Interactions of Ultrahigh-Energy Neutrinos

C. Quigg, M. H. Reno, and T. P. Walker
Fermi National Accelerator Laboratory, Batavia, Illinois 60510
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Cross sections for the charged-current interactions of ultrahigh-energy neutrinos with nucleons are evaluated in light of recent improvements in our knowledge of nucleon structure functions. For 10^{19}-eV neutrinos, the cross section is an order of magnitude larger than the values traditionally used in astrophysical calculations. Some consequences for event rates from generic astrophysical neutrino sources are noted.

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There is great interest in the search for ultrahigh-energy (UHE: $\gtrsim 1$ TeV) astrophysical neutrinos associated with γ-ray point sources such as Cygnus X-3[1] or for the isotropic ($\sim 10^9$ GeV) neutrino flux produced[2,3] in the interactions of extragalactic cosmic rays with the microwave background. Predictions of the neutrino-induced signal in specific detectors depend upon the assumed neutrino flux and the charged-current cross section. To the extent that uncertainties in the standard-model cross section can be controlled, future experiments can probe the environments of UHE neutrino production. In this Letter we report a new evaluation of the UHE neutrino-nucleon cross section and examine its consequences for the observational study of cosmic neutrinos.

It is straightforward to calculate the inclusive cross section for the reaction $\nu_\mu + N \rightarrow \mu^- +$anything in the renormalization-group–improved parton model. The differential cross section is written in terms of the scaling variables $x = Q^2/2M\nu$ and $y = \nu/E_\nu$ as

$$\frac{d^2\sigma}{dx\,dy} = \frac{2G_F^2 M E_\nu}{\pi}\left(\frac{M_W^2}{Q^2+M_W^2}\right)^2 [xq(x,Q^2) + x(1-y)^2\bar{q}(x,Q^2)], \qquad (1)$$

where $-Q^2$ is the invariant momentum transfer between the incident neutrino and outgoing muon, $\nu = E_\nu - E_\mu$ is the energy loss in the laboratory (target) frame, M and M_W are the nucleon and intermediate-boson masses, and $G_F = 1.166\,32\times 10^{-5}$ GeV^{-2} is the Fermi constant. The quark distribution functions are

$$q(x,Q^2) = d_v(x,Q^2) + d_s(x,Q^2) + s_s(x,Q^2) + b_s(x,Q^2), \quad \bar{q}(x,Q^2) = u_s(x,Q^2) + c_s(x,Q^2) + t_s(x,Q^2), \qquad (2)$$

where the subscripts v and s label valence and sea contributions, and u,d,c,s,t,b denote quark flavors.

At low energies ($E_\nu << M_W^2/2M$) and in the parton-model idealization that quark distributions are independent of Q^2, the differential and total cross sections are proportional to the neutrino energy. Up to energies $E_\nu \sim 100$ GeV, the familiar manifestation of the QCD evolution of the parton distributions is to decrease the valence component, and so to decrease the total cross section. At still higher energies, the gauge-boson propagator restricts $Q^2 = 2ME_\nu xy$ to values $\lesssim M_W^2$, and so limits the effective interval in x to the region $x \lesssim M_W^2/2ME_\nu$. The first effect of this damping is further to diminish the cross section below the point-coupling, parton-model approximation. Andreev, Berezinsky, and Smirnov[4] have pointed out that the growth with increasing Q^2 of parton distributions at small x enhances the cross section. Using the parton distributions then available, Andreev, Berezinsky, and Smirnov found neutrino cross sections 2–3 times larger than the scaling prediction, for $E_\nu = 10^8$ GeV.

Knowledge of the quark distribution functions has advanced markedly over the seven years since the publication of Ref. 4. For applications to high-energy collider physics, the QCD evolution of the quark distributions has been studied[5] for $10^{-4} < x < 1$ over the range 5 GeV$^2 < Q^2 < 10^8$ GeV^2. The resulting distributions, which include the perturbatively induced heavy flavors, make possible an improved estimate of the neutrino cross section. This is made timely by the appearance of increasingly capable detectors for cosmic neutrinos.

The calculations we report employ Set 2 of the Eichten-Hinchliffe-Lane-Quigg[5] (EHLQ) structure functions for $x > 10^{-4}$. We thus include the full Q^2 evolution of the parton distribution functions for both sea and valence quarks. For neutrino energies up to about 10^8 GeV, the EHLQ parton distributions contain all the information required to evaluate the neutrino cross sections. At higher energies the effect of the intermediate-boson propagator is to emphasize contributions from the region $x < 10^{-4}$, outside the range of validity of the EHLQ distributions.[6] To extrapolate to smaller values of x we follow the suggestion of McKay and Ralston[7] and use the double-logarithmic approximation (DLA) described by Gribov, Levin, and Ryskin[8]:

$$xq_s(x,Q^2)=C(Q^2)[2(\xi-\xi_0)/\rho]^{1/2}\exp\{[2\rho(\xi-\xi_0)]^{1/2}\}, \quad (3)$$

where $\rho=(8N/b_0)\ln 1/x$ and $\xi(Q^2)=\ln\ln(Q^2/\Lambda^2)$. Here $N=3$ is the number of colors and $b_0=(11N-2n_f)/3$ for n_f flavors (for this application, 5). For the EHLQ distributions, the QCD scale parameter is $\Lambda=290$ MeV and $\xi_0=\xi(Q_0^2)=\xi(5\ \mathrm{GeV}^2)$. The small-$x$ extrapolations of the structure functions are normalized so that for $x_0=10^{-4}$, we have $x_0q_s(x_0,Q^2)^{\mathrm{DLA}}=x_0q_s(x_0,Q^2)^{\mathrm{EHLQ}}$. This fixes a normalization C for each value of Q^2. Numerical integrations were carried out with the adaptive Monte Carlo routine VEGAS.[9]

We show in Fig. 1 the contributions to the neutrino-nucleon total cross section (divided by E_ν) from valence quarks and from the various species of sea quarks and antiquarks. The valence component is significant for $E_\nu \lesssim 10^6$ GeV. The contribution from top (anti)quarks, taken to have mass $m_t=30\ \mathrm{GeV}/c^2$, is negligible at all energies considered. The near-equality of the charm and bottom components occurs because at low and intermediate energies the more numerous charm antiquarks contribute with weight $\frac{1}{3}$, whereas the bottom quarks contribute with weight 1. The cross sections calculated with the EHLQ structure functions alone (without the DLA piece at small x) are indistinguishable from these up to 10^8 GeV. By 10^{10} GeV, however, the EHLQ cross section falls about a factor of 2 below the result presented here, because the EHLQ distributions do not increase for $x<10^{-4}$.

Our results for the neutrino and antineutrino total cross sections are compared with earlier work in Fig. 2. At low energies, where the valence contribution is dominant, the familiar difference between $\sigma(\nu N)$ and $\sigma(\bar{\nu}N)$ appears. At high energies, where neutrino and antineutrino cross sections become equal, our result is considerably larger than cross sections used in earlier astrophysical investigations. At 10^{10} GeV, the cross section is a factor of 8 larger than the parametrization of the Andreev, Berezinsky, and Smirnov result used by Kolb, Turner, and Walker.[10] The parametrization used by Gaisser and Stanev[11] is close to our result at the energies (between 10^4 and 10^5 GeV) important for their application to a point source, but falls far below at higher energies. The calculation by McKay and Ralston yields a cross section similar to ours above 10^6 GeV. Those authors calculated analytically an asymptotic approximation to the neutrino-nucleon total cross section using the DLA distributions. Except in the intermediate-boson propagator, they set $Q^2=M_W^2$ and neglected the y-dependence of the differential cross section. Only sea-quark contributions were included, in the approximation $u_s(x)=d_s(x)=s_s(x)=2c_s(x)=2b_s(x)$. The DLA expression was normalized to the EHLQ structure functions at $x=10^{-4}$ and $Q^2=M_W^2$, and then employed at all contributing values of x. The net effect of our refinements is to reduce the cross section at 10^{10} GeV by about 15% below McKay and Ralston's estimate. At low energies, our inclusion of the valence contribution has a dramatic effect.

The differential cross section $(1/E_\nu)d\sigma/dy$ for neutrino-nucleon scattering is shown in Fig. 3(a). The peaking of the cross section near $y=0$, which becomes increasingly prominent with increasing neutrino energy, is a direct consequence of the cutoff in Q^2 enforced by the W propagator. However, because of the growth of the quark distributions at small values of x for large Q^2, the cross section is nonnegligible at finite

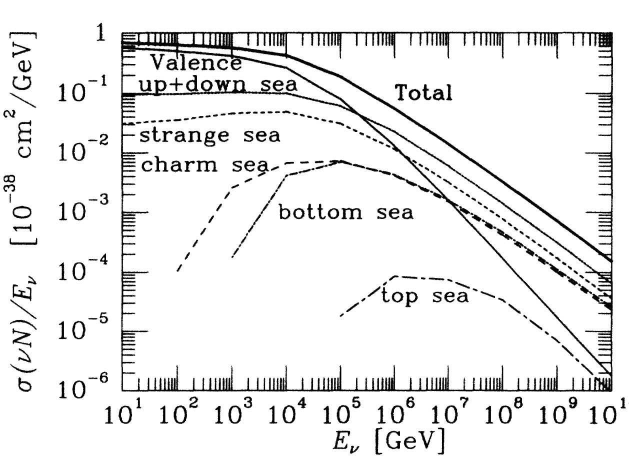

FIG. 1. Components of the neutrino-nucleon total cross section as functions of the neutrino energy.

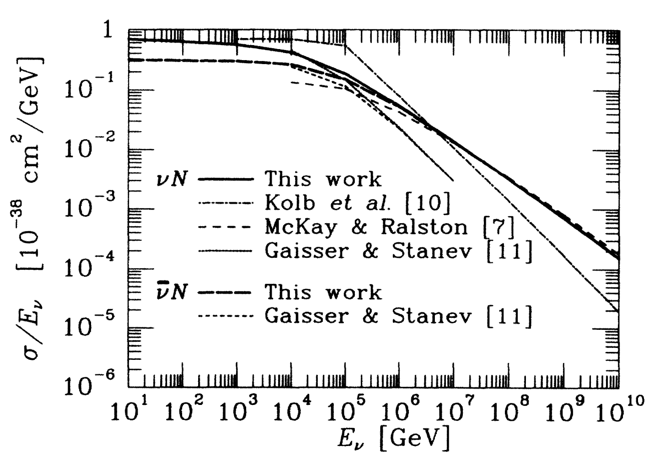

FIG. 2. Comparison of UHE neutrino and antineutrino total cross sections with earlier work.

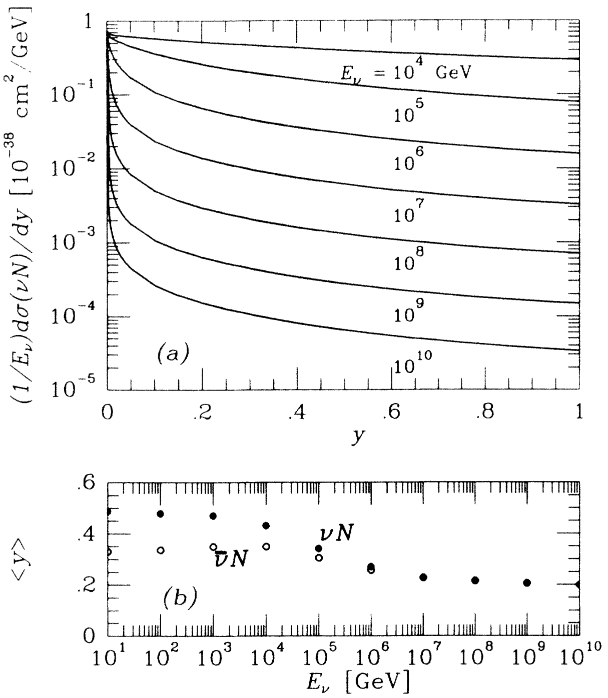

FIG. 3. (a) Differential cross section for νN scattering for neutrino energies between 10^4 and 10^{10} GeV. (b) Mean inelasticity for charged current interactions as a function of incident neutrino energy.

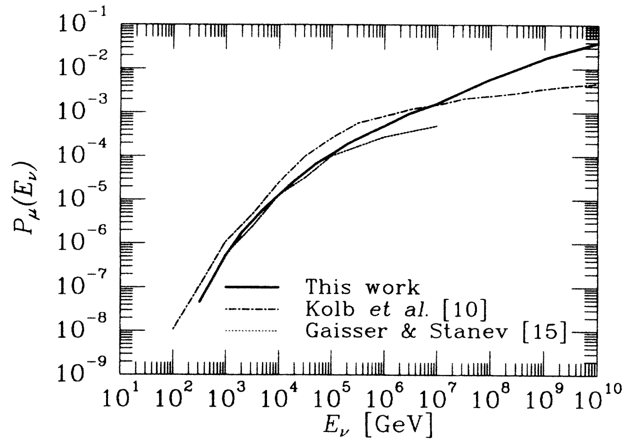

FIG. 4. Probability that a neutrino passing on a detector trajectory creates a muon that traverses an underground detector, vs neutrino energy.

values of y. Accordingly, the mean inelasticity $\langle y \rangle$ does not decrease rapidly as the energy increases. This parameter is shown for both neutrinos and antineutrinos in Fig. 3(b). Again, the contribution of the DLA piece of the parton distributions is important only for $E_\nu > 10^8$ GeV.

Let us now briefly consider the implications of these new results for neutrino astrophysics. Protons emitted and accelerated by the compact object in a binary stellar system interact with matter in the companion star, producing a point source of UHE neutrinos with a power-law differential spectrum[12] $dN_\nu/dE_\nu = N E_\nu^{-\gamma}$ with spectral index γ. It has been suggested[10,11,13] that these neutrinos could be observed in large underground detectors. The event rate in a detector of area A is dominated by muons produced in the rock surrounding the detector, provided that $\gamma < 3$.[10] The rate of muons with $E_\mu > \epsilon E_\nu$ is given by $\Gamma = A \int dE_\nu \times P_\mu(E_\nu) dN_\nu/dE_\nu$, where $P_\mu(E_\nu) = N_A \sigma(E_\nu) \langle R \rangle_{E_\nu}$ is the probability that a neutrino passing on a detector trajectory creates in the rock a muon which traverses the detector. Here N_A is Avogadro's number and

$$\langle R \rangle_{E_\nu} = \frac{1}{\sigma(E_\nu)} \int_0^{1-\epsilon} dy\, R(E_\nu(1-y)) \frac{d\sigma(E_\nu)}{dy},$$

where $R(E_\mu)$ is the range of muon in water-equivalent distance, which follows from the energy-loss relation $-dE_\mu/dx = a(E_\mu) + b(E_\mu)E_\mu$. Here $a(E_\mu)$ represents ionization losses and $b(E_\mu)$ the energy losses from photonuclear processes, bremsstrahlung, and pair production. Using the prescription of Bezrukov and Bugaev[14] for the energy-loss coefficients, we have calculated the probability $P_\mu(E_\nu)$ for muons with $E_\mu > 100$ GeV using our new values for the neutrino-nucleon (differential) cross section. Our results are shown in Fig. 4 together with the earlier calculations of Gaisser and Stanev,[15] who calculated the muon flux by use of similar energy-loss coefficients but used parton-model structure functions without scaling violations. The two curves agree up to about 10^5 GeV. Above that energy the QCD enhancement of the νN cross section increases our prediction relative to that of Gaisser and Stanev. For a point source of UHE neutrinos with spectral index $\gamma = 2$ we find that nearly 70% of the underground muon flux comes from 10^3 GeV $< E_\nu < 10^6$ GeV, so that the rate is changed only by 10%–20% by the new evaluation of $\sigma(\nu N)$. For flatter neutrino spectra the enhanced probability to produce observable muons will be of greater importance. The calculation of Kolb, Turner, and Walker[10] shown in Fig. 4 approximates $d\sigma/dy$ by $\sigma(E_\nu)\delta(y - \frac{1}{2})$ and does not include a cut on E_μ.

We have also considered the effect of the new cross-section estimates (which apply equally for the charged-current interactions of electron neutrinos) on the flux of upward-going air showers produced by a UHE neutrino background. An isotropic background characterized by a power-law spectrum with spectral index $\gamma \simeq 3$ is expected[3,16] as a consequence of cosmic-ray interactions with the microwave background. Berezinsky and Zatsepin[2] and Hill and coworkers[3] have analyzed the possibility of using an air-shower array like the Fly's Eye[17] to detect the upward-going air shower initiated by a neutrino interacting in the Earth, within a few hundred meters of the surface. The rate at which events are seen by a detector of effective area $A_{\rm eff}(E_\nu)$ can be written as

$$\Gamma = \int dE_\nu A_{\rm eff}(E_\nu) P_e(E_\nu) \int d\Omega\, S(E_\nu, \Omega)\, dN_\nu/dE_\nu d\Omega. \quad (4)$$

In this case, $P_e(E_\nu)$ is the probability that an incoming electron neutrino interacts in the Earth and produces a detectable shower, $P_e(E_\nu) = N_A \sigma(E_\nu) L_{\rm LPM}(E_\nu)$, where

$$\mathcal{L}_{\rm LPM}(E_\nu) \simeq (40\ {\rm cmwe})\{[1 - \langle y(E_\nu)\rangle] E_\nu/(62\ {\rm TeV})\}^{1/2}$$

(the abbreviation "cmwe" stands for "centimeters of water equivalent") is the enhanced distance that a UHE electron can travel in earth because of the Landau-Pomeranchuk-Migdal effect.[18] The factor

$$S(E_\nu, \Omega) = \exp[-2R_\oplus N_A \cos\theta\, \sigma(E_\nu)],$$

where the radius of the Earth is $R_\oplus = 1.7 \times 10^9$ cmwe and θ is the angle from the detector's zenith, accounts for the shadowing of UHE neutrinos by the Earth. Integration over the solid angle above the horizon yields a factor

$$[10\pi/\sigma_{34}][\exp(-0.2\Delta\theta\sigma_{34}) - \exp(-0.2\sigma_{34})],$$

where $\sigma_{34} = \sigma(E_\nu)/(10^{-34}\ {\rm cm}^2)$ and $\Delta\theta$ represents the angular resolution of the detector which limits the observer's ability to distinguish air showers produced by neutrinos at grazing incidence from those produced by neutrinos incident within $\Delta\theta$ above the horizon.

Following Hill and co-workers,[3] we calculate the rate (4) corresponding to neutrino energies in the interval $5 \times 10^8\ {\rm GeV} < E_\nu < 10^{10}$ GeV, and $\Delta\theta = 0.1$ rad. The larger value of the νN cross section found in this investigation for $E_\nu \sim 10^9$ GeV increases the opacity of the Earth to UHE neutrinos. Although the total rate of neutrino-induced air showers actually increases, the *observable* event rate decreases by a factor of 2 compared to the estimate of Ref. 3 as the upward-going air showers are forced into the cone within $\Delta\theta$ of the horizon. For example, the rate of upward-going air showers assuming a bright-phase epoch at red shift $\bar{z} = 7$ is about one per year for a detector with an effective area of 100 km^2, like the Fly's Eye.

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