation. This result, contained in Eq. (4), has previously been demonstrated with a modified version of the apparatus in which separate coaxial cables extended from each junction to outside the cryostat.[12] This permitted the creation of a small frequency difference by Doppler shifting of the radiation going to one of the junctions and served to demonstrate that the sensitivity of the apparatus is indeed correctly predicted by Eq. (4).

The sensitivity limit of the apparatus at a loop emf of 1×10^{-22} V over a typical 10-h run implies that the change in the loop flux Φ due to any extraneous sources must be less than $2\times 10^{-4}\Phi_0$/h. To provide shielding against such small changes in the ambient flux Φ_a, Mu-metal shielding was used to reduce the ambient field to about 1 mG. To stabilize the remaining flux the superconducting loop was enclosed in a superconducting NbTi box which was placed inside a large lead can. The temperature of the box was regulated to $\pm 50\ \mu K$ to prevent thermally induced flux motion. In addition both junctions were fabricated on sapphire substrates to minimize any thermal gradients across the junction and prevent a thermoelectric current from being induced in the loop. Deliberate temperature shifts of 1 mK caused changes in Φ of less than of $1\times 10^{-4}\Phi_0$ even during temperature transients.

Additional factors which could cause ΔI_1 or ΔI_2 to change and generate a loop current I_l are changes in the microwave power or frequency or in the junction bias currents I_i. Since there are common sources of microwave and bias current for both junctions, the changes mentioned above, except for the gravitational red shift in frequency, will affect the junctions equally. For identical junctions these extraneous effects would then produce equal changes in ΔI_1 and ΔI_2 causing no change in I_l. Even with the small asymmetries present in the actual apparatus, it has proven possible—as detailed below—to reduce the effects of variations in the bias current and microwave sources to negligible levels.

Current from the common dc bias source was divided between the junctions so that to first order a change in the source current produced no change in I_l. The drift in the current source of 1 part in 10^4 during a run corresponded to an apparent voltage of about 10^{-23} V. The frequency drift of the microwave source, which was phase locked to a quartz-crystal oscillator, was 1 kHz/d. This resulted in a coherent drift in the voltages of the two Josephson batteries of about a picovolt over the course of the measurement. However, the symmetric configuration, with equal microwave path lengths to the two junctions, reduced the flux change through the loop because of this coherent drift to a rate of $4.2\times 10^{-5}\Phi_0$/d, corresponding to an apparent loop emf of less than 10^{-24} V. The power output of the microwave source was regulated to about 2 parts in 10^3 by use of a leveling loop. However, the power reaching the junctions varied by about 1.5% during the measurement because of a change in the attenuation of the coaxial cable as the helium level in the cryostat dropped. This variation, which was the largest source of sample-related flux drift, produced an estimated change in Φ of $5\times 10^{-4}\Phi_0$ over 12 h to give an apparent loop emf of 0.4×10^{-22} V.

The loop flux was monitored for continuous 10-h periods with use of a commercial rf SQUID magnetometer located in the helium bath. The magnetometer output Φ_m (referred to the sample loop) versus time is shown in Fig. 2, curve *a*, for the loop junctions phase locked on their ninth-order steps to 16.8-GHz radiation. This gives a junction voltage of $V \simeq 300 \mu V$, and a difference between the two junction voltages, as a result of the gravitational red shift, of 2.35×10^{-21} V. Φ_m is essentially time independent except near the end of the run when the helium level is near the rf SQUID. This

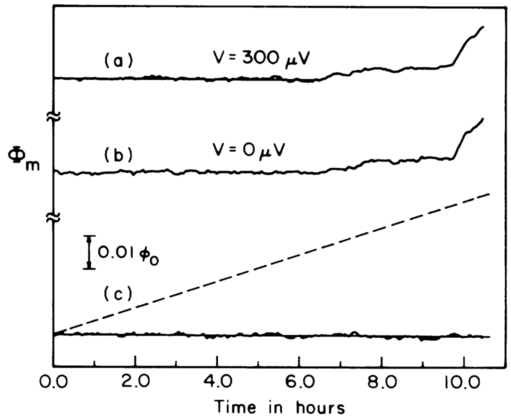

FIG. 2. Curve *a*, Φ_m vs time with the junctions phase locked to 16.8-GHz radiation biased at 300 μV on the ninth-order step. Curve *b*, Φ_m vs time with the junctions biased on the zeroth step. Curve *c*, $\Phi_m(V=300\ \mu V)-\Phi_m(V=0\ \mu V)$ vs time. The dashed line shows the expected signal if μ rather than $\tilde{\mu}$ remained constant along the superconducting wire. The solid lines in *a* and *c* are the respective best linear fits as described in the text.

dependence of Φ_m on time is highly reproducible from run to run. To check that the residual signal is due to the rf SQUID, the measurement was repeated with the junction bias current reduced so that the junctions were on their zeroth step (i.e., $V=0\ \mu V$). The resulting variation of Φ_m with time, shown in Fig. 2, curve *b*, is essentially identical to that in Fig. 2, curve *a*, indicating that there is no measurable change in the flux Φ through the sample loop during the run. The maximum flux change through the sample loop as a result of relativistic effects is just the difference between curves *a* and *b*, i.e., $\Phi_m(V=300\ \mu V)-\Phi_m(V=0\ \mu V)$ as shown in Fig. 2, curve *c*.

A linear-regression analysis of the data in Fig. 2, curve *c*, taking into account the presence of $1/f$ noise in the SQUID output, gives for the magnitude and error limits (2σ) of the loop voltage $(3\pm 6)\times 10^{-23}$ V. Nearly the same limits are obtained from the data in Fig. 2, curve *a*, if the final 4 h of data at low helium level are discarded. In this case we obtain $(1\pm 8)\times 10^{-23}$ V.

We thus conclude that the net emf in a superconducting loop containing two Josephson-effect batteries at different gravitational potentials is not the difference in the battery electrochemical potentials, $\Delta V=\lambda V$, but rather is indistinguishable from zero, consistent with the relativistic prediction that $\tilde{\mu}$ rather than μ is constant along the superconducting wires. In particular we have tested the hypothesis that the potential difference between the wires varies with height as $V(z)=V(0)\times(1-\lambda_q)$. Our results imply that $\lambda/\lambda_q=1\pm 0.04$, as compared with the predicted equality of λ and λ_q.

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