Direct Measurement of Vortex Diffusivity in Thin Films of ^{4}He

P. W. Adams and W. I. Glaberson
Serin Physics Laboratory, Rutgers University, Piscataway, New Jersey 08854
(Received 15 April 1986)

We report direct measurements of vortex diffusivity, D, as a function of temperature through the Kosterlitz-Thouless transition. We find that D is a rapidly varying function of temperature near the transition. It is very small well below the transition, increases to $\sim \hbar/m$ at the static transition temperature, and apparently diverges at the point where the superfluid density vanishes. The diffusivity appears to be insensitive to the presence of ^{3}He impurities and to the nature of the substrate.

PACS numbers: 67.40.Vs, 67.40.Pm, 67.70.+n

In recent years there has been a great deal of interest in 2D superconducting and superfluid ^{4}He systems. In these systems, thermally activated defects (i.e., vortex-antivortex pairs) are the dominant fluctuations and mediate the transition to the respective superconducting and superfluid phases. The static theory of these 2D phase transitions, which fall in the same universality class as the X-Y model, has been a great success. The theory, first developed by Kosterlitz and Thouless,[1] associates the transition from the superfluid to the normal phase with the unbinding of thermal vortex-antivortex pairs at the static transition temperature, T_{KT}. This unbinding is a cooperative effect which destroys the algebraic long-range order of the system at a nonzero superfluid density predicted by the theory.[2]

Kosterlitz and Thouless used renormalization-group techniques to solve the problem of a dilute gas of logarithmically interacting vortex-antivortex pairs. By considering the effects of smaller pairs on the interaction between the members of larger pairs they were able to extract a scale-dependent dielectric constant, $\tilde{\epsilon}$, and vortex-pair excitation probability, y^2. The scale dependence of these parameters is given by the Kosterlitz-Thouless recursion relations. For an unbounded dc experiment, the recursion relations are iterated out to $l=\infty$, where $l=\ln(r/a_0)$, r is the pair separation, and a_0 is the vortex core radius. The transition temperature is determined by

$$\lim_{l\to\infty} \tilde{\epsilon}(T_{\mathrm{KT}},l)=\pi K_0/2, \tag{1}$$

where K_0 is proportional to the unrenormalized areal superfluid density, σ_s^0,

$$K_0-(\hbar^2/m^2)\sigma_s^0/kT. \tag{2}$$

The measured superfluid density is $\sigma_s(T)=\sigma_s^0/\tilde{\epsilon}(T,\infty)$.

To interpret experiments at finite frequencies, the Kosterlitz-Thouless static theory must be incorporated into a more comprehensive theory that accounts for the dynamic response of the vortex plasma to an oscillating field. Ambegaokar and Teitel[3] have shown that the vortex diffusion length, r_D, is the characteristic separation beyond which pairs can no longer equilibrate to the external field. This leads to two modifications of the static theory.[4] First, the recursion relations are not iterated to $l=\infty$ but to a finite cutoff, $l_\omega=\ln(r_D/a_0)$. This, in effect, shifts the transition temperature up from T_{KT} to a new, frequency-dependent, dynamic transition temperature, T_c. Second, $\tilde{\epsilon}$ becomes complex to account for dissipative vortex motion,

$$\tilde{\epsilon}\to\epsilon=\tilde{\epsilon}(l_\omega)+i\pi^4K_0y^2(l_\omega)+i\tfrac{2}{7}\pi^2r_D^2K_0n_f, \tag{3}$$

where the first imaginary term is due to bound pairs and the second to free vorticity of density n_f. Since r_D is related to the vortex diffusivity, D, by $r_D^2=14D/\omega$, where ω is the frequency of oscillation, it becomes apparent that D is the primary transport parameter of this theory and is of fundamental interest. In the present Letter, we report the results of an investigation of the effects of rotation on the Kosterlitz-Thouless transition in thin films of ^{4}He and present direct measurements of D as a function of temperature through the transition.

We have utilized the oscillating-substrate method of Bishop and Reppy[5] in which we monitor the period and amplitude of a high-Q ($\sim 10^5$) torsional oscillator ($\omega\sim 3000$ rad/s), containing a small amount of ^{4}He, as the temperature is swept through the transition. Changes in the period, ΔP, and internal damping, ΔQ^{-1}, of the oscillator are related to the complex dielectric constant, ϵ, by the following[6]:

$$2\Delta P/P=(A\sigma_s^0/M)\,\mathrm{Re}(\epsilon^{-1}), \tag{4}$$

$$\Delta Q^{-1}=(A\sigma_s^0/M)\,\mathrm{Im}(\epsilon^{-1}), \tag{5}$$

where A/M (~ 1500 cm^2/g) is the ratio of substrate area, consisting of a stack of ~ 9000 0.1-mil Mylar disks, to the effective mass of the oscillator. Since we measure $2\Delta P/P$ and ΔQ^{-1}, the real and imaginary parts of ϵ are easily extracted from the data. The vortex diffusivity is determined by measuring the contribution of a known free-vortex density, n_Ω, to $\mathrm{Im}(\epsilon)$. This is done by first measuring $\mathrm{Im}(\epsilon)$ as a function of

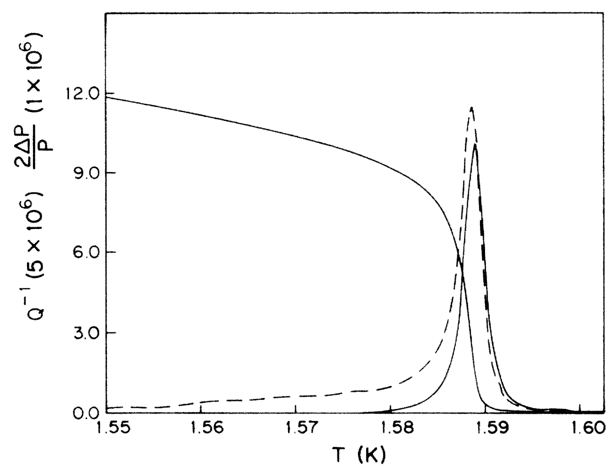

FIG. 1. Solid lines, reduced period and excess dissipation of a 11.4-Å film with the cell at rest. Dashed lines, reduced period and excess dissipation while rotating at $\Omega = 8$ rad/s.

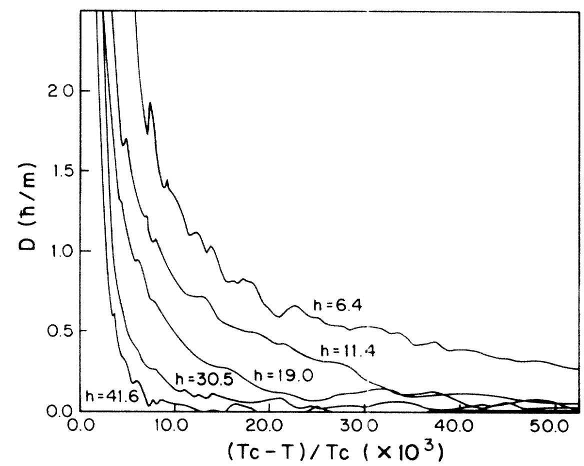

FIG. 2. Diffusivity plotted as a function of the reduced temperature for various film thicknesses, h (Å).

temperature with the cell at rest and then repeating the measurement with the cell rotating at a frequency Ω, for which, $n_\Omega = m\Omega/\pi\hbar$. According to Eq. (3), the difference of these two measurements is proportional to D,

$$D = [\mathrm{Im}(\epsilon)_\Omega - \mathrm{Im}(\epsilon)_0]\hbar\omega/4\pi m K_0 \Omega. \quad (6)$$

Shown as solid lines in Fig. 1 are the reduced period and excess dissipation of the oscillator for a typical nonrotating transition. This is essentially equivalent to the results of Bishop and Reppy.[5] The reduced period is roughly proportional to the superfluid density and qualitatively agrees with what is predicted by Kosterlitz-Thouless theory. The large dissipation peak at the dynamic transition is due in part to pairs with a separation of order r_D and in part to free vorticity from pair dissociation. Superimposed on the nonrotating data is the same transition at a rotation speed of $\Omega = 8$ rad/s ($n_\Omega \sim 1.6\times10^4\,\mathrm{cm}^{-2}$). Note that there is no discernible change in the reduced period with rotation. This is expected since n_Ω does not contribute to $\mathrm{Re}(\epsilon)$. The obvious effect is a substantial rotation-induced damping on the cold side of the dissipation peak that grows dramatically as T approaches the dynamic transition temperature. We believe that this excess damping, which is linear in Ω for $T < T_{\mathrm{KT}}$, is a direct measure of vortex diffusivity.

The diffusivity measurement for the transition in Fig. 1, along with measurements for various other film thicknesses, are shown in Fig. 2. Note that in all the films D seems to be approaching zero well below T_c and apparently diverges at T_c. The systematic increase in the sharpness of this behavior with increasing film thickness may be related to the sharpening of the transition itself. We have investigated the effect of changing the film substrate by predepositing a 100-Å layer of argon onto the Mylar. The smaller van der Waals constant of argon (or perhaps a smoothing of the substrate associated with the predeposition) resulted in a thinner "dead" layer and a corresponding increase in T_c for a given film thickness. There was, however, no observed change in either the magnitude or temperature dependence of D. We have also made measurements in which a small amount of ^{3}He was added so as to form a 5% solution. Although this shifted T_c downwards, the values of D obtained were identical, within experimental error, to the values obtained in pure ^{4}He for a film having the same transition temperature.

Figure 3 displays a plot of $D(T_{\mathrm{KT}})$ as a function of T_{KT}, where T_{TK} is taken to be the temperature at which our measured σ_s satisfies Eq. (1). Though D varies dramatically near the transition, $D(T_{\mathrm{KT}}) \sim \hbar/m$

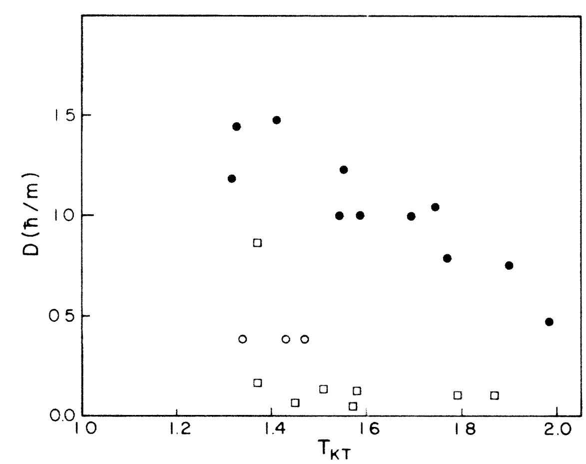

FIG. 3. Diffusivity measured at T_{KT} as a function of T_{KT}. Solid circles, present data; open circles, Ref. 7; squares, Ref. 8.

in the thinner films and falls to $\sim \frac{1}{2}\hbar/m$ in the thickest films (>25 Å).[9] These values compare favorably with those of Kim and Glaberson[7] who obtained $D(T_{\mathrm{KT}}) \sim 0.4\hbar/m$ for $1.3 \mathrm{~K} < T_{\mathrm{TK}} < 1.5 \mathrm{~K}$ from third-sound attenuation on a quartz substrate measured at a frequency $\omega \sim 6000$ rad/s. Finotello and Gasparini[8] have also reported $D(T_{\mathrm{KT}})$ vs T_{KT} obtained from dc thermal-conductivity measurements on Mylar. Their values, however, are roughly an order of magnitude smaller than ours and show substantial scatter. This may be a consequence of their experimental method which we believe may not give a reliable measure of the diffusivity at the transition temperature. The thermal conductance of their films is proportional to $(n_f D)^{-1}$ so that near the transition it is difficult to separate the temperature dependence of n_f from that of D. Their crucial assumption that D is independent of temperature in the vicinity of the transition does not appear to be justified. It is possible, of course, that the diffusivity below the transition temperature as determined in our experiments may not have the same physical origin as the diffusivity above the transition as determined in the thermal-conductivity experiments.

There are few theoretical predictions for diffusivity to which we can compare our data. Ambegaokar, Halperin, Nelson, and Siggia[4] used the definition of D,

$$D = \int_0^\infty \langle \bar{V}_L(t) \bar{V}_L(0) \rangle \, dt, \tag{7}$$

where $\bar{V}_L$ is the vortex-line velocity, to show that D is finite at T_{KT}. They made the simplifying assumption that $\bar{V}_L$ is proportional to the local superfluid velocity arising from surrounding vortex pairs and heuristically derived an integral expression for D that implies that $D \propto (\tilde{\epsilon} - 1)^{1/2}$. Though this dependence diverges at T_c its behavior is too sharp when compared with our data ($\tilde{\epsilon}$ changes little until $T \sim T_c$). Huber[10] used the analysis of Taylor and McNamara,[11] which relates the diffusivity of a charge in a 2D plasma to the fluctuations in the electric field, to show that

$$D = 2^{-3/2} \frac{\hbar}{m} \left[\ln \left(\frac{L}{\xi_+ (1 + \pi^2)^{1/2}} \right) \right]^{1/2}, \tag{8}$$

where ξ_+ is the correlation length (defined for $T > T_{\mathrm{KT}}$) and L is a typical dimension of the system. Though Eq. (8) theoretically increases as $T \rightarrow T_c$ and seems to predict the correct order of magnitude, we do not have an independent determination of ξ_+ nor do we understand how this theory applies to $T < T_{\mathrm{KT}}$. Furthermore, with the assumption of reasonable values for ξ_+,[12] our measured D diverges much more strongly than predicted by Eq. (8). Finally, Petschek and Zippelius[13] have calculated the effect of bound pairs on D and predict that D should be renormalized downwards from its "bare" value, D_0, roughly as $D_0/\tilde{\epsilon}$. Either this prediction is wrong or else our data reflect a rapid variation in D_0, unaccounted for in their theory.

We believe that the Ambegaokar-Halperin-Nelson-Siggia analysis is qualitativaely correct and that diffusivity arises from the fluctuating velocity field of the vortex plasma via the dissipation-fluctuation theorem,[14] $D \propto \langle \eta^2 \rangle$, where η is a Gaussian noise source acting on a test vortex. As the plasma is heated, pairs in the vicinity of this test vortex become larger and more numerous, thereby increasing the magnitude of η which in turn contributes to D. Thus, except for some small background contribution from rotons, phonons, and/or substrate, D seems to be a manifestation of the nature of the phase transition.

We have made an unsuccessful attempt to find a functional dependence of D which collapses all of our data onto a single curve. This conflicts with Kim and Glaberson's[7] report that $D \propto (T/\sigma_s)^2$, independent of film thickness. Their experiment, however, was complicated by the fact that their film had a thickness which was a relatively strong function of temperature. They also only reported measurements over a rather small range of T_c's and were unable to measure D significantly above T_{KT}. Our data do, however, suggest that D diverges via a power law in reduced temperture. Shown in Fig. 4, is a log-log plot of d versus the inverse of the reduced temperature, $(T_c - T)/T_c$, for several film thicknesses. The upper curves are for the thinnest films and have a slope of ~ 1. The behavior of the lower curves is much more rapid and may be a consequence of vortex pinning. Thus it appears that, in thin films, D diverges as $T_c/(T_c - T)$. This temperature dependence also seems to be consistent with the nonlinear superfluid dissipation data of Gillis,

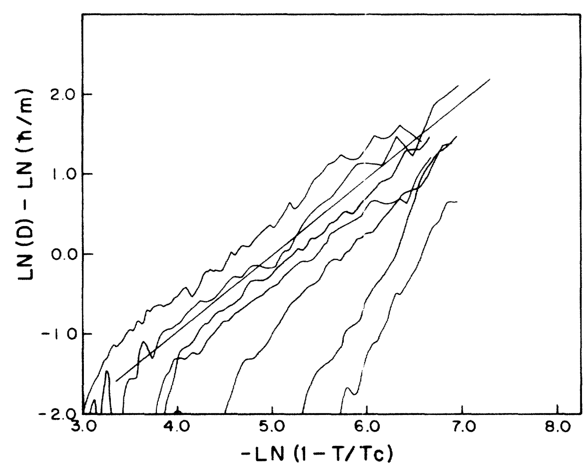

FIG. 4. The logarithm of the diffusivity plotted as a function of the logarithm of the reduced temperature. The upper curves are for the thinnest films. The straight line has unity slope to aid the eye.

Volz, and Mochel.[15] They measure the onset of nonlinear dissipation in a Helmholtz resonator, operated at a frequency $\omega \sim 14\,000$ rad/s, and numerically integrate a set of modified recursion relations for which D is varied to fit their data. Their measurements fall in the range $T/T_{\mathrm{KT}} < 0.5$ ($T_{\mathrm{KT}} \sim 1.2$ K) and are best fitted with $D \sim 0.01\hbar/m$ at $T/T_{\mathrm{KT}} \sim 0.3$. Given that T_{KT} is typically about 5 mK below T_c, this value of D is quite close to what one would expect by extrapolation from our high-temperature data.

We have considered the possibility that the behavior of our rotational data does not entirely represent a temperature dependence in D. It is not unreasonable to assume that pairs in the vicinity of a free vortex are "stretched" by its local velocity field, thus increasing the average pair separation at all temperatures. Since the cold side of the dissipation peak arises from pairs with $r \sim r_D$, this effect could cause a widening of the peak similar to what we observe. We have used the finite-flow recursion relations of Gillis, Volz, and Mochel[15] to estimate the magnitude of this effect. We have calculated $\Delta Q^{-1}(\bar{v})$, where $\bar{v}$ is the velocity field of an isolated vortex, and averaged it over the extent of the field. This calculation predicts a widening of the peak which is an order of magnitude smaller than what we observe. It also does not account for the extent of the rotating dissipation tail. We therefore believe that we are indeed measuring diffusivity.

In summary, we have made direct measurements of vortex diffusivity, a parameter of crucial importance in describing the dynamical 2D phase transition as well as an in interpreting thermal-conductance experiments, and observe a rather strong divergence at T_c. This divergence suggests that the dynamics of the phase transition itself is responsible for the diffusivity. No adequate theory now exists which accounts for the observed behavior.

This work was supported in part by the National Science Foundation under Grant No. DMR-83-19941.

[1]J. M. Kosterlitz and D. J. Thouless, J. Phys. C **6**, 1181 (1973).

[2]D. R. Nelson and J. M. Kosterlitz, Phys. Rev. Lett. **39**, 1201 (1977).

[3]V. Ambegaokar and S. Teitel, Phys. Rev. B **19**, 1667 (1979).

[4]V. Ambegaokar, B. I. Halperin, D. R. Nelson, and E. D. Siggia, Phys. Rev. B **21**, 1806 (1980).

[5]D. J. Bishop and J. D. Reppy, Phys. Rev. Lett. **40**, 1727 (1978).

[6]D. J. Bishop and J. D. Reppy, Phys. Rev. B **22**, 4171 (1980).

[7]M. Kim and W. I. Glaberson, Phys. Rev. Lett. **52**, 53 (1984).

[8]D. Finotello and F. M. Gasparini, to be published.

[9]The decrease in $D(T_{\mathrm{KT}})$ with increasing film thickness has also been observed in thin-film superconductors. See A. F. Hebard and M. A. Paalanen, Phys. Rev. Lett. **54**, 2155 (1985).

[10]D. L. Huber, Phys. Lett. **79A**, 331 (1980).

[11]J. B. Taylor and B. McNamara, Phys. Fluids **14**, 1492 (1971).

[12]See Ref. 4 and A. T. Fiory, A. F. Hebard, and W. I. Glaberson, Phys. Rev. B **28**, 5075 (1983).

[13]R. G. Petschek and A. Zippelius, Phys. Rev. B **23**, 3483 (1981).

[14]Our measurements are probably not sensitive to fluctuations with a time scale much greater than a period of oscillation.

[15]K. A. Gillis, S. Volz, and J. M. Mochel, J. Low Temp. Phys. **61**, 172 (1985).