NMR in Normal ^{3}He with a Meander-Line Coil

S. A. Bogacz and J. B. Ketterson

Department of Physics and Astronomy, Northwestern University, Evanston, Illinois 60201, and Materials Science and Technology Division, Argonne National Laboratory, Argonne, Illinois 60439

(Received 1 April 1986)

A simple technique is proposed for probing the single-particle and collective excitations in a Fermi liquid via a standing magnetic surface wave of arbitrary ω and fixed k generated by a meander-line coil. The calculated power absorption spectrum for ^{3}He displays singularities associated with the $l=0$ spin-wave mode and a Doppler-shifted spin resonance of the single-particle excitations.

PACS numbers: 67.50.−b, 07.58.+g

For the most part, magnetic resonance experiments are usually carried out in the presence of a spatially uniform rf excitation field. An important exception is a light scattering study[1] where in-plane wave-vector conservation places restrictions on the wave vector of excitations generated in a medium. A second exception is the case of metallic samples where the skin effect concentrates the Fourier components of the wave vector of excitations generated in the medium to the vicinity of δ^{-1}, where δ is the skin depth; this property allows the generation of magnetostatic modes in ferromagnets[2] and Silin spin waves in pure metals[3] (at low temperatures). In the case of standard NMR, spin diffusion is generally studied by application of an inhomogeneous static field,[4] rather than an inhomogeneous rf field.[5,6]

In this paper we explore rf excitation using a periodic meander line which has the property of having a specified wave vector lying in the plane of the surface of a sample. This situation is shown schematically in Fig. 1. By symmetry this structure restricts the in-plane component of the wave vector of excitations generated in the medium to match that of the meander line; it places no restrictions on the normal component. Qualitatively, we expect the meander line to behave as a diffraction grating (or, more precisely, left- and right-moving diffraction gratings). The k vectors of the excitations generated in the medium would then be expected to satisfy the grating equation. In particular there will be a "cutoff frequency" for the grating which can be probed experimentally, to gain information on the excitation spectrum. A 3D analog of the meander-line transducer has been proposed by Corruccini[7] as a way of setting up a nonuniform rf field to couple the spin modes. The kinematics of this device would be governed by the Bragg law; as discussed above, the diffraction-grating law applied to our 2D structure.

We recently applied this idea to the case of an insulating ferromagnet[8] and indeed found cutoff-frequency phenomena for the magnetostatic Damon-Eshbach surface wave and the bulk spin-wave mode. We also examined the acoustic analog of this idea[9] (where an interdigital transducer replaced the meander line) for the case of normal ^{3}He and the cutoff phenomena for the zero sounds were again encountered.

Here we propose a relatively simple technique for measuring both the Fermi velocity and the phase velocity of the spin waves in ^{3}He which in turn yield the amplitudes of the spin-antisymmetric Landau interaction function, F_0^a and F_1^a. Although our model calculation was done for normal ^{3}He, it may be generalized to the superfluid phases and to other Fermi liquids such as spin-polarized ^{3}He, or the electrons in pure metals.

We consider a sample of normal ^{3}He occupying the $z>0$ half space. We direct a uniform static magnetic field, H_0, along the z axis, and excite the surface with a standing magnetic surface wave, which can be decomposed into left- and right-running waves propagating along the x axis. As was shown in Ref. 8, this kind of disturbance can be generated by an array of parallel, equally spaced wires carrying antiparallel alternating currents $i=i_0e^{-i\omega t}$. The standing-surface-wave

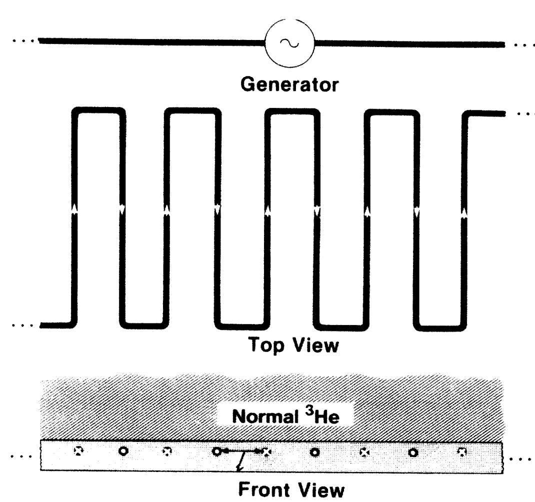

FIG. 1. A schematic of the meander-line coil used to excite a standing magnetic surface wave.

solution for the magnetic scalar potential has the form[8]

$$\phi(t,\mathbf{r}) = \frac{4 i_0}{c}\cos(\omega t)\sum_{m=0}^{\infty}(-1)^m \frac{1}{(2m+1)}\cos(k_m x)e^{-k_m z}, \quad (1)$$

where $k_m = (2m+1)k_x$. The value of k_x is fixed by the geometry of the transducer and is given by $k_x = \pi/l$, where l is the spacing between adjacent elements (carrying oppositely directed currents). Each Fourier component of the driving field can be written as a linear combination of four running waves and the problem reduces to finding the response of the system to the driving field

$$\phi^{\pm}(r,t) = \phi_0 \exp(-i\omega t \pm i k_x x - k_x z) \quad (2)$$

for both positive and negative ω.

The spin dynamics of a Fermi liquid are governed by the Landau-Silin kinetic equation.[10] For normal ^{3}He only the transverse components of the spin density $\sigma_{\hat{\mathbf{p}}}(\mathbf{r},t)$ are associated with propagating modes. The inhomogeneous form of the Landau-Silin equation can be written as

$$\{\partial_t + (v\hat{p}^{\alpha}\partial_{\alpha} \pm i\Omega_0^a)(1+F^a)\}\sigma_{\hat{\mathbf{p}}}^{\pm} = (v\hat{p}^{\alpha}\partial_{\alpha} \pm i\Omega_0^a)\partial_x\phi; \quad (3)$$

here the operator F^a represents the spin-antisymmetric part of the molecular field interaction; its kernel is expressed as a series of Legendre polynomials $P_l(\hat{\mathbf{p}}\cdot\hat{\mathbf{p}}')$, with respective Landau amplitudes F_l^a. Furthermore, $\sigma_{\hat{\mathbf{p}}}^{\pm} = \sigma_{\hat{\mathbf{p}}}^{x} \pm i\sigma_{\hat{\mathbf{p}}}^{y}$, $\Omega_0^a = \Omega_0/(1+F_0^a)$, where Ω_0 is the Larmor frequency, and ϕ is the magnetic scalar potential generated by the transducer.

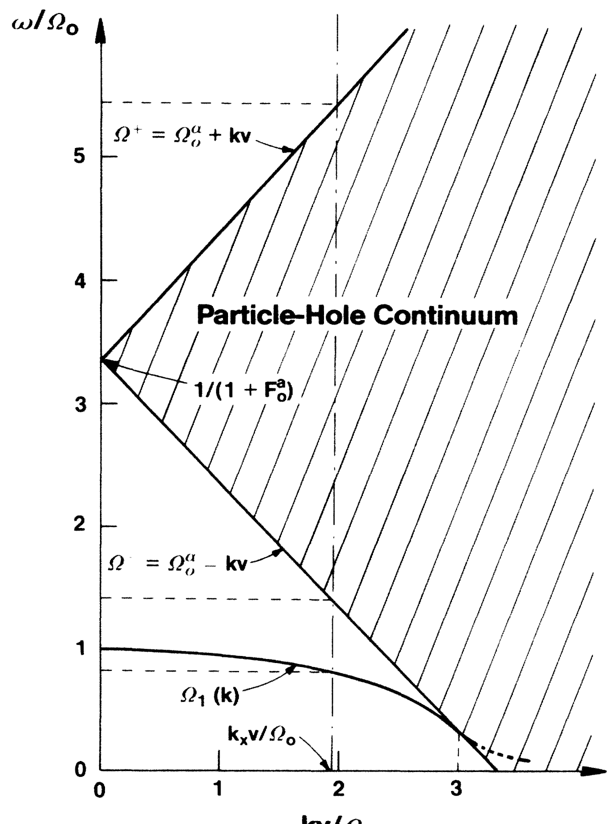

FIG. 2. Locations of the dispersion branch for the $l=0$ spin wave and the particle-hole continuum in the dimensionless ω-k plane. Ω_0 is the Larmor frequency. A value of $k_x v/\Omega_0$ sets the probing profile.

This equation will be solved for the potential given by Eq. (2) under the assumption that the ^{3}He quasiparticles reflect specularly[11] from the transducer interface.

The analytic solution for $\sigma_{\hat{\mathbf{p}}}^{\pm}$ allows us to formulate the response of the system in terms of the power absorbed per unit area of the transducer-liquid interface. It turns out that we can only couple to the spherically symmetric ($l=0$) mode of the spin density. The bulk dispersion relation for this mode is illustrated in Fig. 2 along with the boundaries of the particle-hole continuum, as modified by the magnetic field.

The calculations to be presented show that the power absorbed by the liquid involves both the single-particle excitations and the (collective) $l=0$ spin mode. The calculated spectrum is shown in Fig. 3 and

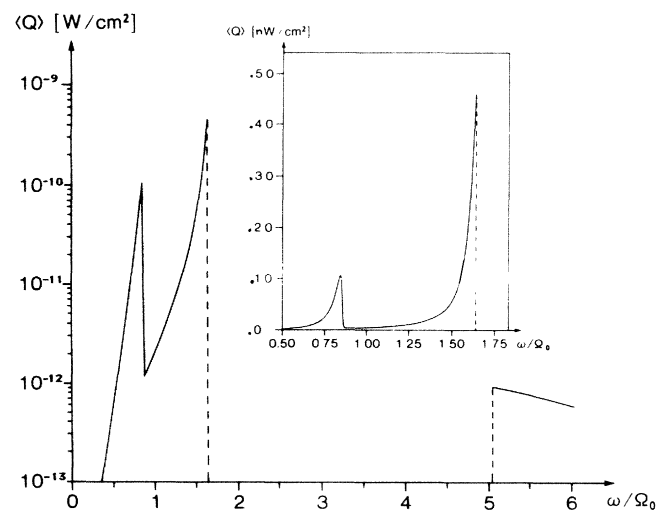

FIG. 3. Power absorbed per unit area of the ^{3}He-transducer interface as a function of the frequency of the driving field.

displays singularities associated with various thresholds for both of the above processes. The locations of these singularities (or cutoff frequencies) are illustrated in Fig. 2 and are associated with the following physical phenomena.

(a) $\Omega_1(k_x)$ is the threshold frequency for the propagation of the $l=0$ mode. (Note that this mode crosses into the particle-hole continuum at large k whereupon Landau damping sets in.)

(b) $\Omega_\pm = \Omega_0^a \pm k_x v$ define the limits of a region where the direct generation of single-particle excitations by the driving field is forbidden. Physically, these latter frequencies correspond to the so-called Doppler-shifted spin-resonance (DSSR).[12]

To understand DSSR consider a ^{3}He quasiparticle moving parallel to the surface with the Fermi velocity, $\pm v$, and interacting with a magnetic surface disturbance of phase velocity ω/k_x; the quasiparticle will sense a *Doppler-shifted* frequency given by $\omega_D = \omega(1 \pm k_x v/\omega)$. We anticipate from Fig. 2 (and the detailed solution will bare it out) that when ω_D matches the spin-enhanced Larmor frequency, $\Omega_0^a = \Omega_0/(1+F_0^a)$, a DSSR singularity will occur; this is equivalent to condition (b) above.

Experimental measurements of $\Omega_1(k)$ would allow one to probe the dispersion relation for the transverse, $l=0$, spin mode and would determine the Landau parameters (F_0^a, F_1^a) with spectroscopic precision. Furthermore, observation of the DSSR thresholds, $\Omega^\pm$, would provide an independent determination of the Fermi velocity and F_0^a.

We now briefly present the analytic solution of our problem. The assumption of specular reflection of the quasiparticles at the interface imposes the requirement that $\sigma_{\hat{p}}$ be invariant under reflection (changing the sign of $\hat{p}_z$ at $z=0$). We seek a solution of Eq. (3) with the potential given by Eq. (2), which satisfies the above boundary condition in the following form:

$$\sigma_{\hat{p}}^\pm(\mathbf{r},t) = e^{-i\omega t + ik_x x}\int_{-\infty}^{\infty} \sigma_{\hat{p}}^\pm(k_z) e^{ik_z z}\, dk_z.$$

Using a Fourier-transform method we solve our equation in the full space, and by appropriate (odd) analytic continuation of the potential ϕ into the lower half space,[11] we obtain a solution satisfying the specular boundary condition. A full solution is given in terms of different moments of $\sigma_{\hat{p}}$ in p space and has the following form:

$$\sigma_0^\pm(z) = \frac{1}{2\pi i}\int_{-\infty}^{\infty} dk_z\, 2\phi_0 \frac{k_x^2}{k^2}\frac{e^{ik_z z}}{D^\pm}\left(1+\frac{F_1^a}{3}\right)\left\{\frac{\omega}{kv}\Lambda^\pm(\omega,k) - 1\right\},$$
$$\sigma_\alpha^\pm(z) = \frac{1}{2\pi i}\int_{-\infty}^{\infty} dk_z\, 2\phi_0 \frac{k_\alpha k_x^2}{k^3}\frac{e^{ik_z z}}{D^\pm}\frac{\omega}{kv}\left\{\frac{\omega \mp \Omega_0^a}{kv}\Lambda^\pm(\omega,k) - 1\right\}; \tag{4}$$

here $\sigma_0 = \langle \sigma_{\hat{p}} \rangle_{\hat{p}}$ and $\sigma_\alpha = \langle \sigma_{\hat{p}} \hat{p}^\alpha \rangle_{\hat{p}}$, $k = (k_\alpha k_\alpha)^{1/2}$, where $\langle \ldots \rangle_{\hat{p}} = (1/4\pi)\int d\Omega_{\hat{p}} \ldots$, and

$$D^\pm = (1+F_0^a)\left(1+\frac{F_1^a}{3}\right) + \omega\frac{\omega\mp\Omega_0}{(\omega\mp\Omega_0^a)^2}F_1^a - \left\{F_0^a\left(1+\frac{F_1^a}{3}\right) + \frac{\omega\mp\Omega_0}{\omega\mp\Omega_0^a}F_1^a\right\}\frac{\omega}{kv}\Lambda^\pm(\omega,k) \tag{5}$$

with

$$\Lambda^\pm(\omega,k) = \tfrac{1}{2}\ln\left[\frac{(\omega\mp\Omega_0^a + kv)}{(\omega\mp\Omega_0^a - kv)}\right]. \tag{6}$$

We note that our solutions represent the response of the liquid to a single running component of the standing wave of Eq. (1). To obtain the full response one has to add the three remaining solutions, which can be easily obtained from the analytic solution Eq. (4) by alternation of the signs of ω and k_x. We identify $D^\pm = 0$ with the dispersion relation for the $l=0$ spin wave.

The remaining integration in Eq. (4) can be carried out explicitly by use of Cauchy's integral theorem. Since we are interested in the solution in the region $z>0$, our contour of integration must be closed in the upper half of the complex k_z plane. To avoid multivaluedness of the integrand connected with the logarithmic terms of $\Lambda^\pm$, we introduce a cut line connecting the branch points of $\Lambda^\pm$. Therefore the contour will include a detour which excludes the cut line and the branch points. Finally, the integrals evaluated will contain two contributions: a contribution from a simple pole arising from $D=0$ (given by the respective residue) and a contribution from the branch point of Λ (represented by the integral along the cut line). Both contributions can be expressed as a sum of plane waves with k vectors given by the location in the complex k_z plane of the pole or the branch point, respectively.

An energy-flow theorem can be obtained from the Landau-Silin equation and it allows us to express the energy-current density as

$$\langle Q \rangle = F_0^a \langle\langle \sigma_0^+, \sigma_0^- \rangle\rangle + F_1^a \langle\langle \sigma_\gamma^+, \sigma_\gamma^- \rangle\rangle, \tag{7}$$

where

$$\langle\langle A,B\rangle\rangle = \tfrac{1}{2}\int_0^\infty dz\,\{\langle\dot{A},B\rangle_t + \langle A,\dot{B}\rangle_t\} \quad (8)$$

and the time average, $\langle\ \ldots\ \rangle_t$, is evaluated as

$$\langle A,B\rangle_t = \tfrac{1}{2}\,\mathrm{Re}(A^*B). \quad (9)$$

A straightforward algebraic consequence of Eqs. (7)–(9) is that the only nontrivial contributions to the energy flow, $\langle Q\rangle$, come from σ's with real k_z vectors. Since both D and Λ are functions of ω, the location of the simple pole and the branch point depends on frequency. Certain critical values of ω, when the singularities move from the real to the imaginary axis in the complex k_z plane, represent the previously discussed cutoff frequencies for the energy flow. To make a connection between the above analytic results and our previous physical picture, we identify those frequencies where the simple pole lies on the real axis with the allowed region for $l=0$ spin-wave generation. Similarly, the frequency range in which the branch points of $\Lambda^\pm$ lie on the real axis defines the situation where a direct coupling between the single-particle excitations and the driving field occurs.

We have yet to account for dissipative effects. Phenomenologically, one can modify our collisionless treatment by adding a small imaginary part to the frequency in our final result according to the substitution $\omega \rightarrow \omega + i/\tau$. This is only a qualitatively correct picture, since certain conservation theorems must be built into the true collision integral. We now present the power absorption spectrum obtained numerically from Eq. (7) and the integrated solutions given by Eq. (4). We use $F_0^a = -0.7$ and $F_1^a = -0.55$ for the Landau amplitudes ($p=0$ atm) and assume 10^3 G for the dc field. The spacing between wires in the meander-line coil carrying a current of 0.01 μA is assigned a value 10 μm. The spin-diffusion relaxation time, $T^2\tau_D$, has the experimentally measured value of 3.0×10^{-7} sec mK^2.[13] This evaluated at 2 mK is identified with our phenomenological parameter τ. Figure 3 shows the graphical results. Note that singularities occur involving all of the kinematical features contained in our qualitative discussion. The peak power absorbed by a 1-cm^2 meander-line transducer at the singularity associated with the $l=0$ spin-mode generation is equal to about 10^{-10} W. Similarly, the peak power at the edges connected with the particle-hole excitations is about 5×10^{-10} W. Both magnitudes of power are readily detectable. In both cases we satisfy the condition that the dynamic spin density is much less than the static spin density (induced by the static external dc magnetic field), a necessary condition for our linearized theory to apply.

In conclusion, we have presented the results of a detailed calculation concerning a new experimental technique which is capable of directly probing single-particle and collective excitations of an interacting Fermi liquid. An expanded version of these model calculations will be presented in a future publication.[14]

We would like to acknowledge financial support from National Science Foundation Grant No. DMR 81-07385 and from the U.S. Department of Energy through Contract No. W-31-109-ENG-38.

[1]R. E. Camley and D. L. Mills, Phys. Rev. B **18**, 4821 (1978).

[2]P. C. Fletcher and C. Kittle, Phys. Rev. **120**, 2004 (1960).

[3]G. Dunifer, S. Schultz, and P. H. Schmidts, J. Appl. Phys. **39**, 397 (1968).

[4]H. C. Torrey, Phys. Rev. **104**, 563 (1956).

[5]N. Masuhara, D. Candela, D. O. Edwards, R. F. Hoyt, H. N. Scholtz, and D. S. Sherrill, Phys. Rev. Lett. **53**, 1168 (1984).

[6]G. Tastevin, P. J. Nacher, M. Leduc, and F. Laloë, J. Phys. (Paris), Lett. **46**, L249 (1985).

[7]L. R. Corruccini, Ph. D. thesis, Cornell University, 1971 (unpublished).

[8]S. A. Bogacz and J. B. Ketterson, J. Appl. Phys. **58**, 1935 (1985).

[9]S. A. Bogacz and J. B. Ketterson, J. Low Temp. Phys. **60**, 133 (1985).

[10]V. P. Silin, Zh. Eksp. Teor. Fiz. **33**, 495 (1958) [Sov. Phys. JETP **6**, 387 (1958)].

[11]A. I. Fomin, Zh. Eksp. Teor. Fiz. **61**, 2562 (1971) [Sov. Phys. JETP **34**, 1371 (1972)].

[12]H. N. Spector and J. B. Ketterson, Phys. Rev. B **11**, 3724 (1971).

[13]L. R. Corruccini, D. D. Osheroff, D. M. Lee, and R. C. Richardson, J. Low Temp. Phys. **8**, 229 (1972).

[14]S. A. Bogacz and J. B. Ketterson, to be published.

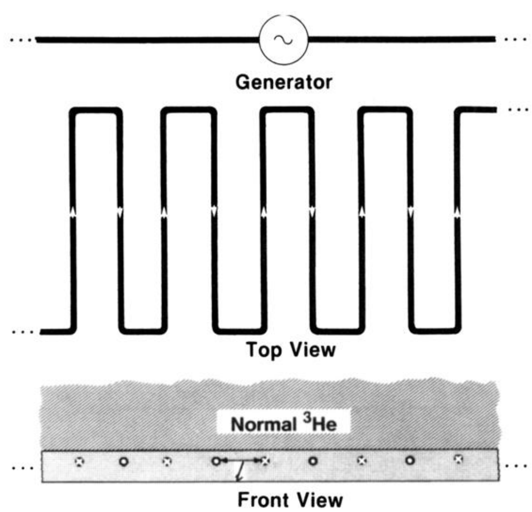

FIG. 1. A schematic of the meander-line coil used to excite a standing magnetic surface wave.