Evidence for Partial Occupancy of the $3s_{1/2}$ Proton Orbit in ^{208}Pb

E. N. M. Quint,(1) B. M. Barnett,(4) A. M. van den Berg,(1,3) J. F. J. van den Brand,(1) H. Clement,(4) R. Ent,(1,2) B. Frois,(6) D. Goutte,(6) P. Grabmayr,(4) J. W. A. den Herder,(1) E. Jans,(1) G. J. Kramer,(1) J. B. J. M. Lanen,(1,3) L. Lapikás,(1) H. Nann,(5) G. van der Steenhoven,(1,2) G. J. Wagner,(4) and P. K. A. de Witt Huberts(1)

(1)*Nationaal Instituut voor Kernfysica en Hoge-Energiefysica–Sektion K, 1009 AJ Amsterdam, The Netherlands*
(2)*Free University, Amsterdam, The Netherlands*
(3)*Fysisch Laboratorium, Rijksuniversiteit Utrecht, 3508 TA Utrecht, The Netherlands*
(4)*Physikalisches Institut, D-7400 Tübingen, Federal Republic of Germany*
(5)*Indiana University Cyclotron Facility, Bloomington, Indiana 47405*
(6)*Département de Physique Nucléaire et de Hautes Energies, Centre d'Etudes Nucléaires de Saclay, 91191 Gif-sur-Yvette Cedex, France*

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The pronounced $l=0$ selectivity of the $(e,e'p)$ proton knockout reaction has been exploited to determine the relative occupation of the $3s_{1/2}$ proton orbit in ^{205}Tl and ^{206}Pb. This value, when combined with previous electron scattering data, enables us to extract in a largely model-independent fashion the absolute $3s_{1/2}$ occupation probability in ^{208}Pb. The depletion of the $3s_{1/2}$ proton orbit is found to be $(18 \pm 9)\%$, compatible with predictions of many-body theories.

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In the mean-field approximation to the nuclear many-body problem it is assumed that shells are occupied up to and empty above the Fermi level (ε_F). This model is fairly successful in the description of ground-state charge densities and in providing the basis for shell-model calculations of nuclear dynamics at low excitation energy. However, the basic assumption of closed shells is an approximation only. One should add dynamical corrections to the static mean field in order to account for nucleon-nucleon correlations. These correlations are predicted to smear out the Fermi surface and thus induce a depletion of states below ε_F. Theoretical estimates of correlations[1-3] predict a typical depletion of (20–30)% of valence orbits in ^{208}Pb, such as the $3s_{1/2}$ orbit. Since the $3s_{1/2}$ orbit contributes about 50% to the charge density at the center of ^{208}Pb, such a depletion would explain the observed discrepancy[4] of the mean-field predictions with the charge distribution. In addition the empirical observation of quenching and missing strength in various electromagnetic observables[5] may in part be understood in terms of partial occupancy. However, no sufficiently accurate experimental information on the $3s_{1/2}$ occupation probability in ^{208}Pb has yet been obtained.

In this paper we discuss a high-resolution proton knockout experiment on ^{206}Pb and ^{205}Tl that addresses the question of the occupancy of the $3s_{1/2}$ proton orbit in the lead region. In order to avoid the difficulties in obtaining an absolute occupation probability directly from the $(e,e'p)$ spectral function, a sum-rule method is used, in which only relative spectroscopic factors are needed. Using the well-known fact that the number of $3s_{1/2}$ protons $n(A)$ in system A equals the spectroscopic strength $S_f(A)$ for $3s_{1/2}$ removal summed over final states (f) in the $(A-1)$ system, we can write

$$n(A)-n(A-1)=\sum_f S_f(A)-\sum_f S_f(A-1). \qquad (1)$$

For the nuclides ^{205}Tl ($J^\pi=\frac{1}{2}^+$) and ^{206}Pb the spectroscopic strength for $3s_{1/2}$ proton removal can be related[6] to the number z of $3s_{1/2}$ protons contributing to the charge-density difference $\Delta\rho=\rho(206)-\rho(205)$. One then has $z=n(206)-n(205)$ or, equivalently,

$$n(206)=z/[1-\sum_f S_f(205)/\sum_f S_f(206)]. \qquad (2)$$

Therefore both $n(206)$ and $n(205)$ can be determined for a given value of z by measurement of the ratio of spectroscopic strengths. With the ratio $n(208)/n(206)$ from a previous $(e,e'p)$ experiment,[7] $n(208)$ can also be determined. From electron scattering studies at Saclay[8] it has been found that $z=0.7$ is compatible with the charge-form-factor data. We note that this sum-rule analysis of combined $(e,e'p)$ and (e,e') data avoids the difficulty that the interpretation of z in terms of absolute $3s_{1/2}$ occupancies depends on the amount of configuration mixing.

The experimental method has been described in Ref. 7. The present measurement of the reactions ^{205}Tl$(e,e'p)$ and ^{206}Pb$(e,e'p)$ was performed at an incoming electron energy of 410 MeV and the kinematical conditions were chosen such that the missing momentum (p_m) was centered at 15, 80, and 160 MeV/c.

In the reaction ^{205}Tl$(e,e'p)^{204}$Hg at $p_m=15$ MeV/c three excited states in ^{204}Hg were identified at $E_x=1.64(1)$, 2.37(2), and 2.62(3) MeV (see Fig. 1). The p_m distributions for the transitions to these states show the same p_m dependence as the transition to the

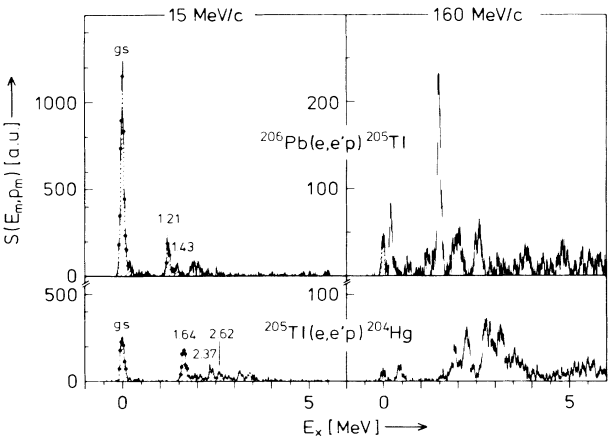

FIG. 1. The experimental spectral function vs excitation energy for ^{206}Pb and ^{205}Tl at low and high values of the missing momentum. The states due to $3s_{1/2}$ proton knockout have been crosshatched.

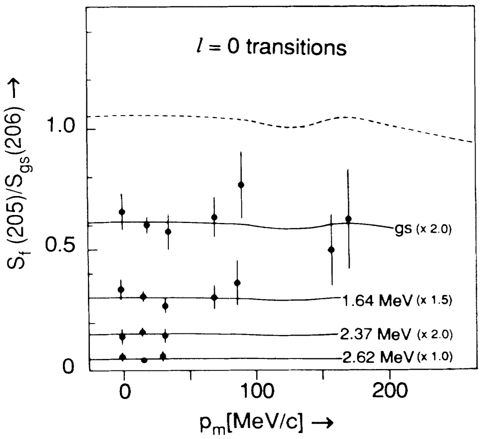

FIG. 2. Ratio of the $3s_{1/2}$ strength between the transition to the ground and excited states in ^{204}Hg and the transition to the ground state of ^{205}Tl. The dashed curve corresponds to equal spectroscopic factors for the two isotones. The solid curves are scaled to fit the experimental ratios.

ground state (see Fig. 2). These transitions are therefore due to $3s_{1/2}$ proton knockout. In the simultaneously measured reaction ^{206}Pb$(e,e'p)^{205}$Tl the fragment of $3s_{1/2}$ strength found in the transition to the ^{205}Tl ground state relative to the total observed strength was found to be 0.80(3). Averaging this with the previously determined[7] value 0.78(4) yields the value of 0.79(2). Apart from the ground-state transitions and the transitions to the two known $\frac{1}{2}^+$ states in ^{205}Tl at 1.21(1) and 1.43(2) MeV and to the three excited states in ^{204}Hg, no transitions with a $3s_{1/2}$ character were observed. The resulting spectroscopic strengths relative to that of the ground-state transition in ^{206}Pb are listed in Table I. The ratio of the summed strengths found in the individual transitions in the two isotones is

$$\textstyle\sum_f S_f(205)/\sum_f S_f(206) = 0.49(4).$$

Assuming for the moment that the sum exhausts the $3s_{1/2}$ strength, one can deduce the occupation numbers by substituting the summed ratio in Eq. (2). By adopting the value $z = 0.7$ given in Ref. 8, we obtain $n(206) = 1.37(10)$ and $n(205) = 0.67(10)$. In the quoted errors the systematical uncertainties were added linearly to the statistical errors, but no estimate of the uncertainty in z was included. With the ratio $\sum_f S_f(206)/S_{gs}(208) = 0.83(7)$ for the lead isotopes ^{206}Pb and ^{208}Pb determined in Ref. 7, one obtains the $3s_{1/2}$ occupancy of ^{208}Pb as $n(208) = 1.65(18)$.

Inherent to the above analysis is the assumption that the identified discrete transitions represent all $3s_{1/2}$ strength. We have checked this assumption by carrying out a multipole decomposition[7] of the spectral function up to 5.5 MeV (see Fig. 3). The ratio of the $l = 0$ strength integrated up to 5.5 MeV is $\int S(205)/\int S(206) = 0.46(5)$, in agreement with the ratio of the summed strengths mentioned above. A similar agreement has been observed[7] for the lead isotopes, where we found $\int S(206)/\int S(208) = 0.84(11)$. Obviously the l-decomposition method yields less accurate results, because it lacks the additional information on the number of discrete transitions. Since there is no evidence from the energy distribution of $3s_{1/2}$ strength (see Fig. 3) that we have missed any $l = 0$ transition of sizable strength, we base our final result $n(208) = 1.65(18)$ on the peak-fitting method. Hence the present analysis yields an occupation probability of 82(9)% for the $3s_{1/2}$ orbit in ^{208}Pb.

TABLE I. $3s_{1/2}$ strength in ^{205}Tl and ^{206}Pb relative to the ground-state transition for ^{206}Pb. Only statistical errors are given. The 4% error in the denominator of $S_f/S_{gs}(206)$ has not been included, since it cancels in the further analysis. The spectroscopic factors (S) for the $3s_{1/2}$ strength in ^{206}Pb and ^{205}Tl have been deduced with Eq. (2) (see text) with use of $z = 0.7$ (Ref. 8). The systematic error from this experiment on the spectroscopic factors for ^{205}Tl is 6% and for ^{206}Pb 3%; no contribution from the error in the value of z has been included.

E_x (MeV)	$S_f/S_{\mathrm{g.s.}}(206)$	S_f
	^{205}Tl$(e,e'p)^{204}$Hg	
0.00	0.29(1)	0.32(3)
1.64(1)	0.20(1)	0.22(2)
2.37(2)	0.08(1)	0.08(1)
2.62(3)	0.05(1)	0.05(1)
Total	0.61(2)	0.67(6)
	^{206}Pb$(e,e'p)^{205}$Tl	
0.0	1.00(4)	1.10(5)
1.21(1)	0.18(2)	0.20(2)
1.43(2)	0.07(1)	0.08(1)
Total	1.25(4)	1.37(6)

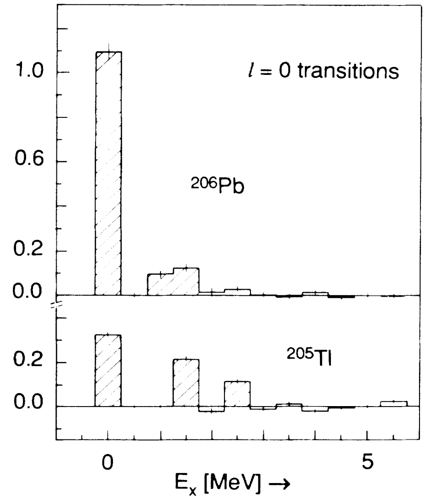

FIG. 3. Fragmentation of the $3s_{1/2}$ proton-hole strength in ^{205}Tl and ^{204}Hg. The absolute scale of S has been calibrated by use of $z=0.7$ (Ref. 8).

The question arises as to how much $3s_{1/2}$ strength could possibly reside at excitation energies higher than 5.5 MeV. First, the single-particle spreading width for orbits near the Fermi surface is expected to be only a few megaelectronvolts.[2,9] Second, the fragmentation of single-particle states into the continuum at high excitation energies has been calculated with Jastrow correlations.[10] The result is that for all nuclei involved only very small amounts (about 5%) of $3s_{1/2}$ strength are fragmented above 5.5 MeV. To investigate this we have applied the following method. At low missing momentum the observed strength is predominantly $l=0$ with a small contamination of $l=2$ ($2d$) strength. This allows the determination of the difference D in $3s_{1/2}$ strength between the two isotones relative to the ground-state transition from ^{206}Pb to ^{205}Tl by a direct bin-by-bin subtraction of the two spectral functions:

$$D(E_x)=\frac{\int_0^{E_x} S(206)-(1-\varepsilon)\int_0^{E_x} S(205)}{S_{\text{g.s.}}(206)}. \tag{3}$$

Here ε is a small correction ($\approx 5\%$) due to the difference in separation energies of the two nuclei, which makes both the $2d$ and $3s_{1/2}$ momentum densities near $p_m=15$ MeV/c about 5% larger for Tl than for Pb (see, e.g., the dashed curve in Fig. 2 for the $3s_{1/2}$ wave function). The subtraction procedure of Eq. (3) would yield an exact result if ^{205}Tl and ^{206}Pb contained the same number of $2d$ protons. Even if this were not the case the error would still be less than 4% if we assume the extreme of one $2d$ proton for the difference, since at $p_m=15$ MeV/c the momentum density for one $3s_{1/2}$ proton is, according to the distorted-wave impulse approximation calculations, a factor of 30 larger than that for one $2d$ proton. Applying the subtraction (3) to the present data we find $D(E_x=5\text{ MeV})=0.67(4)$ and $D(E_x=15\text{ MeV})=0.67(6)$. This is an indication that there is no, or equal, $3s_{1/2}$ strength fragmented between excitation energies of 5 and 15 MeV in ^{205}Tl and ^{206}Pb.

The same method has been applied to the data of an $(e,e'p)$ experiment[7] involving the lead isotopes ^{206}Pb and ^{208}Pb. Here $D(E_x=15\text{ MeV})$, evaluated relative to $S_{\text{g.s.}}(208)$, is 19(8)%, which is in excellent agreement with the observed difference[7] 17(7)% for the discrete transitions alone.

We conclude that there is no significant difference in $3s_{1/2}$ strength at excitation energies from 5 up to 15 MeV between the three nuclei involved in the present analysis. As a consequence the observed difference between the summed $3s_{1/2}$ knockout strength up to $E_x=5.5$ MeV is equal to z and *absolute* spectroscopic factors for the individual transitions can be deduced in the described way.

A possible source of error in our analysis is the question of whether the $3s_{1/2}$ bound-state wave function for a transition from the ground state in ^{205}Tl to an excited state in ^{204}Hg should be calculated while keeping either the radius of the potential well, or the rms radius of the wave function, constant. In this analysis we have chosen for a constant rms radius, an assumption supported by results of a recent $(e,e'p)$ experiment.[11] If the analysis is done with a fixed-well geometry the value for $n(208)$ increases from 1.65(18) to 1.70(18).

We emphasize that all $3s_{1/2}$ occupation probabilities deduced from the present sum-rule analysis scale linearly with the value of z as obtained from the absolute elastic electron scattering form factors of ^{206}Pb and ^{205}Tl. In the framework of the mean-field theory the obtained value of $z=0.7$ is almost independent of the effective interaction employed in the calculations.[8] Further theoretical work which includes dynamical correlations in self-consistent calculations is needed in order to delineate the possible limitations of the present interpretation in the framework of mean-field theory.

Combining the results from the present relative experiment on ^{206}Pb and ^{205}Tl with the experimentally determined $3s_{1/2}$ component in the charge-density difference of these isotones,[8] we conclude that these nuclei have a 68(5)% occupancy for the $3s_{1/2}$ proton orbit relative to the sum-rule prediction. Using this number and the $3s_{1/2}$ occupation ratio of ^{206}Pb and ^{208}Pb deduced previously,[7] we obtain an 82(9)% occupation probability of the $3s_{1/2}$ orbit in ^{208}Pb. It should be noted that these occupancies remove to a large extent the discrepancy between the measured[4,8] charge densities for ^{205}Tl, ^{206}Pb, and ^{208}Pb and mean-field calculations.[12] When a partial occupancy of the $3s_{1/2}$ orbit is introduced in Hartree-Fock calculations, the required reduction of the charge density in the nuclear interior is indeed obtained. Depletions of a similar magnitude have also been invoked in order to explain the systematic quenching of high-spin magnetic and electric transition form factors for nuclei in the Pb region.[5]

The derived 82(9)% occupancy of the $3s_{1/2}$ orbit of the doubly magic nucleus ^{208}Pb is compatible with the occupation probability of 0.92(13) deduced with a similar sum-rule method from $(d,{}^3\mathrm{He})$ experiments.[13] Note that both errors given here do not account for the uncertainty in z. These results support the predictions from many-body theory that the occupancy for orbits near the Fermi surface in ^{208}Pb is 70–80%, when random-phase approximation and short-range correlations are included.[2,14–16] Separate (e,e') and $(e,e'p)$ studies yield indirect information on the depletion of the $3s_{1/2}$ proton orbit in ^{208}Pb; the combined, largely model-independent analysis presented here has given direct evidence for a depletion that is compatible with theoretical predictions.

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[1] G. E. Brown, J. H. Gunn, and P. Gould, Nucl. Phys. **46**, 598 (1963).

[2] V. R. Pandharipande, C. N. Papanicolas, and J. Wambach, Phys. Rev. Lett. **53**, 1133 (1984).

[3] M. Jaminon, C. Mahaux, and H. Ngo, Nucl. Phys. **A440**, 228 (1985).

[4] B. Frois *et al.*, Phys. Rev. Lett. **38**, 457 (1977).

[5] C. N. Papanicolas, *Nuclear Structure at High Spin, Excitation, and Momentum Transfer—1985,* edited by Hermann Nann, AIP Conference Proceedings No. 142 (American Institute of Physics, New York, 1986), p. 110.

[6] P. Grabmayr *et al.*, Phys. Lett. **164B**, 15 (1985).

[7] E. N. M. Quint *et al.*, Phys. Rev. Lett. **57**, 186 (1986).

[8] J. M. Cavedon *et al.*, Phys. Rev. Lett. **49**, 978 (1982); B. Frois *et al.*, Nucl. Phys. **A396**, 409c (1983).

[9] C. Mahaux *et al.*, Phys. Rep. **120**, 1 (1985).

[10] M. C. Birse and C. F. Clement, Nucl. Phys. **A351**, 112 (1981).

[11] J. W. A. den Herder *et al.*, Phys. Rev. Lett. **57**, 1843 (1986).

[12] J. Dechargé and D. Gogny, Phys. Rev. C **21**, 1568 (1980).

[13] H. Clement *et al.*, Phys. Lett. **183B**, 127 (1987).

[14] D. Gogny, as quoted by J. Dechargé and L. Sips, Nucl. Phys. **A407**, 1 (1983).

[15] Z. Y. Ma and J. Wambach, Nucl. Phys. **A402**, 275 (1983).

[16] C. Mahaux and H. Ngo, Nucl. Phys. **A431**, 486 (1984).