One-Dimensional Electron-Electron Scattering with Small Energy Transfers

S. Wind,[a] M. J. Rooks, V. Chandrasekhar, and D. E. Prober
Section of Applied Physics, Yale University, New Haven, Connecticut 06520
(Received 28 March 1986)

We report magnetoresistance studies of Al and Ag wires of width 35 to 110 nm which probe the electron phase-breaking rate. We find that this rate at low temperatures is determined by one-dimensional electron-electron scattering with small energy transfers. This confirms the importance of this mechanism for electron energy loss in one-dimensional systems, as suggested by Al'tshuler *et al.*, and defines clearly the relevant dimensional length scales.

PACS numbers: 71.55.Jv, 72.15.Gd

In the past decade the understanding of electron scattering processes in disordered metals at low temperatures has advanced dramatically through studies of electron localization effects. Localization studies are now a reliable tool for the probing of electronic processes which are inaccessible to other measurement techniques. A prime example is the electron phase-breaking (phase decoherence) rate, τ_ϕ^{-1}. Experiments on two-dimensional (2D) systems[1,2] have shown that phase breaking is due to two mechanisms: 2D electron-electron scattering,[3,4] dominant at low temperatures ($T \lesssim 5$ K), and electron-phonon scattering. Thus, $\tau_\phi^{-1} = \tau_{ee}^{-1} + \tau_{ep}^{-1}$.

For one-dimensional (1D) systems the electron phase-breaking mechanisms are not well understood. While several experiments[5-8] have studied 1D localization effects,[9] the predicted 1D electron-electron phase-breaking rate was not clearly observed. For some studies[6,7] the wires were too wide to be in the one-dimensional limit with respect to electron-electron scattering. (The dimensional requirements for observing 1D localization are much less stringent.) In a study of narrower wires of quench-condensed Li, another scattering mechanism of unknown but non-1D origin appeared to be dominant.[8] A recent study[10] has claimed to observe the 1D electron-electron rate, but the wire studied was not clearly in the 1D size regime.

There are a number of reasons why it is important to resolve the issue of the 1D phase-breaking rate. First, the 1D electron phase-coherence length

$$l_\phi = (D\tau_\phi)^{1/2},$$

with D the diffusion constant, sets the dimensional scale for the observation and understanding of Aharonov-Bohm quantum interference effects in metal rings,[11] and universal conductance fluctuations[12] in rings and wires. Second, an understanding of the 1D rate may help settle a significant open issue which has not been resolved by the 2D experiments. There are two different predictions[3,4] for the 2D electron-electron phase-breaking rate. These predictions use different physical approaches and have a different form, but (for 2D systems) are numerically similar. The 1D experiments may settle which approach is correct. Finally, for future ultrasmall devices the temporal and spatial limits on electron phase coherence may be of paramount importance in device operation.

The Al and Ag wires we have studied are extremely narrow, and are small enough to be in the 1D size regime for electron-electron scattering.[3] We find that the electron phase-breaking rates fully confirm the theoretical predictions for 1D electron-electron scattering with small energy transfers, as regards the magnitude, temperature dependence, and width dependence.

Electron-electron scattering in disordered metals[13] leads to energy relaxation ($\Delta E \sim k_B T$) and to destruction of phase memory ($\Delta\phi \sim 2\pi$). The energy relaxation rate τ_ϵ^{-1} is the inelastic rate. An inelastic collision with large energy change ($\Delta E \sim k_B T$) certainly destroys the electron phase memory and contributes to the phase-breaking rate. Al'tshuler and co-workers[3] calculated another contribution to the phase-breaking rate in disordered systems, from multiple collisions with *small* energy transfers (quasielastic collisions, with $\Delta E \ll k_B T$). This quasielastic rate is equivalent to the Nyquist rate, τ_N^{-1}, due to the scattering of electrons by electromagnetic fluctuations. The relative magnitudes of τ_N^{-1} and τ_ϵ^{-1} depend on system dimensionality. In three dimensions, the inelastic rate dominates the Nyquist rate.[3] In 2D, τ_N^{-1} and τ_ϵ^{-1} have the same temperature dependence and similar magnitudes,[3,4] and therefore are difficult to distinguish. In 1D systems, the Nyquist rate is predicted to be greater than the energy relaxation rate,[3] so that $\tau_{ee}^{-1} = \tau_N^{-1}$. Thus, in 1D systems the theoretical prediction for phase breaking due to small energy transfers can be clearly tested.

The prediction for the 1D electron-electron scattering rate is[3]

$$\tau_{ee}^{-1} = \left[\frac{R_\square}{\sqrt{2}(\hbar/e^2)}\left(\frac{k_B}{\hbar}\right)\frac{\sqrt{D}}{W}\right]^{2/3} T^{2/3}. \quad (1)$$

$R_\square$ is the film sheet resistance, D is the diffusion constant equal to $v_F l/3$ with l the mean-free path, and W is the wire width. For a wire to be in the 1D regime for this mechanism, both the wire width and thickness

TABLE I. Sample parameters. A_{ee} was determined by fitting with Eq. (3), with τ_ϕ^{-1} inferred from the magnetoresistance data. A_{ee}^{th} is calculated from Eq. (1). For sample groups Al1, Al2, and Al3, $l_{s.o.}=0.45$, 0.55, and 0.56 μm, respectively. For samples Ag1 and Ag2, $l_{s.o.}=0.32$ and 0.52 μm, respectively, and $l_s=1.80$ and 1.19 μm, respectively.

Sample	W[a] (nm)	$R_\square$[b] (Ω)	D[c] (cm^2/sec)	$\frac{A_{ee}}{A_{ee}^{th}}$	$10^{-7}A_{ep}$ (K^{-3} sec^{-1})
Al1*a*	35	1.8	39	1.02	2.0
Al1*b*	46	1.8	39	1.03	2.1
Al2*a*	40	1.4	49	0.94	2.4
Al2*b*	60	1.4	49	1.04	2.1
Al2*c*	62	1.4	49	1.32	2.0
Al2F	Film	1.3	53	Film	1.6
Al3	110	1.1	63	1.11	2.0
Ag1	60	2.6	48	1.10	4.4
Ag2	100	1.5	85	1.06	2.2

[a]Width determined directly from examination in a JEOL-100CX scanning transmission electron microscope in conjunction with electrical measurements.

[b]At 4.5 K.

[c]D was determined from the ρl product established by previous measurements of the superconducting critical field, dH_{c2}/dT (see Ref. 16), with the resistivity, ρ, taken at 4.5 K. For the Ag samples, $\rho l=5.36\times10^{-12}\Omega$-cm^2 was used.

must be less than the phase-breaking length $l_\phi=(D\tau_\phi^{1/2})$ *and* the thermal diffusion length $l_T=(\hbar D/k_BT)^{1/2}$, in analogy to the 2D case.[3] In the 2D case, one requires the film thickness to be less than both l_ϕ and l_T.

The wires studied in this work were made of Al or Ag. They were fabricated by electron-beam lithography with use of a novel bilayer resist technique[14] and deposited from a point source by thermal evaporation onto oxidized Si substrates. The wires were in the form of meander lines ~ 50 μm long with widths ranging from 35 to 110 nm (± 5 nm) and thickness 20 nm. Groups of Al wires were evaporated at the same time so that the electrical properties of each group (e.g., group Al2) are essentially the same; they differ only in width. In most depositions, a 2D film was coevaporated along with the wires for comparison of measured quantities. T_c was 1.44 K, 1.36 K, and 1.32 K for sample groups Al1, Al2, and Al3, respectively, as determined from the coevaporated 2D films.[15] Other relevant sample parameters are listed in Table I. The experiment consisted of resistance measurements at $1.5\text{ K}\lesssim T\lesssim 20$ K in a perpendicular magnetic field with a four-terminal ac bridge. Small currents were used to avoid self-heating.

Figure 1 shows the magnetoresistance of sample Al2*a*, of width $W=40$ nm, at three representative temperatures. The smooth curves are fits by the 1D magnetoresistance theory[9] including contributions from localization and from Maki-Thompson superconducting fluctuations (for Al only). (Aslamasov-Larkin superconducting fluctuations are negligible.[17]) The formula for these contributions at fixed temperature and magnetic field is

$$\frac{\Delta R}{R}(T,H)=\left[\frac{R_\square}{\pi(\hbar/e^2)W}\right]\{\tfrac{3}{2}[l_2^{-2}+l_H^{-2}]^{-1/2}-(\beta+\tfrac{1}{2})[l_1^{-2}+l_H^{-2}]^{-1/2}\}. \quad (2)$$

Here $l_2^{-2}=l_\phi^{-2}+\frac{4}{3}l_{s.o.}^{-2}+\frac{2}{3}l_s^{-2}$, $l_1^{-2}=l_\phi^{-2}+2l_s^{-2}$ with $l_{s.o.}=(D\tau_{s.o.})^{1/2}$ the spin-orbit scattering length, and $l_s=(D\tau_s)^{1/2}$ the diffusion length for magnetic scattering. $l_H=\sqrt{3\hbar}c/eHW$, and $\beta(T/T_c)$ is the superconducting fluctuation parameter.[6,7] $\beta=0$ for Ag. The dimensionality of each term is determined by the appropriate length scale: The triplet term (prefactor $\frac{3}{2}$) is 1D as long as $W<l_2$; the Maki-Thompson term and the singlet term (prefactor $\frac{1}{2}$) are 1D if $W<l_1$.

Fitting of the theory to the data was carried out as described in Ref. 6, with l_ϕ and $l_{s.o.}$ as fitting parameters. In the case of the Ag wires, l_s was an additional fitting parameter since there was evidence of magnetic scattering in these samples. Values of these parameters are given in the caption of Table I. An important self-consistency check is that the inferred dimensional length scales l_1 and l_2 are greater than the wire width. Our wires are well within this 1D size regime, ensuring that the 1D localization theory has been properly applied.

The phase-breaking rate inferred from the magne-

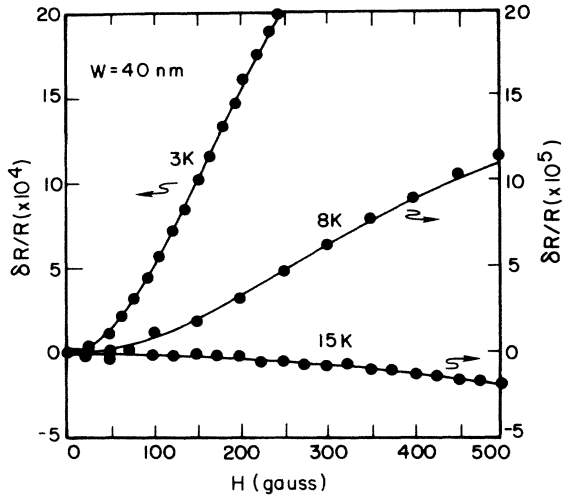

FIG. 1. Normalized magnetoresistance, $\delta R/R$, of wire Al2a at three temperatures; $\delta R(T,H)=\delta R(T,H)-\Delta R(T,0)$. Fitting parameters for the 1D theory are $l_\phi=1.19$ μm at 3 K, 0.52 μm at 8 K, and 0.24 μm at 15 K.

toresistance data is plotted in Fig. 2 for several samples. The solid lines for the wires are fits by the form

$$\tau_\phi^{-1}=A_{ee}T^{2/3}+A_{ep}T^3, \tag{3}$$

corresponding to a combination of 1D electron-electron scattering [Eq. (1)] and 3D electron-phonon scattering.[9,16] The rate found in the codeposited 2D film is also included in the figure. For the film the solid line is a fit by the theoretical form[16] $A_{ee}'T+A_{ep}T^3$. As can be seen, all the fits are excellent. It is even more significant that below ~ 6 K the magnitude of τ_ϕ^{-1} for the wires is significantly greater than for the codeposited 2D film, with τ_ϕ^{-1} increasing as W decreases. The coefficients A_{ee} determined by fitting with Eq. (3) can be compared with the 1D theoretical prefactor of Eq. (1); the agreement is excellent (see Table I). At high temperatures, electron-phonon scattering dominates; it is essentially the same magnitude for the film and the wires, and is consistent with previous studies.[6,7,16]

An important further test of the 1D electron-electron theory is the predicted width dependence at fixed temperature. This is shown in Fig. 3. Here we have subtracted the electron-phonon scattering rate. The solid lines correspond to the theoretical magnitude *and* width dependence directly from Eq. (1), with *no adjustable parameters.* The quantitative agreement is excellent. The predicted width dependence is seen clearly. We note that all three groups of aluminum wires, Al1, Al2, and Al3, have similar properties, so that such intercomparison is sensible.

The results in the present study may be contrasted with results on Al wires in previous studies.[6,7] Those

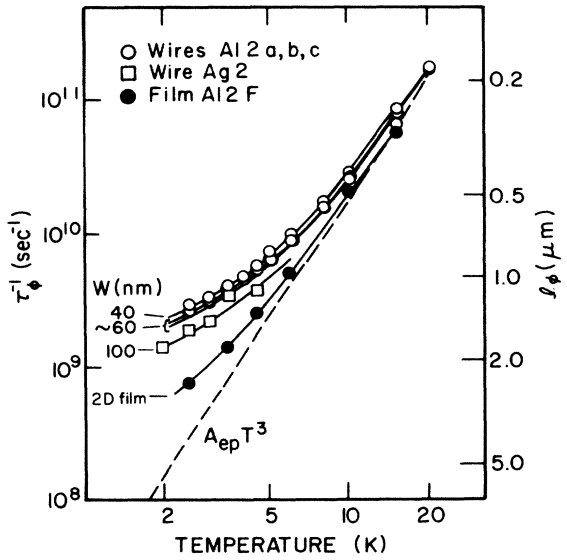

FIG. 2. Phase-breaking rate vs temperature. The solid lines for the wires are fits by Eq. (3). The data for wire Ag2 ($W=100$ nm) from 2 to 4.5 K are normalized to the $R_\square$ and D of the Al samples according to Eq. (1), to allow comparison with results for the Al wires. The solid line for the 2D Al film is a fit by the form $A_{ee}'T+A_{ep}T^3$, with $A_{ee}'=3.9\times10^8$ K^{-1}-sec^{-1}. The dashed line plots the electron-phonon rate, $\propto T^3$. The scale for l_ϕ applies for the Al samples only.

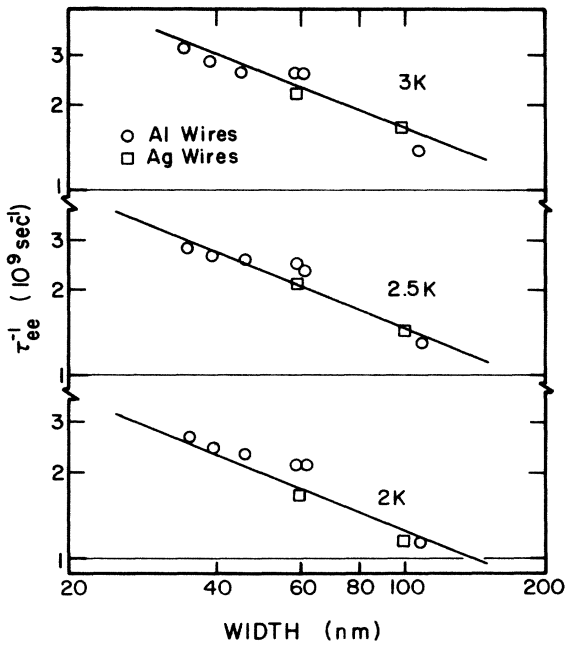

FIG. 3. Electron-electron contribution to $\tau_\phi^{-1}=$ [(total phase-breaking rate) − (electron-phonon rate $=A_{ep}T^3$)] as a function of wire width. The solid lines give the theoretical prediction of Eq. (1). The data are normalized to the $R_\square$ and D of samples Al2, according to Eq. (1).

wires, while one-dimensional with respect to the localization contribution and Maki-Thompson fluctuations, showed electron-electron scattering rates identical to those of their coevaporated 2D films, because the wires were *wider* than l_T and thus in the 2D electron-electron scattering regime. An example is a wire 240 nm wide.[6] Its measured rate is ~ 2.5 times smaller than the 1D prediction. The wires in the present study are all narrower than l_T up to ~ 10 K. Our present results with those of Ref. 6 confirm that l_T is the length scale determining dimensionality for electron-electron scattering.

In another recent study[10] it was claimed that the 1D Nyquist rate was observed in one Au-Pd wire of cross-sectional width 46 nm. This wire showed strong magnetic scattering. The authors analyzed the wire magnetoresistance data using the 1D localization theory. However, the wire was wider than the dimensional length scales for both the triplet and singlet terms: $l_2 = 10$ nm and $l_1 \sim 30$ nm. This implies that the wire is *not* in the 1D limit of the localization theory. In addition, the wire width is greater than l_T throughout the entire temperature range ($l_T \approx 18$ nm at 5 K), so that the wire is also not in the one-dimensional limit with regard to electron-electron scattering. We conclude that Ref. 10 does not unambiguously demonstrate the 1D Nyquist rate.

In summary, we find that the electron phase-breaking rate at low temperatures is due to one-dimensional electron-electron scattering with small energy transfers. Our results also confirm that the thermal diffusion length l_T is the relevant length scale determining dimensionality for this mechanism.

We thank P. Santhanam, E. Abrahams, and R. G. Wheeler for useful discussions, P. Male for assistance with scanning electron microscopy, and P. L. McEuen for assistance with microfabrication. This work was supported in part by National Science Foundation Grant No. DMR-8505539, and for one of us (M.J.R.) by National Science Foundation Grant No. ECS-8509135. Support of the microlithography facilities at Yale University was provided by National Science Foundation Grant No. DMR-8213080, IBM Corp., and Shipley, Inc.

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[a]Also at Department of Physics, Yale University, New Haven, CT 06520.

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