Nonlinear Power Saturation and Phase (Wave Refractive Index) in a Collective Free-Electron Laser Amplifier

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We report measurements of the nonlinear radiation intensity and phase (wave refractive index) of a free-electron laser amplifier operating in the collective (Raman) regime. The laser generates up to ~ 100 kW of rf power at a frequency of 9.3 GHz and with an efficiency of $\sim 10\%$. Power saturation, efficiency, synchrotron oscillations, and the rf phase are studied as functions of electron beam energy, current, and axial distance within the helical wiggler. Excellent agreement with non-linear theory that takes cognizance of electron trapping in the combined ponderomotive and space-charge potential well is obtained.

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Free-electron lasers (FEL's) are currently under investigation because of several remarkable properties, including their high efficiencies and output powers, and their inherent frequency tunability. The purpose of this paper is twofold: First, a study of the nonlinear physics which determines the efficiency and power output of the collective (Raman) regime free-electron laser; and second, an investigation, in both the linear and nonlinear regimes, of the change in the effective refractive index experienced by the electromagnetic wave. Changes in refractive index produce a cumulative phase shift in the wave which is measured experimentally. Not only do phase measurements constitute an interesting test of FEL theory, but they are also relevant to the predicted phenomenon of optical guiding.¹ An FEL employing optical guiding would rely on the electron beam to guide the emitted radiation in the same manner as light is guided on an optical fiber, thereby extending the FEL interaction length beyond that allowed by the Rayleigh diffraction limit.

Previously, the small-signal linear gain behavior of the device was investigated,² and is now well understood. Here, the nonlinear saturation of the FEL instability is examined by observing the dependence of the FEL output power on the wiggler length, electron beam current, and beam energy. We find that the output power agrees well with the predictions of the nonlinear theory of Kroll, Morton, and Rosenbluth³ (KMR) and others^{4,5} when the theory is suitably extended for use with this experiment. Power output and efficiency are found to be governed by electron trapping in the potential well formed by the combined action of the ponderomotive and self-consistent space-charge forces. In contrast, the KMR model was previously examined for a Compton regime FEL with a different wiggler and waveguide configuration than ours,⁶ where the space-charge effects were small.

The FEL is illustrated in Fig. 1. A thermionically emitting Pierce gun removed from a SLAC klystron

(model 343) is energized by the Physics International Pulserad 615MR high-voltage facility. The ensuing 0.25-cm-radius electron beam is guided magnetically into a copper plated, 2.54-cm-i.d. stainless-steel evacuated drift tube which also acts as the cylindrical waveguide. Beam integrity is maintained by a uniform axial guide field B_{\parallel} . Experimental measurements and electron-gun computer simulations² indicate that the beam-energy spread is less than 0.5%. This, together with the relatively high current density and modest beam voltage, assures operation in the collective (Raman) regime. The 50-period circularly polarized wiggler has a period of 3.3 cm, and is generated by bifilar conductors wound on the drift tube.

The FEL is operated in the amplifier mode. A microwave *E*-plane bend launcher superimposes the input microwave signal onto the electron beam and converts the TE_{01} waveguide mode of the rectangular input waveguide to a linearly polarized, TE_{11} mode of the circular guide in the FEL. At the end of the wiggler known fractions of the power in the incident and orthogonal polarizations are extracted by directional couplers, and measured by calibrated crystal



FIG. 1. Schematic of the collective FEL experimental setup.



FIG. 2. The small-signal gain and relative phase shift as functions of beam voltage. The solid lines are from the experiment, the dashed lines are from theory (Ref. 9). $B_{\rm w} = 122$ G, $B_{\rm H} = 1510$ G, I = 3.5 A, f = 11.1 GHz.

detectors. The calibration error is estimated to be less than $\pm 0.5 \text{ dB}$.

Figure 2 shows a comparison between experiment and theory^{7,8} for the FEL gain and the phase shift, as a function of the beam voltage, when the device is operating in the small-signal linear regime with rf power input not exceeding 10 W. Gain is observed at the operating point where the FEL instability can be modeled as an interaction between an electromagnetic wave and a negative energy, slow space-charge wave. A decrease in the output signal is obtained at a somewhat lower energy where the electromagnetic wave interacts with a positive energy, fast space-charge wave. The fine structure in Fig. 2 is due to alternating constructive and destructive interference between the aforementioned waves.⁹ This interference results from the relative shifts in the wave phases.

To measure these phase shifts directly, we use a microwave interferometer which mixes the rf output signal from the FEL with the rf input signal. As shown in Fig. 2(b), the phase shifts extend over a wide range of beam energies, and they can be quite large, even when the FEL gain is near unity. A characteristic of the collective (Raman) regime is that the phase shift is near zero when the FEL interaction is strongest [i.e., at the large maximum and minimum of Fig. 2(a)]. This is unlike the phase behavior of an FEL operating in the strong-pump Compton regime, where, as is well



FIG. 3. Microwave power as a function of wiggler length z for four currents, with $B_w = 188$ G, $B_{\parallel} = 1512$ G, and f = 9.3 GHz. The points are from experiment, the lines from simulation: circles, I = 5.5 A; plusses, I = 3.7 A; squares, I = 2.5 A; triangles, I = 0.9 A.

known,⁸ the phase shift is large and positive at maximum gain. In fact, the small positive phase shift observed for our operating parameters at maximum gain is a reflection of the very slight strong-pump nature to the interaction.

To study the nonlinear regime, we saturate the FEL within a distance of approximately one meter by injecting a high-power (~ 30 -kW) signal from a short-pulse magnetron operating at a frequency 9.3 GHz. Figure 3 shows the FEL output power as a function of the axial distance z for four different values of the electron beam current I and the corresponding power predicted by the theory. The length of the interaction region is adjusted by an axially movable "kicker" magnet that deflects the beam into the waveguide wall. The current varies between 0.9 and 5.5 A.

At the beginning of the wiggler, launching losses^{2, 8} cause the microwave output power to grow quadratically with distance. This is followed by a section of exponential growth. Beyond this distance, the beam electrons are trapped in a "ponderomotive" potential well produced by the beating of the wiggler and the electromagnetic fields. Once the electron beam falls to the bottom of the well, the output power saturates. Near the wiggler end, the microwave power decreases as the microwaves reaccelerate the electrons towards the top of the ponderomotive well, causing a synchrotron oscillation to occur.

In Fig. 3, the axial distance to reach saturation is seen to be only weakly dependent on the beam

current, because the gain and the maximum saturated output power decrease simultaneously when the beam current is decreased. In contrast, we observe (not shown) that the saturation wiggler length is strongly dependent on the gain when the gain is changed without greatly changing the maximum saturated output power (for instance, by varying the wiggler field).

The input rf power of 30 kW is large enough compared with the electron beam power (≤ 1 MW) that the electron beam is influenced by the nonlinear forces of the ponderomotive well even at the beginning of the wiggler. This manifests itself by the fact that the computed beam kinetic energy that maximizes the unsaturated gain is approximately 2% higher than is found with the small-signal, linear theory. Another consequence is that the well-known¹⁰ theoretical collective regime efficiency scaling, $\eta \propto (\text{current})^{1/2}$, is not observed. Because the electron beam is affected by the trapping potential, even at the wiggler entrance, the measured efficiency of about 10% does not depend strongly on the beam current.

The output powers shown in Fig. 3 are determined at the fixed beam voltage V that gives the greatest gain

at the beginning of the wiggler ($z \leq 60$ cm). This voltage is not the voltage that gives the maximum smallsignal gain because nonlinear effects are already important at $z \leq 60$ cm (see above). In addition, this maximum gain voltage is not the voltage that gives the maximum output power along the entire wiggler length. In fact, the voltage that yields the maximum output power increases steadily for wiggler lengths longer than the saturation length. This shift is due to the synchrotron oscillation that follows saturation. At the peak gain voltage the FEL saturates at approximately $z \approx 115$ cm, after which the output power decreases. At higher beam voltages, the FEL interaction proceeds more slowly, and saturation is delayed past z = 115 cm. Thus, at large z the output power is maximized at higher beam voltages. This phenomenon is shown in Fig. 4 where we plot the output power as a function of beam voltage for three axial positions. We see that both in the data and in the theory the voltage corresponding to peak power increases $\sim 2\%$ as z increases from 81 to 155 cm.

In Fig. 5(a) we show the phase shift as a function of distance within the wiggler, for parameters where the



FIG. 4. Power as a function of beam energy for three values of wiggler length z. The solid lines are from the experiment, the dashed lines are from simulation. Here $B_w = 187$ G, $B_{\parallel} = 1470$ G, I = 4.5 A, $P_{in} = 32$ kW, and f = 9.3 GHz.



FIG. 5. Nonlinear phase measurements: (a) phase vs z at $\gamma = 1.301$; (b) phase vs voltage at z = 104 cm; (c) power vs voltage at z = 104 cm. The solid lines are from the experiment; the dashed lines are from simulation. $B_w = 176$ G, $B_{\parallel} = 1450$ G, I = 4.1 A, $P_{in} = 27$ kW, and f = 9.3 GHz.

FEL saturates at $z \sim 100$ cm. However, even in the saturated regime the phase continues to change, thus demonstrating continued bunching of the electon beam by the FEL interaction. In Figs. 5(b) and 5(c) we plot the power and phase shift as a function of beam voltage at a fixed distance z = 104 cm. Because of the previously mentioned energy shifts, the phase is now slightly negative at the maximum output power. This is in contrast to the slightly positive phase shift found at maximum linear growth.

The phase shifts illustrated in Figs. 1 and 5 are relative to the phase the electromagnetic wave would have in the absence of the FEL interaction. A positive phase shift denotes a reduction in the phase velocity of the wave, and thus an increase in the effective refractive index, and vice versa for a negative phase shift. A refractive index greater than unity is required for the electromagnetic radiation to be guided by the FEL interaction itself.

The experimental measurements in Figs. 2-5 are compared to the predictions of a modified version of the KMR (Ref. 3) equations. We note that major modifications are required for the particular parameter regime of this experiment. They include corrections for the excitation of a collective space-charge wave on a finite radius beam, the transverse structure of the waveguide mode,¹¹ three-dimensional wiggler fields, the presence of an axial guide magnetic field, and the relatively low velocity ($v/c \sim 0.6$) of the electron beam. Three-dimensional effects are included by use of appropriately calculated input parameters and analytic expressions for the transverse electron motion.¹² The equations are solved by use of a numerical simulation that tracks 1024 particles in a single ponderomotive wavelength. The simulation itself is one dimensional, permits the propagation of only the TE_{11} waveguide mode, and contains no adjustable parameters. The details of the model and simulation will be reported elsewhere.

In conclusion, we find that our measurements of the spatial rf growth, saturation, and subsequent decay in our collective (Raman) free-electron laser amplifier are fully explained by nonlinear theory which takes into account electron trapping in potential wells formed by the combined action of the ponderomotive and beam space-charge forces. Scaling of the power and efficiency with electron beam current and voltage are likewise in agreement with theoretical predictions. Detailed measurements of the rf phase (i.e., wave refractive index) in both the linear and nonlinear regimes are the first of their kind. They not only provide a good test of FEL physics, but also are relevant to the predicted phenomena of optical guiding.

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