Chiral Hierarchies and Flavor-Changing Neutral Currents in Hypercolor

Thomas Appelquist, Dimitra Karabali, and L. C. R. Wijewardhana
Department of Physics, Yale University, New Haven, Connecticut 06511
(Received 2 June 1986)

Dynamical chiral-symmetry breaking in asymptotically free theories with a slowly running coupling is analyzed. When the confinement scale Λ is much less than the cutoff M beyond which the theory cannot be used in isolation, the dynamical mass $\Sigma(p)$ starts from a value $\approx \Lambda$ for momenta $p \lesssim \Lambda$ and falls slowly for a significant range $p > \Lambda$. It then takes on the asymptotic form $(\ln p)^a/p^2$ where $a > 1$. In hypercolor theories this behavior generates sufficiently large fermion masses for a higher value of M than naively expected. This in turn could allow for an adequate suppression of flavor-changing neutral currents.

PACS numbers: 12.50.Lr, 11.30.Qc, 11.30.Rd, 12.15.Mm

The possibility that an asymptotically free gauge theory is responsible for electroweak-symmetry breaking[1] leads to some important questions about the dynamics of chiral-symmetry breaking. Among these is the question of hierarchy. If chiral symmetry is spontaneously broken, the size of the chiral-symmetry-breaking scale relative to the fundamental scales of the theory is of great interest.

We shall consider an asymptotically free gauge theory with a vectorlike coupling to fermions in some representation of the gauge group. Explicit fermion masses will be neglected so that the theory possesses a global chiral symmetry. In this limit, there are two dimensioned parameters that play a role in determination of the physical properties of the theory. The first is the confinement scale Λ, the inverse size of the gauge-singlet bound states. In QCD, this parameter is about 200 MeV and the scale of spontaneous chiral-symmetry breaking is about the same.[2,3]

The other important scale is the cutoff M beyond which the theory can no longer be used in isolation. In an asymptotically free theory, nothing within the theory itself dictates this scale; it can be arbitrarily large. The cutoff is instead determined by the onset of new physics, such as grand unification in the case of QCD. In a dynamical theory of electroweak-symmetry breaking such as hypercolor, the cutoff must be much lower to allow for the existence of new interactions[4] responsible for producing the masses of the known fermions.

In this paper, the spontaneous breaking of chiral symmetry will be discussed under the assumption that $\Lambda \ll M$. The fermion self-energy function $\Sigma(p)$ that develops will have some nonzero value Σ_0 at $p=0$ and then fall monotonically as $p \to \infty$.[5] There are then two possible hierarchies to consider: Σ_0/Λ and M/Σ_0.

A hierarchy between Λ and Σ_0 might develop if the theory contains some small parameter. A possible example is a gauge theory with the fermions in a higher representation of the gauge group.[6] The interaction strength responsible for chiral condensation is $C_2(R)\alpha(q)$, where $C_2(R)$ is the quadratic Casimir operator for representation R and $\alpha(q)$ is the running coupling. Chiral condensation should then set in when $C_2(R)\alpha(q)$ reaches a value of order unity, and if $C_2(R) \gg 1$, this might take place at a scale q well beyond Λ. For all the gauge theory examples considered here, however, Σ_0/Λ will be of order unity. Our primary concern will be the M/Σ_0 hierarchy.

In a hypercolor theory the cutoff M will represent the onset of the new interactions responsible for both the ordinary-fermion masses and possibly for flavor-changing neutral currents (FCNC's).[7,8] Our conclusion will be that for reasonable values of the gauge-theory parameters, it is possible for M to be large enough to suppress FCNC's adequately and yet still give large enough values for the ordinary-fermion masses. The focus of attention will be both on the size of the hierarchy between Σ_0 and M and on the behavior of $\Sigma(p)$ as a function of p. These two features together determine the size of ordinary-fermion masses in hypercolor theories.

In the perturbative regime, the renormalization-group equation is

$$q\,\partial\alpha(q)/\partial q = -(b-\Delta b)\alpha^2(q)+\ldots, \qquad (1)$$

where b represents the contribution of gauge fields and Δb is the contribution of whatever fermions are approximately massless at momentum q. With $\Delta b < b$ to preserve asymptotic freedom, $\alpha(q)$ will continue to increase with decreasing q according to Eq. (1) until fermion condensation takes place. A variety of studies[2,3] show that for any $b-\Delta b > 0$, fermions in the R representation of the gauge group will condense when the coupling constant has grown to exceed a critical value α_c given by

$$3\alpha_c C_2(R)/\pi = 1. \qquad (2)$$

The slowly running limit $b-\Delta b < 1$ will be of special interest to us. It must then be checked that the order-

$\alpha^3(q)$ term in Eq. (1) does not enter with a larger coefficient and overwhelm the leading term. The gap equation governing fermion condensation has also been studied with a fixed coupling constant.[9] It is again found that the coupling constant must be larger than α_c [Eq. (2)] if chiral condensation is to take place.

For any value of $b-\Delta b>0$, there will thus exist some scale μ at which $\alpha_\mu \equiv \alpha(\mu)$ is of order α_c and at which chiral condensation will be assumed to take place. If lowest-order perturbation theory can be applied down to this scale, the running coupling at scales $q \geqslant \mu$ will be given by

$$\alpha(q)=\alpha_\mu/[1+\alpha_\mu(b-\Delta b)\ln(q/\mu)]. \tag{3}$$

Since $C_2(R)$ is greater than one, it is quite possible that Eq. (3) can be reliably used in the neighborhood of condensation, or at least not far beyond it. For $p \lesssim \mu$, $\Sigma(p)$ will be of order Σ_0. For $p \gtrsim \mu$, $\Sigma(p)$ will begin its monotonic fall. The relative size of μ and Σ_0 is determined by the detailed dynamics in the momentum range below μ. That μ cannot be much smaller than Σ_0 is clear. It will now also be argued that μ cannot be large compared to Σ_0.

The perturbative gap equation is

$$\Sigma(p)=C_2\int \frac{d^4k}{4\pi^3}\frac{\alpha(p,k)\Sigma(k)}{k^2+\Sigma^2(k)}\frac{\gamma^\mu[g_{\mu\nu}-(p-k)_\mu(p-k)_\nu/(p-k)^2]\gamma^\nu}{(p-k)^2}, \tag{4}$$

where $\alpha(p,k)$ represents the one-loop corrections. If μ were large compared to Σ_0, the region $p,k \sim \mu$ would be controlled by a linearized version of the equation. Furthermore since then $p,k >> \Sigma_0$, it can be seen that in Landau gauge, $\alpha(p,k)$ becomes $\alpha(p)$ for $k << p$ and $\alpha(k)$ for $p << k$ with $\alpha(q)$ given by Eq. (3) both above and well below μ. But then the solutions are well known (and described below) and do not exhibit a transition corresponding to the onset of chiral-symmetry breaking. The nonlinearities are crucial for chiral-symmetry breaking. Thus we will take $\mu \simeq \Sigma_0(\simeq \Lambda)$.[10]

To describe the region $p>\Sigma_0$ it is reasonable to neglect angular dependence[3] in $\alpha(p,k)$ and to reduce Eq. (4) to the following approximate one-dimensional form:

$$\Sigma(p)=\frac{1}{2\alpha_c}\left\{\int^p \frac{k\,dk}{p^2}\alpha(p)\frac{k^2\Sigma(k)}{k^2+\Sigma^2(k)}+\int_p^\infty \frac{k\,dk}{k^2}\alpha(k)\frac{k^2\Sigma(k)}{k^2+\Sigma^2(k)}\right\}. \tag{5}$$

Analytical solutions will first be discussed and used to estimate ordinary-fermion masses in hypercolor theories. In these theories, dynamical breakdown of chiral symmetry is communicated to the ordinary fermions through the existence of effective four-fermion couplings whose strength we take to be g_M^2/M^2. It will be assumed that at momenta $p \sim M$, the four-fermion vertex opens up into something that damps rapidly with further increases in p. The ordinary-fermion mass is then given approximately by

$$m_f \simeq \frac{g_M^2}{4\pi^2}\frac{N}{M^2}\int^M p\,dp\,\Sigma(p), \tag{6}$$

where N is the number of hyperfermions circulating in the closed loop. The coupling strength $g_M^2/4\pi^2$ will be taken to be of order unity.

The asymptotic form of $\Sigma(p)$ is most easily derived from Eq. (5) by writing the running coupling in its conventional form

$$\alpha(q)=[(b-\Delta b)\ln(q/\Lambda')]^{-1}, \tag{7}$$

where Λ' is related to μ by

$$\Lambda'=\mu e^{-1/(b-\Delta b)\alpha_\mu}, \tag{8}$$

The parameter Λ' is, in fact, what is conventionally called the intrinsic scale of the gauge theory. However, if $(b-\Delta b)\alpha_\mu << 1$, Λ' will be much smaller than the physical confinement scale Λ ($\simeq \Sigma_0 \simeq \mu$).

The solution $\Sigma(p)$ corresponding to dynamical chiral-symmetry breaking is then[11]

$$\Sigma(p)\simeq \frac{\Sigma_0^3}{p^2}\left[\frac{\ln(p/\Lambda')}{\ln(\mu/\Lambda')}\right]^{A/2-1} \tag{9}$$

for $p >> \mu$, where $A=1/\alpha_c(b-\Delta b)$. While this is the conventional form used as input into Eq. (6), it is important to stress that it is only correct asymptotically. A careful analysis of the gap equation shows that, for Eq. (9) to describe $\Sigma(p)$ correctly, $\alpha_c(b-\Delta b) \times \ln(p/\mu)$ must be large compared to unity.

To make this point in a rather different way, we return to Eq. (3). If $\alpha_\mu(b-\Delta b)$ is small compared to unity, there will be a substantial range of momenta p above μ for which $\alpha(p)\simeq\alpha_\mu$. For p in this range, the integral in Eq. (5) will be dominated by k in the same range, and the equation can then be written in the linearized, constant-coupling, approximate form

$$\Sigma(p)=\frac{1}{2}\frac{\alpha_\mu}{\alpha_c}\left\{\int^p \frac{k\,dk}{p^2}\Sigma(k)+\int_p^\infty \frac{k\,dk}{k^2}\Sigma(k)\right\}. \tag{10}$$

If we assume that α_μ is larger than α_c and yet small enough to justify the use of Eq. (5), the solution is

$$\Sigma(p) \simeq \frac{\Sigma_0^2}{p}\cos\left[\left(\frac{\alpha_\mu}{\alpha_c}-1\right)^{1/2}\ln\frac{p}{\mu}+\delta\right], \quad (11)$$

where δ depends on the gauge-theory parameters. Because this solution can only be used when $\alpha_\mu(b-\Delta b)\ln(p/\mu) << 1$, the oscillations should not be seen in the exact solution to Eq. (5).[12] They are in fact not seen in the examples to be described here.

Thus, while $\Sigma(p)$ eventually damps rather rapidly with p [Eq. (9)], there could be a substantial range over which it falls much less rapidly[13] [Eq. (11)]. For any given set of gauge-theory parameters, it should be possible to bound the true value of the fermion mass [Eq. (6)] by use of these two expressions. A carrying out of the integral in each case gives

$$\frac{m_f}{\Sigma_0} \geqslant N\left(\frac{\Sigma_0}{M}\right)^2 \frac{2}{A}\frac{[\ln(M/\Lambda')]^{A/2}}{[\ln(\mu/\Lambda')]^{A/2-1}} \quad (12)$$

and

$$m_f/\Sigma_0 \leqslant N\Sigma_0/M, \quad (13)$$

where the second expression comes from setting the cosine equal to unity in Eq. (11).

To convert these bounds into numerical estimates, we take $N=3$ and $\Sigma_0 \simeq 800$ GeV,[14] and impose the condition $M \geqslant 10^3$ TeV in order adequately to suppress the possible FCNC's. One then finds $m_f \lesssim 2$ GeV, a number somewhat larger than typical lepton and light-quark masses. The lower bound depends sensitively on the gauge-theory parameters. With the hyperfermions in the fundamental representation of SU(3), we have $C_2(\mathbf{3})=\frac{4}{3}$, $\alpha_c=\pi/4\simeq 0.8$, and $b-\Delta b=(11-\frac{2}{3}N_3)/2\pi$, where N_3 is the number of fundamentals which must be at least 2. For this minimal theory, then, $b-\Delta b\simeq 1.5$. Finally we suppose that $\alpha_\mu \simeq 0.9$. Then $A\simeq 0.8$, $\Lambda'\simeq 0.5\mu$, and $m_f \geqslant 3$ MeV. In this case, the asymptotic solution [Eq. (9)] takes over very quickly and therefore the exact result is very close to this lower bound. This is probably too small to account for the masses of most of the ordinary fermions. Furthermore, the convergence of the perturbation expansion is doubtful in this case. The coefficient of the $\alpha^3(q)$ term in Eq. (1) is -0.97.

The above estimates and the question of convergence are, however, quite sensitive to the gauge-theory parameters. If $\alpha_\mu(b-\Delta b)$ is small, the range over which the more slowly falling solution [Eq. (11)] is relevant will expand, and furthermore the lower bound itself will increase. Suppose, for example, that the gauge theory of interest is SU(3) and there are two condensing fermions in the **6** representation. Then $N=6$, $C_2(\mathbf{6})=\frac{10}{3}$, $\alpha_c=\pi/10$, and $b-\Delta b=(11-\frac{20}{3})/2\pi\simeq 0.7$. Taking $\alpha_\mu\simeq 0.4$, we have $A\simeq 4.5$, $\Lambda'\simeq 0.05\mu$, and then $m_f \gtrsim 50$ MeV. Thus the lower bound is already far above the previous value. Furthermore, the more slowly falling solution should persist much longer than in the previous example. Thus the actual numerical result for m_f should be well above 50 MeV. The coefficient of the $\alpha^3(q)$ term in Eq. (1) for this theory is $+0.82$. Thus, at $q\simeq\mu$ where $\alpha_\mu\simeq 0.4$, the convergence appears to be reasonably good. The opposite sign will in fact make the coupling run even more slowly than in the above lowest-order estimate. The effective value of $b-\Delta b$, at $q\simeq\mu$, is $\simeq 0.3$. This will increase the value of m_f even more. Clearly what is happening here is that as $b-\Delta b$ decreases with α_μ/α_c roughly constant, $\Sigma(p)$ drops more slowly over a longer range[15] giving rise to a larger value of m_f through Eq. (6).

We conclude by presenting the results of a numerical analysis for a somewhat more extreme set of gauge-theory parameters without connecting them to a specific theory. The full nonlinear integral equation (5) will determine not only the form of $\Sigma(p)$ for $\Sigma_0 < p < M$ but also the value of Σ_0 relative to the confinement scale Λ. Setting the scale of course involves the use of Eq. (5) at momentum scales where it should be relied on for nothing more than estimates to within, say, a factor of 2. For this purpose, it will be assumed that if Λ is less than μ,[16] the coupling $\alpha(p)$ will run more rapidly below μ, increasing like[17] $\alpha(q)=\alpha_\mu/[1+\alpha_\mu b\ln(q/\mu)]$ as q decreases. The parameter b does not include the effect of the condensing fermions and should be of order unity. It is this feature that will prevent a large hierarchy between Λ and Σ_0 in the example to be discussed and in the sextet case described above. It will be assumed that at $q\simeq\Lambda$, $\alpha(q)$ will have reached a value $\alpha_\Lambda \gtrsim \alpha_c$. Then μ and α_μ will be determined in terms of α_Λ and Λ. Both will in turn be related to Λ' through Eq. (8). Below the scale Λ, the screening mechanism associated with confinement will, in effect, prevent further increase in $\alpha(q)$ and we shall simply take $\alpha(q)$ to be constant.

For the parameters to be discussed here, it will turn out that $\Lambda \lesssim \mu$. For other values of the parameters, it can happen that $\mu \lesssim \Lambda$ so that there is no intermediate region. It will always turn out, however, that $\mu/\Lambda \sim 1$. It is only this order-of-magnitude result that we will make use of in addressing the problem of FCNC's in hypercolor theories.

We present numerical results for the case $N=3$, $C_2(R)=3$, and $b-\Delta b=0.2$. Then $\alpha_c\simeq 0.35$ and $A\simeq 14.3$. We take $\alpha_\Lambda\simeq 0.6$ although similar results would be found for smaller values of α_Λ. For numerical work, the ultraviolet cutoff $M\simeq 10^3$ TeV will be imposed on Eq. (5). The numerical solution to Eq. (5) then gives $\mu=2\Sigma_0\simeq 1.4\Lambda$, $\alpha_\mu=0.5$, and

$\Lambda' \simeq 10^{-4}\Sigma_0$. These values are quite insensitive to the cutoff $M >> \Lambda$. Higher-order corrections to the β function will be neglected here. The small value of $b - \Delta b$ might, in fact, effectively arise partly from a positive $\alpha^3(q)$ term. $\Sigma(p)$ falls nearly as slowly as $1/p$ initially and begins to drop like Eq. (9) as p approaches the cutoff M. The analytical estimates for the lower and upper bounds, from Eqs. (12) and (13), respectively, are

$$90 \text{ MeV} \lesssim m_f \lesssim 2 \text{ GeV}, \quad (14)$$

where $\Sigma_0 \simeq 800$ GeV and $M \simeq 10^3$ TeV. The numerical evaluation of Eq. (6) gives $m_f \simeq 570$ MeV. This is typical of the kind of result that can be obtained for ordinary-fermion masses. With, say, $\alpha_\mu \lesssim 0.5$ and $b - \Delta b \lesssim 0.4$ (a range not difficult to obtain), fermion masses in the range above 100 MeV are to be expected. If the cutoff M can be taken to be somewhat below[8] 10^3 TeV, these estimates can be increased even more.

Another important phenomenological problem facing hypercolor theories is the presence of light pseudo-Goldstone bosons. With low enough masses they can be produced in existing accelerators and also mediate FCNC's. The mechanism discussed in this paper to raise fermion masses will also increase the masses of pseudo-Goldstone bosons. These masses will be estimated in a future paper.

In this paper we have studied dynamical chiral-symmetry breaking in asymptotically free gauge theories with a slowly running coupling. For a reasonable range of gauge-theory parameters the dynamical mass falls like $1/p$ for a significant range of p above the confinement scale, and then takes on the asymptotic form $1/p^2$ times a large positive power of $\ln p$. In hypercolor theories, this yields larger values for the ordinary-fermion masses than might be expected naively. Even for cutoffs of order 10^3 TeV ordinary fermion masses above 100 MeV can be obtained. This mechanism will also raise the masses of pseudo-Goldstone bosons.

This work is supported in part by the U.S. Department of Energy under Contract No. DE-AC02-76ERO 3075.

[1]S. Weinberg, Phys. Rev. D **13**, 974 (1976), and **19**, 1277 (1979); L. Susskind, Phys. Rev. D **20**, 2619 (1979).

[2]H. Hamber and G. Parisi, Phys. Rev. Lett. **47**, 1792 (1981); J. B. Kogut, J. Polonyi, H. W. Wyld, and D. K. Sinclair, Nucl. Phys. **B225 [FS9]**, 326 (1983), and Phys. Rev. Lett. **54**, 1475 (1985).

[3]K. Higashijima, Phys. Rev. D **29**, 1228 (1984); P. Castorina and S. Y. Pi, Phys. Rev. D **31**, 411 (1985); M. Peskin, in *Recent Advances in Field Theory and Statistical Mechanics*, edited by J. B. Zuber and R. Stora, Les Houches Summer School Proceedings Vol. 39 (North-Holland, Amsterdam, 1984).

[4]S. Dimopoulos and L. Susskind, Nucl. Phys. **B155**, 237 (1979); E. Eichten and K. Lane, Phys. Lett. **90B**, 125 (1980).

[5]Our discussion will be restricted to Euclidean momenta.

[6]W. Marciano, Phys. Rev. D **21**, 2425 (1980).

[7]S. Dimopoulos, S. Raby, and P. Sikivie, Nucl. Phys. **B176**, 449 (1980).

[8]S. Dimopoulos and J. Ellis, Nucl. Phys. **B182**, 505 (1981).

[9]T. Maskawa and H. Nakajima, Prog. Theor. Phys. **52**, 1326 (1974), and **54**, 860 (1975); R. Fukuda and T. Kugo, Nucl. Phys. **B117**, 250 (1976); V. Miransky, Nuovo Cimento **90A**, 149 (1985).

[10]The relation $\Sigma_0 \simeq \Lambda$ will emerge in the numerical work to be described shortly. A somewhat more precise definition for the chiral-symmetry–breaking scale might be q_c: $\Sigma(q_c) = q_c$. This scale, too, will be of order Σ_0.

[11]K. Lane, Phys. Rev. D **10**, 2605 (1974); H. D. Politzer, Nucl. Phys. **B117**, 397 (1976).

[12]This will be discussed in detail in a future publication.

[13]The slower rate of fall resembles the behavior found in theories where the existence of an ultraviolet stable fixed point is assumed. See B. Holdom, Phys. Rev. D **24**, 1441 (1981). A similar assumption in the context of a scale-invariant model, with the fixed point located at α_c, is made by K. Yamawaki, M. Bando, and K. Matumoto, Phys. Rev. Lett. **56**, 1335 (1986). T. Akiba and T. Yanagida [Tohoku University Report No. 85/292 (to be published)] also address the problem in the same context by simply assuming that α is a constant and close to α_c.

[14]This corresponds to an SU(3) hypercolor theory with one weak SU(2) doublet of hyperquarks, yielding $F_\pi = 250$ GeV. If there are n doublets the corresponding F_π is reduced by a factor $1/\sqrt{n}$.

[15]There are some similarities between the present analysis and that of B. Holdom [Phys. Lett. **150B**, 301 (1985)]. His goal was to arrange a hierarchy between Λ and q_c (see Ref. 10) but his analysis focused on the range between q_c and the cutoff M. He stressed the variation of $\alpha(q_c)$ with $b - \Delta b$, but our analysis concludes that there is little significance to the small variation found. In particular, we find no evidence for the development of a Λ/μ hierarchy.

[16]In the numerical work, the choice $\mu = 2\Sigma_0$ was made, corresponding to a "θ-function" threshold approximation for the decoupling of fermions of mass Σ_0.

[17]The use of this running coupling below the chiral condensation scale in Eq. (5), although reasonable, has not actually been justified. It is being used here to give a qualitative measure of the physics in this region in order to set the relative order of magnitude between Λ and μ.