Comment on "Chaotic Rabi Oscillations under Quasiperiodic Perturbation"

In a recent Letter,[1] the occurrence of quantum chaos in a system consisting of a spin-$\frac{1}{2}$ particle, with the two levels coupled through a time-dependent perturbation $S(t)$, was reported. The equations of motion for the three components[2] of the polarization $\mathbf{P}$ represent a two-dimensional autonomous system, since the norm is conserved ($P^2=1$). In addition, there are one or two forcing terms, depending on whether $S(t)$ is chosen periodic or quasiperiodic. The authors observed that the motion in the latter case, in which $S(t)=g\cos(\omega t)\times\cos(\omega' t)$, looked quite "complex" for sufficiently high coupling amplitude g. To support the conjecture that some sort of chaos occurred, they computed power spectra for the variable P_z and autocorrelation functions for the wave function $\Psi(t)=(\psi_1,\psi_2)$.

The purpose of this Comment is to give evidence that actually no chaotic motion exists, but that the solutions are completely regular. Motivated by the interest of the model for muon-spin-rotation experiments,[3] we integrated the equations of motion using a fourth-order Runge-Kutta-Gills method, with a time step $\Delta t=2\pi/1000\omega$. Moreover, the conservation of the norm was guaranteed in the calculation by rescaling of variables after each time step. We first computed Poincaré sections (three-dimensional) of the flow with the period $2\pi/\omega$ of the first forcing term, observing that they indeed looked rather complex, and that regular structures only emerged after about 10^4 points were plotted. To demonstrate the non-chaotic character of the solutions, we then computed double Poincaré sections, i.e., we recorded a point on the section only if the argument of the second forcing term $[\omega' t(\mathrm{mod}2\pi)]$ belonged to the interval $[0,\varepsilon]$ (with $\varepsilon\approx 2\times10^{-3}$). Double Poincaré sections have already been shown to be useful in the investigation of strange attractors with more than one expanding direction.[4] In Fig. 1, we show a double Poincaré section (for $g=2.5$ and frequencies as in Ref. 1), in spherical coordinates (θ,ϕ). From the picture (corresponding to $\approx 2\times10^5$ periods of the first forcing term), the absolute regularity of the solution (a three-torus) is evident. Furthermore, no qualitative changes have been observed for larger values of g (up to $g=10$) and different choices of the (incommensurate) frequencies.

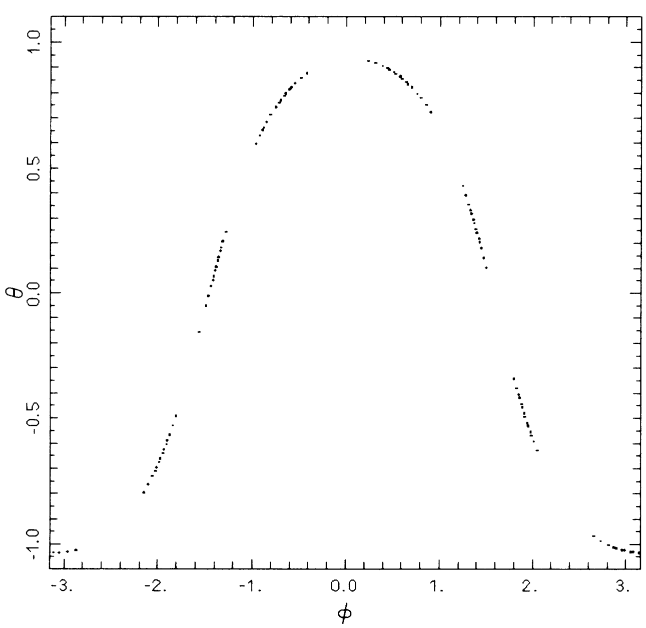

FIG. 1. Double Poincaré section of the flow describing a spin-$\frac{1}{2}$ system with the two levels coupled through a quasiperiodic force of amplitude $g=2.5$. ϕ and θ are the spherical coordinates of the polarization $\mathbf{P}$.

We also considered a modification of the model in which $S(t)$ represented a kicking force with quasiperiodically varying amplitude. This allowed the construction of a map which describes the evolution of variables after successive kicks, with a considerable improvement in computational speed and precision. This new system displays the same behavior as that of Ref. 1. Moreover, it has been possible to study quantitatively effects such as the linear increase in time of Lyapunov numbers (exponential divergences are not possible, as already mentioned in Ref. 1), and the growing complexity of power spectra with increasing g.[5] The latter phenomenon can be characterized by computing the spectral entropy, interpreting the power spectrum as a probability in the space of frequencies.[5]

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[1] Y. Pomeau, B. Dorizzi, and B. Grammaticos, Phys. Rev. Lett. **56**, 681 (1986).

[2] In the notation of Ref. 1, the components of the polarization are given by $P_x=C$, $P_y=B$, and $P_z=A$, if the static and the oscillating fields are applied, respectively, along the z and the x directions.

[3] J. A. Brown, R. H. Heffner, M. Leon, S. A. Dodds, T. L. Estle, and D. A. Vanderwater, Phys. Rev. B **27**, 3980 (1983); S. A. Dodds, private communication.

[4] R. Badii and A. Politi, in *Dimensions and Entropies in Chaotic Systems*, edited by G. Mayer-Kress (Springer-Verlag, Berlin, 1986).

[5] R. Badii and P. F. Meier, unpublished.