New Test of General Relativity: Measurement of de Sitter Geodetic Precession Rate for Lunar Perigee

Bruno Bertotti[(a)]
Institute of Astronomy, University of Cambridge, Cambridge, United Kingdom

Ignazio Ciufolini
Center for Space Research, University of Texas at Austin, Austin, Texas 78712

and

Peter L. Bender[(b)]
Joint Institute for Laboratory Astrophysics, University of Colorado and National Bureau of Standards, Boulder, Colorado 80309
(Received 12 January 1987)

According to general relativity, the calculated rate of motion of lunar perigee should include a contribution of 19.2 msec/yr from geodetic precession. We show that existing analyses of lunar-laser-ranging data confirm the general-relativistic rate for geodetic precession with respect to the planetary dynamical frame. In addition, the comparison of Earth-rotation results from lunar laser ranging and from very long-baseline interferometry (VLBI) shows that the relative drift of the planetary dynamical frame and the extragalactic VLBI reference frame is small. The estimated accuracy is about 10%.

PACS numbers: 04.80.+z, 95.10.Jk, 96.20.−n

General relativity predicts that a gyroscope attached to a test body orbiting around the sun precesses with respect to the distant standard of rest—distant galaxies—with the (prograde) angular velocity

$$\mathbf{\Omega}_G = -\tfrac{3}{2}\mathbf{v}\times\mathbf{r}M_\odot/r^3, \tag{1}$$

where $\mathbf{v}$ is the velocity of the body, $\mathbf{r}$ the radius vector from the sun, and $M_\odot$ the gravitational radius of the sun.[1,2] This is called the geodetic (or de Sitter) precession. On the basis of the theory of Fermi transport[3,4] the spin vector $\mathbf{S}$ of a gyroscope moves according to the formula

$$d\mathbf{S}/dt = \mathbf{\Omega}\times\mathbf{S}. \tag{2}$$

For a general post-Newtonian metric and for a body which moves with proper (i.e., nongravitational) acceleration $\mathbf{a}$, Eq. (1) generalizes to[5-7]

$$\mathbf{\Omega} = -\tfrac{1}{2}\mathbf{v}\times\mathbf{a} + (\gamma+\tfrac{1}{2})\mathbf{v}\times\nabla U; \tag{1$'$}$$

here U is the Newtonian potential. The first term is present also when $U=0$ and space-time is flat; it describes the Thomas precession, well known in special relativity. The Lense-Thirring effect[8-10] and geodetic precession are the main examples of the peculiar (and Machian!) influence of mass motions on the inertial frames, and have never been tested.

A space experiment to measure the relativistic precessions is in preparation in the United States.[11] It involves flying superconductive gyroscopes on a low Earth satellite. The rotation axes are measured with respect to a stellar reference system. The experiment is based on the principle of equivalence, according to which the dynamical laws governing a system of laboratory size are those of ordinary electromagnetism and special relativity; hence the gyroscope behavior determines parallel transported directions along a world line.

We shall use here instead a gravitationally bound system, the earth-moon system, small with respect to the rest of the planetary system (and we shall use the world "local" in this sense). Similarly to the case of laboratory experiments, the motion due only to interactions within the system determines a privileged local frame; for example, the frame of ideal Keplerian motion. In this frame one can expect that the gravitational effects of the sun and of planets other than the Earth are essentially reduced to their tidal forces and show up only through the curvature tensor. One can say that this is a generalized principle of equivalence. Because of the nonlinear character of gravitation, however, this is by no means a trivial statement.

The work by Ashby and Bertotti[12,13] is essentially a proof, in a suitable approximation, of this principle. They extend Fermi's construction of a local inertial frame to the neighborhood of the (gravitating) Earth. A local metric is obtained in which the Earth—whose center is at rest—appears solely through its own Schwarzschild solution to the appropriate order and its corrections due to higher harmonics and its rotational gravitomagnetic field. The sun shows up in the tidal terms, with very small relativistic corrections, and in a minute nonlinear interaction with the Earth. Both these corrections are negligible to the level required in the present paper. They also compute explicitly the coordinate transformation from the usual, post-Newtonian (PN) frame, in which the center of gravity of the solar system is at rest, to this "Ptolemaic" frame. In an expansion with respect to the distance r from the earth, it turns out, as expected, that the linear part of this coordinate transformation corresponds to geodetic precession; in other words, the spatial base vectors $\mathbf{\Lambda}_{(i)}$ of the new frame precess with the angular velocity $\mathbf{\Omega}_G$ with respect to the old "Copernican," post-Newtonian frame; there is no Thomas precession. In the new frame the dynamics of the Earth and its satellites are governed by the usual, classical laws, corrected with the locally generated relativistic effects, if needed. These are the advance of the perigee and the Lense-Thirring precession due to the Earth's angular momentum. The Lense-Thirring effect for the moon is much smaller than the geodetic precession.

If we introduce the angular momentum per unit mass, $\mathbf{L}=\mathbf{r}\times\mathbf{v}$, of the Earth in its solar orbit, Eq. (1) shows that $\mathbf{\Omega}_G$ is directed along the pole of the ecliptic, in the direction of $\mathbf{L}$. $\mathbf{\Omega}_G$ has a constant part

$$\Omega_0=\tfrac{3}{2}\,\omega_\odot M_\odot/a_\oplus=19.2\text{ msec/yr}, \tag{3}$$

and a correction with a 1-yr period due to the eccentricity $e_\oplus$ of the Earth's orbit,

$$\Omega_1\cos\omega_\odot t=\tfrac{9}{2}\,(\omega_\odot M_\odot e_\oplus/a_\oplus)\cos\omega_\odot t$$
$$=0.96\cos\omega_\odot t\text{ msec/yr}. \tag{4}$$

$\omega_\odot$ is the sidereal frequency and t is reckoned from a perihelion passage; $a_\oplus$ is the semimajor axis of the Earth.

If geodetic precession were not present, the sidereal mean motion of the moon and the lunar perigee and node rates would all be changed by the same amount. de Sitter, in his classical paper of 1916,[1] predicted this effect for the node and the perigee and found that at that time the observations were not accurate enough. In principle we could determine the geodetic precession rate by comparing in the PN frame the observed and calculated rates for any of these motions. We choose to consider the 8.85-yr-period sidereal rate of perigee motion, for which the nonrelativistic part of the calculated rate is determined with high accuracy from the observed mean motions of the moon and Earth and other known quantities. The observed rate of perigee motion $\dot{\tilde{\omega}}$ minus the mean motion rate $\dot{L}'$ for the sun can be determined accurately from variations in the Earth-moon distance.

Currently, numerically integrated ephemerides for the moon are calculated with the use of programs developed at the Jet Propulsion Laboratory and Massachusetts Institute of Technology which include all known general relativistic effects of significant magnitude.[14,15] A recent lunar-ephemeris fit to lunar-laser-range observations from August 1969 to July 1986 is described by Newhall *et al.*[16] The lunar motion is tied in to the PN planetary dynamical frame[17] via other kinds of solar-system observations, including particularly microwave tracking data for the Viking landers on Mars. The mean motion of the Earth with respect to the planetary dynamical frame (PN) is essentially determined by its motion with respect to the perihelion of Mars plus the calculated perihelion precession rate for Mars. Since the general relativistic contribution to the perihelion precession rate for Mercury has been verified to 0.5% accuracy[15] and the corresponding rate for Mars is much less, the uncertainty in the rate is small. The overall uncertainty in the Earth's mean motion, or equivalently in the mean motion of the sun $\dot{L}'$, is 0.25 msec/yr.[18]

The accuracy of the observed value of $\dot{\tilde{\omega}}-\dot{L}'$ can be determined from how well the lunar-range data fit the calculated emphemeris. We consider the quantity

$$\dot{l}-\dot{D}=\dot{L}'-\dot{\tilde{\omega}}, \tag{5}$$

where L is the lunar mean longitude, $l\equiv L-\tilde{\omega}$, and $D\equiv L-L'$. From the leading terms in the known expressions[19,20] for the partial derivatives of the lunar range with respect to l and D, we can find the change in range $\Delta\rho$ which would correspond to perturbations $\Delta(\dot{l}-\dot{D})$ and $\Delta(\dot{l}+\dot{D})$:

$$\Delta\rho\simeq at\{[0.0272\sin l+0.0144\sin(l-2D)-0.0077\sin 2D]\Delta(\dot{l}-\dot{D})$$
$$+[0.0272\sin l-0.0048\sin(l-2D)+0.0077\sin 2D]\Delta(\dot{l}+\dot{D})\}. \tag{6}$$

Here a is the lunar semimajor axis, t is the time, and the periods of the three periodic terms are 27.55, 31.81, and 14.77 d. Thus the signature of an error in $\dot{l}-\dot{D}$ is an oscillating error in range which has an envelope increasing linearly in time. We have not been able to find any other parameter in the lunar problem which would give error terms that mimic the $t\sin(l-2D)$ and $t\sin(2D)$ terms from Eq. (6).

The present Jet Propulsion Laboratory lunar ephemeris fits the range normal points for the McDonald Observatory from the end of 1975 to early 1982 with an rms residual of about 18 cm.[21] Similarly, the rms residuals for the McDonald and Haleakala Observatories since December 1985 are about 6 cm.[21] We have calculated the maximum value of $\Delta(\dot{l}-\dot{D})$ which would give no more than twice the observed rms range residuals for the first halves of 1976 and 1986, and find

$$\delta\,|\dot{l}-\dot{D}|\lesssim 1.5\text{ msec/yr}. \tag{7}$$

This appears to be a reasonable estimate for the present uncertainty in the difference between the observed and calculated values for $\dot{\tilde{\omega}}-\dot{L}'$.

The uncertainty in the calculated value of $\dot{\tilde{\omega}}$ based on data through May 1982 was given by Dickey *et al.*[22] as 0.45 msec/yr. The single largest source of error was the uncertainty in the inclination of the lunar orbit to the ecliptic. Combining this with the uncertainties in the observed values of $\dot{\tilde{\omega}} - \dot{L}'$ and $\dot{L}'$ gives

$$\delta[\dot{\tilde{\omega}}_{\rm obs} - \dot{\tilde{\omega}}_{\rm calc}] \lesssim 1.6 \text{ msec/yr}. \tag{8}$$

Since the general-relativistic rate of 19.2 msec/yr for geodetic precession was included in the calculated value, this indicates that an accuracy of $\pm 10\%$ for the general-relativistic rate appears to be achievable with presently available data. However, detailed computer studies of the data are needed in order to determine the actual accuracy, as discussed later.

The agreement between the calculated post-Newtonian motion of the lunar perigee and the observed motion is an indirect verification of the generalized equivalence principle[12,13]: Without such agreement it would not be possible to transform away locally the relativistic effects of the sun with a geodetic rotation. It is also important, in the light of Mach's principle, to connect the local dynamical frame with the extragalactic very long-baseline-interferometry (VLBI) frame. According to the standard theory the local, parallel-propagated Fermi axes—with respect to which the local motion is inertial—rotate with respect to the VLBI frame with the geodetic precession $\mathbf{\Omega}_G$. Hence *the rotational angular velocity* $\mathbf{\Omega}^{(E)}$ *of the Earth relative to the VLBI frame is the sum of its angular velocity with respect to the local Fermi axes and the geodetic precession velocity* $\mathbf{\Omega}_G$:

$$\mathbf{\Omega}^{(E)}_{\rm VLBI} = \mathbf{\Omega}^{(E)}_{\rm local} + \mathbf{\Omega}_G. \tag{9}$$

If we assume, as usual, that the distant standard of rest of the PN planetary frame—connected to the Fermi frame with the "geodetic" rotation—agrees with the VLBI frame, Eq. (9) is equivalent to the agreement between the kinematical (VLBI) and planetary (PN) determinations of the angular velocity of the Earth.

This can be approximately checked by comparing values for Earth rotation time UT1 as determined by VLBI with values with respect to the PN planetary dynamical frame determined from lunar laser ranging. The Polaris-IRIS VLBI results[23] are available from late 1980 to the present, and results from the NASA Deep Space Network also are available. The UT1 results from lunar laser ranging are available up through May 1982[22] and from early 1985 until July 1986.[16] Although a thorough comparison including the latest data has not been done recently, the UT1 rates from the two techniques appear to agree to better than 1.5 msec/yr.[24] If we estimate the uncertainty in the lunar-laser-ranging results since late 1980 as being roughly 1 msec/yr and the VLBI uncertainty as considerably less, the uncertainty in the relative rotation rate between the planetary dynamical frame and the extragalactic VLBI frame would be about 2 msec/yr. Thus an accuracy for the contribution from geodetic precession to the mean motion of perigee with respect to the VLBI frame of about 10% of the geodetic precession rate appears to be achievable.

It also would be possible in principle to determine the geodetic precession rate by use of other observable quantities besides the mean motion of lunar perigee. The general precession of the Earth's rotation axis has been determined by observations of stellar proper motions,[25] by astrometric observations of the planets and the sun,[26] by lunar laser ranging,[16] and by VLBI.[27] However, comparison with a calculated value is not possible because of our poor knowledge of the mean moment of inertia of the Earth. Similarly, the contribution of geodetic precession to the mean motion of the moon cannot be determined accurately because we do not know the mass of the Earth-moon system and the lunar semimajor axis accurately enough to calculate the classical contribution to the mean motion. For the node of the lunar orbit, the contribution of geodetic precession to the mean motion is poorly determined because the motion of the node has relatively little effect on the lunar range.

Another possibility would be to use an artificial earth satellite such as LAGEOS[28,29] instead of the moon. The node of the LAGEOS orbit, for example, can be determined accurately enough, but, it precesses as a result of the Earth's oblateness and other perturbations. Its classical precession is determined by the even zonal harmonic coefficients of the Earth's gravity field, and the uncertainty[9] in these coefficients is far too large. For example, an uncertainty in J_2 of a part in a million produces an uncertainty of 450 msec/yr in the nodal precession. The moon, which is 30 times farther away than LAGEOS, is very little affected by the uncertainty in J_2 or in higher terms.

As explained in Refs. 9 and 10, the uncertainties in the Earth's gravitational field could be overcome by means of a new LAGEOS-type satellite, coupled with the present LAGEOS. They must have the same semimajor axis a and inclinations which add up to 180°. With this orbital configuration, the point lying half way between the two nodes provides a locally inertial direction. The sum of the geodetic and Lense-Thirring precessions for this locally inertial direction with respect to the extragalactic VLBI reference frame can be determined by combining satellite laser ranging and VLBI measurements of UT1.

In summary, it has been shown that the data are consistent with the general relativistic rate for the geodetic precession with respect to both the planetary dynamical frame and the extragalactic VLBI frame. The estimated accuracy is about 10%.

Since submitting this Letter, we have learned that detailed studies to determine the geodetic precession rate from lunar-laser-ranging data are being carried out at the Harvard-Smithsonian Center for Astrophysics.[30] The preliminary results indicate worse accuracy than our estimates in this Letter. The results are sensitive to the set of parameters which is solved for and to the editing of the data. We therefore caution readers that independent determinations of the accuracy based upon full computer studies are needed, in which the geodetic precession rate is specifically parametrized and all other relevant parameters are solved for simultaneously.

We wish to thank N. Ashby, R. J. Eanes, R. A. Matzner, J. Ries, B. E. Schutz, B. D. Tapley, and M. A. Vincent for helpful discussions. We particularly wish to thank J. G. Williams for extensive discussions concerning the accuracy of determining the lunar perigee motion, and T. M. Eubanks for information on the comparison of UT1 rates. Earlier discussions with R. W. King, J. O. Dickey, and X. X. Newhall concerning the analysis of lunar–laser-ranging data and with R. D. Reasenberg and R. W. Hellings concerning the Viking lander tracking results also are appreciated. And finally, we are indebted to R. D. Reasenberg and I. I. Shapiro for the information they provided about the unpublished Center for Astrophysics results concerning geodetic precession. One of us (I.C.) was supported by National Academy of Sciences under Grant No. NAS 5-28192. Another of us (P.L.B.) was partially supported by NASA under Order No. S-56150-D.

[(a)]Permanent address: Departimento di Fisica, Nucleare e Teorica, Università di Pavia, 27100 Pavia, Italy.

[(b)]Quantum Physics Division, National Bureau of Standards.

[1]W. De Sitter, Mon. Not. R. Astron. Soc. **77**, 155 (1916).

[2]A. D. Fokker, Proc. K. Ned. Akad. Wet. **23**, 729 (1920).

[3]E. Fermi, Atti Accad. Naz. Lincei, Cl. Sci. Fis. Mat. Nat., Rend. **31**, 21, 51 (1922).

[4]A. G. Walker, Proc. Roy. Soc. Edinburgh **52**, 345 (1932).

[5]C. W. Misner, K. S. Thorne, and J. Wheeler, *Gravitation* (Freeman, San Francisco, 1973), p. 1066.

[6]C. M. Will and K. Nordtvedt, Jr., Astrophys. J. **177**, 757 (1972).

[7]C. M. Will, *Theory and Experiment in Gravitational Physics* (Cambridge Univ. Press, Cambridge, 1981).

[8]J. Lense and H. Thirring, Phys. Z **19**, 156 (1918) [English translation by B. Mashhoon *et al.*, Gen. Rel. Gravit. **16**, 711 (1984)].

[9]I. Ciufolini, Phys. Rev. Lett. **56**, 278 (1986).

[10]I. Ciufolini, Celest. Mech. (to be published).

[11]J. P. Turneaure *et al.*, in *Proceedings of the Fourth Marcel Grossmann Meeting on General Relativity, Rome, Italy, 1985,* edited by R. Ruffini (Elsevier, Amsterdam, 1986), p. 411; D. Bardas *et al.*, in *Cryogenic Optical Systems and Instruments II,* edited by R. K. Melugin, SPIE Technical Publication No. 619 (SPIE, Bellingham, Washington, 1986), p. 29; see also related papers in this volume.

[12]N. Ashby and B. Bertotti, Phys. Rev. Lett. **52**, 485 (1984).

[13]N. Ashby and B. Bertotti, Phys. Rev. D **15**, 2246 (1986).

[14]J. G. Williams *et al.*, Phys. Rev. Lett. **36**, 551 (1976).

[15]I. I. Shapiro *et al.*, Phys. Rev. Lett. **36**, 555 (1976).

[16]X. X. Newhall *et al.*, in *Proceedings of the International Astronomical Union Symposium Number 128, Earth Rotation and Reference Frames for Geodesy and Geodynamics, Berkeley, West Virginia, 1986* (Reidel, Dordrecht, The Netherlands, 1986).

[17]J. Kovalevsky, in *Proceedings of the International Astronomical Union Colloquial Number 26, Reference Coordinate Systems for Earth Dynamics, Warsaw, Poland, 1975,* edited by B. Kolaczek and G. Weiffenbach (Polish Academy of Science, Warsaw, 1975), p. 123.

[18]J. G. Williams, Icarus **57**, 1 (1984).

[19]E. W. Brown, Mem. R. Astron. Soc. **53**, 39 (1987), and **57**, 51 (1905).

[20]J. P. Hauser, Ph.D. thesis, University of Colorado, 1974 (unpublished).

[21]J. G. Williams, private communication, based on results of Newhall *et al.* (Ref. 16).

[22]J. O. Dickey *et al.*, J. Geophys. Res. **90**, 9353 (1985).

[23]D. S. Robertson and W. E. Carter, in *Proceedings of the International Conference on Earth Rotation and the Terrestrial Reference Frame, Columbus, Ohio, 1985,* edited by I. Mueller (Department Geodetic Science and Surveying, Ohio State University, 1985), p. 296.

[24]T. M. Eubanks, private communication.

[25]W. Fricke, Astron. J. **72**, 1368 (1967); W. Fricke, in *Reference Coordinate Systems for Earth Dynamics,* edited by E. M. Gaposchkin and B. Kolaczek (Reidel, Dordrecht, The Netherlands, 1981), p. 331.

[26]J. H. Lieske *et al.*, Astron. Astrophys. **58**, 1 (1977).

[27]T. A. Herring *et al.*, J. Geophys. Res. **91**, 4745 (1986).

[28]D. E. Smith and P. J. Dunn, Geophys. Res. Lett. **7**, 437 (1980).

[29]S. C. Cohen and D. E. Smith, J. Geophys. Res. **90**, 9217 (1985).

[30]R. D. Reasenberg and I. I. Shapiro, private communication.