Electromagnetic Radiation from Superconducting Cosmic Strings

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The power of electromagnetic radiation from a current-carrying loop of superconducting string is much greater than that given by a naive estimate. Most of the radiation originates near the cusps where the string velocity reaches the speed of light, and is emitted in short and sharply directed pulses.

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Cosmic strings are linear defects that could be formed at a phase transition in the early Universe and could have played an important role in galaxy formation.[1,2] Strings predicted in some grand unified models behave as superconducting wires.[3] Such strings moving through magnetized cosmic plasmas can develop large currents and can be observed as sources of synchrotron radiation.[3,4] Ostriker, Thompson, and Witten[5] have suggested that, for sufficiently large currents, the electromagnetic radiation from superconducting closed loops can sweep away the surrounding plasma. The resulting bubbles of radiation can be used to explain the voids observed in the large-scale distribution of galaxies. In this paper we calculate the rate and the angular distribution of electromagnetic radiation from oscillating loops, with somewhat unexpected results.

Witten has shown that the current in a superconducting loop can be expressed in terms of a scalar field $\phi(\zeta^a)$ which is defined on the two-dimensional world sheet of the loop, $x^\mu(\zeta^a)$. The field equations can be obtained from the action[3,6]

$$S=\int d^2\zeta\left(-\mu\sqrt{-g}+\tfrac{1}{2}\sqrt{-g}g^{ab}\phi_{,a}\phi_{,b}-eA_\mu x^\mu{}_{,a}\varepsilon^{ab}\phi_{,b}\right)-\frac{1}{16}\pi\int d^4x\,F_{\mu\nu}F^{\mu\nu}. \tag{1}$$

Here, $g_{ab}=x^\mu{}_{,a}x_{\mu,b}$ is the metric on the string world sheet; ζ^0 and ζ^1 are a timelike and a spacelike parameter on the sheet; Greek indices take values from 0 to 3, Latin indices from the beginning of the alphabet take values 0 and 1, and Latin indices from the middle of the alphabet take values 1,2,3. The coupling e is model dependent; typically $e^2\sim 0.1$ for fermionic superconductivity and $e^2\sim 10$ for bosonic superconductivity.[3] The action (1) is invariant under arbitrary reparametrizations of the world sheet and under electromagnetic gauge transformations. To fix the gauge, it is convenient to impose the conditions

$$\dot{x}^\mu x'_\mu=0,\quad \dot{x}^\mu\dot{x}_\mu+x'^\mu x'_\mu=0,\quad \partial_\mu A^\mu=0, \tag{2}$$

where dots and primes stand for derivatives with respect to ζ^0 and ζ^1, respectively. Varying the action (1) with respect to $x^\mu(\zeta^a)$, $\phi(\zeta^a)$, and $A^\mu(x)$ and using Eqs. (2) we obtain

$$\mu\Box_2 x^\nu=-eF^\nu{}_\sigma\varepsilon^{ab}\phi_{,a}x^\sigma{}_{,b}+\varepsilon^{ab}\phi_{,b}(\partial/\partial\zeta^a)(h^{-1}\varepsilon^{cd}x^\nu{}_{,c}\phi_{,d})-\tfrac{1}{2}(\partial/\partial\zeta^a)(h^{-1}x^{\nu,a}\phi_{,b}\phi^{,b}), \tag{3}$$

$$\Box_2\phi=-\tfrac{1}{2}e\varepsilon^{ab}F_{\mu\nu}x^\mu{}_{,a}x^\nu{}_{,b}, \tag{4}$$

$$\Box_4 A^\mu=4\pi j^\mu, \tag{5}$$

where the indices are raised and lowered by use of the Lorentzian metric η^{ab},

$$\Box_2=\eta^{ab}(\partial/\partial\zeta^a)(\partial/\partial\zeta^b),\quad \Box_4=\eta^{\mu\nu}\partial_\mu\partial_\nu,$$
$$h\equiv\dot{x}^\mu\dot{x}_\mu=-x'^\mu x'_\mu, \tag{6}$$
$$j^\mu=-e\varepsilon^{ab}\int d^2\zeta\,\delta^{(4)}(x-x(\zeta))x^\mu{}_{,a}\phi_{,b}.$$

It is easily verified that the current is conserved, $\partial_\mu j^\mu=0$.

In the absence of external electric and magnetic fields, terms on the right-hand side of Eq. (3) describe the back reaction of the fields A_μ and ϕ on the motion of the loop, and the right-hand side of Eq. (4) describes the back reaction of electromagnetic field on the current in the loop. Assuming that for sufficiently small currents the back reaction is negligible, we have

$$\Box_2 x^\mu=0, \tag{7}$$

$$\Box_2\phi=0. \tag{8}$$

The solution of Eq. (7) can be written as

$$t=\zeta^0,\qquad \mathbf{x}(\zeta,t)=\tfrac{1}{2}[\mathbf{a}(\zeta-t)+\mathbf{b}(\zeta+t)], \tag{9}$$

where $\zeta\equiv\zeta^1$, and the gauge conditions (2) give the following constraints for the otherwise arbitrary functions[7] **a** and **b**:

$$\mathbf{a}'^2=\mathbf{b}'^2=1. \tag{10}$$

For a closed loop these functions must be periodic, since $\mathbf{x}(\zeta+L,t)=\mathbf{x}(\zeta,t)$, where $L=M/\mu$ is the invariant length of the loop and M is the loop's mass. This immediately implies that the motion of the loop is periodic in time; the period is[7] $T=L/2$.

The general solution of Eq. (8) is $\phi=\phi_1(\zeta-t)+\phi_2(\zeta+t)$. We will be mainly interested in the special solution,

$$\phi=(i_0/e)t, \tag{11}$$

which corresponds to a constant current in the loop. The current i_0 can take any value between zero and $i_{\max}$, where the critical current $i_{\max}$ is model dependent, but can never exceed the symmetry-breaking scale of the strings, η.

One expects that the radiation rate from an oscillating loop with a current i_0 can be estimated[5] by use of the magnetic dipole radiation formula

$$P\sim\omega^4\mathcal{M}^2\sim i_0^2, \tag{12}$$

where $\mathcal{M}\sim i_0L^2$ is the magnetic moment and $\omega\sim L^{-1}$ is the typical frequency of oscillation. However, the dipole formula is reliable only for nonrelativistic sources. Strings, on the other hand, oscillate with relativistic speeds and tend to form cusps where the string velocity momentarily reaches the speed of light.[8,9] In the cases of gravitational[10] and Goldstone-boson radiation,[11] the high-frequency radiation from the vicinity of the cusps substantially increases the total power (by a factor ~10). We will see that in the case of electromagnetic radiation the effect of cusps is even more dramatic.

The power of electromagnetic radiation from a periodic source is given by

$$P=\textstyle\sum_n P_n,\qquad P_n=-\frac{\omega_n^2}{2\pi}\int d\Omega\, j_\mu^*(\omega_n,\mathbf{k})j^\mu(\omega_n,\mathbf{k}), \tag{13}$$

where the integration is over the directions of **k**, $|\mathbf{k}|=\omega_n$, $\omega_n=2\pi n/T$, T is the period of oscillation, and $j^\mu(\omega_n,\mathbf{k})$ is the Fourier transform of the current density (6). In the case of a constant current (11), the charge density vanishes, $j^0=0$, and

$$\mathbf{j}(\omega_n,\mathbf{k})=\frac{i_0}{T}\int_0^T dt\,e^{i\omega_n t}\int_0^L d\zeta\exp[-i\mathbf{k}\cdot\mathbf{x}(\zeta,t)]\mathbf{x}'(\zeta,t). \tag{14}$$

The asymptotic behavior of $\mathbf{j}(\omega_n,\mathbf{k})$ at large n can be found by use of the method introduced by Srednicki and Theisen.[12] The main contribution to the high-frequency radiation comes from the vicinity of the cusps where $|\dot{\mathbf{x}}|=1$, and so $\mathbf{a}'=-\mathbf{b}'$ [see Eqs. (9) and (10)]. Choosing the origin of space-time coordinates and the parameter on the string so that the cusp is at $\zeta=t=\mathbf{x}=0$, we can expand the functions **a** and **b** near the cusp[9,12]:

$$\mathbf{a}(\zeta)=\mathbf{a}_0'\zeta+\tfrac{1}{2}\mathbf{a}_0''\zeta^2+\tfrac{1}{6}\mathbf{a}_0'''\zeta^3+\cdots,\qquad \mathbf{b}(\zeta)=\mathbf{b}_0'\zeta+\tfrac{1}{2}\mathbf{b}_0''\zeta^2+\tfrac{1}{6}b_0'''\zeta^3+\cdots. \tag{15}$$

Using the constraint equations (10) we obtain the following relations between the expansion coefficients:

$$\mathbf{a}_0'=-\mathbf{b}_0';\quad |\mathbf{a}_0'|=|\mathbf{b}_0'|=1;\qquad \mathbf{a}_0'\cdot\mathbf{a}_0''=\mathbf{b}_0'\cdot\mathbf{b}_0''=0. \tag{16}$$

The coefficients $\mathbf{a}_0'',\mathbf{b}_0''$ and $\mathbf{a}_0''',\mathbf{b}_0'''$ are typically of the order L^{-1} and L^{-2}, respectively. If we take **k** in the direction of the luminal velocity $(-\mathbf{a}_0')$, then

$$\omega_n t-\mathbf{k}\cdot\mathbf{x}=nL^{-3}\times O(t^3,\zeta^3,t^2\zeta,\zeta^2 t). \tag{17}$$

The main contribution to $\mathbf{j}(\omega_n,\mathbf{k})$ comes from the integration region where $|\omega_n t-\mathbf{k}\cdot\mathbf{x}|\lesssim1$,

$$|\zeta|,|t|\lesssim n^{-1/3}L. \tag{18}$$

This gives

$$|\mathbf{j}(\omega_n,k)|\sim i_0Ln^{-1}. \tag{19}$$

Now suppose there is a small angle θ between the directions of **k** and $-\mathbf{a}_0'$. Then

$$\omega_n-|\mathbf{k}\cdot\mathbf{a}_0'|\sim\theta^2,\quad |\mathbf{k}\cdot\mathbf{a}_0''|\sim|\mathbf{k}\cdot\mathbf{b}_0''|\sim\theta L^{-1}\omega_n,$$

and the additional terms in $\omega_n t-\mathbf{k}\cdot\mathbf{x}$ are of the order $\omega_n\theta^2 t$, $\omega_n\theta L^{-1}(\zeta^2,t^2,\zeta t)$. This shows that the estimate (19) remains unchanged for

$$\theta\lesssim n^{-1/3}. \tag{20}$$

Equations (13), (19), and (20) imply that

$$P_n\sim n^2L^{-2}\theta^2|\mathbf{j}(\omega_n,\mathbf{k})|^2\sim i_0^2n^{-2/3}. \tag{21}$$

The numerical coefficient in Eq. (21) can be calculated for some simple loop trajectories by use of the method of Turok,[8] Vachaspati and Vilenkin,[10] and Burden.[13] We did this calculation for several loops from the families of solutions found by Kibble and Turok[7] and by Burden.[13] The result is

$$P_n \approx \kappa i_0^2 n^{-2/3} \quad (n \gg 1), \tag{22}$$

with $\kappa \sim 10$.

The most surprising feature of Eq. (22), which makes the case of electromagnetic radiation very different from that of gravitational or Goldstone boson radiation, is that it gives a divergent total power, P. We expect, of course, that, with back reaction taken into account, the behavior of P_n at large n will be modified. If the series is cut off at some $n \sim n_*$, then the total power is

$$P \approx 3\kappa i_0^2 n_*^{1/3}. \tag{23}$$

An upper bound on n_* can be obtained in the following way. According to Eqs. (18) and (22), the energy $\Delta\varepsilon = P_n nT = \kappa i_0^2 T n^{1/3}$ is radiated in one oscillation period $T = L/2$ from a region with $\Delta\zeta \sim n^{-1/3}L$. The region itself has energy $\mu\Delta\zeta$, and we should obviously require that $\Delta\varepsilon \lesssim \mu\Delta\zeta$. This implies

$$n_* \lesssim (\mu/i_0^2)^{3/2} \sim (\eta/i_0)^3, \tag{24}$$

where we have used the relation $\mu \sim \eta^2$. The corresponding upper bound on the total power of radiation is

$$P \lesssim \kappa i_0 \eta. \tag{25}$$

We see that for $i_0 \ll \eta$ the energy lost in one oscillation period is only a small fraction ($\lesssim \kappa i_0/\eta$) of the total energy of the loop. This suggests that although the effects of back reaction are large in the vicinity of the cusps, they do not substantially alter the large-scale motion of the loop, and so the use of unperturbed equations of motion (7) and (8) in deriving Eq. (22) for $n \ll n_*$ is justified.

A reliable estimate of n_* cannot be obtained without our addressing the difficult problem of the back reaction. In the limit of an infinitely thin string, the first term on the right-hand side of Eq. (3) is infinite and has to be renormalized (one has to extend the method developed by Dirac[14] for a point particle to the case of a string). The effect of the other two terms (describing the inertia of the current) is easier to estimate. We note that in the vicinity of the cusps $h = \dot{x}^\mu \dot{x}_\mu$ is quadratic in ζ^a, and so the most divergent terms as $\zeta^a \to 0$ are of the order $(i_0/e)^2 L^2 \zeta^{-3}$. On the other hand, terms on the left-hand side of Eq. (3) are $\sim \mu L^{-1}$, and so the back reaction is important for $\zeta \lesssim \zeta_* \sim (i_0/e\eta)^{2/3}L$. This corresponds to $n_* \sim (e\eta/i_0)^2$ and

$$P \sim 3\kappa(e\eta i_0^2)^{2/3}. \tag{26}$$

We note that this estimate is consistent with the upper bound (25) and emphasize that it can be changed when the electromagnetic back reaction is properly taken into account. Note also that for $\zeta > \zeta_*$, the current in the local rest frame of the string is $i < (i_0/i_{\max})^{1/3} i_{\max}$ (since $i_0 < i_{\max}$).

The angular distribution of the radiation from a current-carrying loop is highly asymmetric. The dominant part of the energy is emitted from the cusps in narrow beams directed along the luminal velocity. The radiation is not continuous, but comes in periodic bursts. The angular distribution of radiation averaged over the period is

$$dP/d\Omega \sim i_0^2 \theta^{-3}, \tag{27}$$

where θ has the same meaning as above and Eq. (27) applies for $n_*^{1/3} \ll \theta \ll 1$. The shape of the propagating pulse of radiation can be found by use of the method of Vilenkin[9] and Vachaspati.[15] We choose the z axis in the direction of the luminal velocity ($\theta = 0$) and assume, as before, that the cusp occurred at $\mathbf{x} = t = 0$. Then the electric and magnetic fields on the z axis are perpendicular to the axis; their magnitudes are given by $E = B \sim i_0 L r^{-1} |z-t|^{-1}$. The behavior of the fields at small nonzero θ can be found for $z = t$: $E = B \sim i_0 r^{-1} \theta^{-3}$. We expect these equations to be modified by the back reaction for $|z-t| \lesssim n_*^{-1} L$ and $\theta \lesssim n_*^{-1/3}$.

So far we have discussed the radiation from an isolated loop with a constant current. Another case of interest for astrophysical applications is that of a loop oscillating in a constant external magnetic field. The current induced by the field, $i \sim e^2 BL$, oscillates with the period of the loop. For $B \ll \eta/eL$, $i \ll e\eta$ and the Lorentz force on the loop is negligible compared to the string tension. Hence, we can use the free-string solution (9). For a constant field $\mathbf{B}$, Eq. (4) determining the current takes the form

$$\Box_2 \phi = e\mathbf{B} \cdot (\mathbf{x}' \times \dot{\mathbf{x}}), \tag{28}$$

and can be solved exactly,

$$\phi = \tfrac{1}{4} e\mathbf{B} \cdot [\mathbf{a}(\zeta - t) \times \mathbf{b}(\zeta + t)] + \phi_0(\zeta, t), \tag{29}$$

where $\phi_0(\zeta,t)$ is a solution of the homogeneous equation (8). The radiation from a loop with a current given by (29) can be analyzed in the same way as that from a loop with a constant current, with very similar results.

To summarize, we have found that the power of electromagnetic radiation from a superconducting loop of string is much greater that one would naively expect. Most of the radiation is emitted in short and sharply directed pulses originating near the cusps. In an astrophysical setting, the pulses of radiation sweep the plasma they meet on their way, producing relativistic jets. A model of quasars with superconducting strings as the energy source will be discussed elsewhere.[16] (We note that jets can also be produced by bursts of gravitational radiation emanating from the cusps.[16]) The highly asymmetric character of radiation from superconducting strings may present a problem for the Ostriker-Thompson-Witten scenario.[5] It appears that, instead of blowing spherical bubbles in plasma, strings will tend to drill narrow holes in it.

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