Compression Modulus of Nuclear Matter and Charge-Distribution Differences

Giampaolo Co'
Institut für Kernphysik der Kernforschungsanlage Jülich, 5770 Jülich, Federal Republic of Germany

and

Josef Speth
Los Alamos National Laboratory, Los Alamos, New Mexico 87545
(Received 24 March 1986)

We demonstrate that the charge-distribution differences between isotopes of heavy-mass nuclei are the most sensitive experimental quantities for extraction of the compression modulus of nuclear matter. This analysis shows a compression modulus larger than 200 MeV.

PACS numbers: 21.65.+f, 21.10.Ft, 27.80.+w

One of the basic properties of nuclei is their compression modulus. This quantity is not only interesting in its own, but its extrapolation to nuclear matter has important consequences in astrophysics, especially in connection with supernova explosions.[1] So far, this quantity has been extracted mainly from the excitation energy of the giant monopole resonances in heavy-mass nuclei[2] and changes of the mean square charge radii $\delta\langle r^2\rangle$ between different isotopes[3] that can be deduced from isotope-shift measurements. The most commonly accepted value for the compression modulus of nuclear matter is $K=210\pm30$ MeV.[2] However, one has to consider that both breathing mode and isotope shifts are related to the surface of the nucleus. For this reason, the extrapolation to nuclear matter of the compression modulus extracted from these quantities is a process that is strongly model dependent, as we discuss below.

In Landau's Fermi-liquid theory,[4] the scalar-isoscalar particle-hole (p-h) force parameter f_0 is related to the compression modulus in nuclear matter by

$$K=6(\hbar^2 k_F^2/2m^*)(1+2f_0), \qquad (1)$$

where k_F is the Fermi momentum and m^* the effective mass. (Note that our Landau parameters differ by a factor of 2 from those of Brown and Osnes[5] as a result of a different density of states.) All the quantities are determined at the Fermi surface. The extension of Landau's theory to finite Fermi systems, due to Migdal,[4] leads to a p-h force that is density dependent, especially in the scalar-isoscalar channel. It has been shown in various microscopic calculations of the effective p-h interaction that the interaction in this channel is very attractive at low densities, whereas with increasing density the attraction gets reduced and may even become repulsive. It is this density dependence that makes the extrapolation of the breathing mode and $\delta\langle r^2\rangle$ results to nuclear matter very model dependent.

Recently Brown and Osnes[5] have suggested a different procedure by which to extract the force parameter f_0 of Eq. (1). They used one of the sum rules for the Landau parameters[6] relating (all) the scalar-isoscalar Landau parameters f_l to (all) the spin-isospin parameters g_l' and the isovector-tensor contribution δ_t. Brown and Osnes argue that this procedure leads to a compression modulus of 100 MeV or even smaller. The result of Brown and Osnes depends sensitively on the Landau parameter f_1 which is connected with the effective mass in nuclei. If one chooses an effective mass of $m^*/m=0.8$ rather than $m^*/m=0.9$, the compression modulus changes from $K=112.5$ MeV to the much higher value of $K=178.5$ MeV.

Our present analysis of the compression modulus is based on the determination of the Landau parameter f_0 from experiment. Here we show that the charge-distribution differences between ^{207}Pb and ^{208}Pb and between ^{206}Pb and ^{208}Pb that have been obtained from inelastic electron-scattering experiments represent a unique possibility to determine the compression modulus of the interior region of heavy-mass nuclei. These charge-distribution differences represent one of the few experimental bits of information about the interior of heavy-mass nuclei.

Our calculations of the charge-distribution differences are based on the Landau-Migdal theory of finite Fermi systems, which is especially appropriate for the calculation of those quantities. The basic equations for the density-distribution differences between neighboring nuclei can be found in the work of Speth, Werner, and Wild.[7] The theoretical results depend sensitively on the p-h interaction, which is expressed as (we drop the spin-dependent parts that we do not contribute to 0^+ properties)

$$F^{\text{p-h}}\left(\mathbf{r},\mathbf{r}',\rho\left(\frac{\mathbf{r}+\mathbf{r}'}{2}\right)\right)=C_0[f_0(\rho(\mathbf{r}))+f_0'(\rho(\mathbf{r}))\boldsymbol{\tau}\cdot\boldsymbol{\tau}']\delta(\mathbf{r}-\mathbf{r}'). \qquad (2)$$

Here $C_0=\hbar^2\pi^2/k_F m=302$ MeV fm^3 and the density dependence of the force is given by

$$f(\rho(r)) = f^{\text{ex}} + (f^{\text{in}} - f^{\text{ex}})\rho(r)/\rho(0). \quad (3)$$

The density $\rho(r)$ is chosen to have the form of a Fermi distribution with the parameters of Rinker and Speth.[8] We are left with the four parameters f_0^{in}, f_0^{ex}, $f_0'^{\text{in}}$, and $f_0'^{\text{ex}}$, which have to be determined by adjustment to experiments. It is important to mention that the isospin-dependent parameters $f_0'^{\text{in}}$ and $f_0'^{\text{ex}}$ can be determined independently from the $E0$ properties. We used the well-known relation between the symmetry energy (β) and the Landau parameter $f_0'^{\text{in}}$.[4] Here we chose $\beta = 34$ MeV which corresponds to $f_0'^{\text{in}} = 0.83$ and extracted the external parameter $f_0'^{\text{ex}}$ from the excitation energy of the dipole resonance in ^{208}Pb. (We chose a value 2 MeV lower than the experimental energy in order to account for the effects of the effective mass.) Therefore, there remain only two Landau parameters f_0^{in} and f_0^{ex}. Using Landau's definition of the particle-hole interaction, we obtain a relation between the depth of the single-particle potential V_0 and the Landau parameters f_0:

$$V_0 = \int_0^{\rho_0} F^{\text{p-h}}[\rho]\,d\rho = \tfrac{1}{2} C_0 \rho_0 (f_0^{\text{in}} + f_0^{\text{ex}}), \quad (4)$$

which determines f_0^{ex} for a given f_0^{in}. The results denoted by (a) in Table I have been derived under condition (4). In the other cases, (b) f_0^{ex} was adjusted in order to reproduce the excitation energy of the breathing mode.

In the following, we show results of nuclear-structure calculations that have been obtained with four different compression moduli. All these calculations have been performed in the same manner as described in Ref. 3. The corresponding results are shown in Table I. In the cases (b) one realizes that the excitation energy of the breathing mode can be reproduced for all four (very) different compression moduli. This is connected with the density dependence of the central, isospin-independent force, and the fact that the breathing mode is essentially a surface property. It also clearly indicates that one is not able to extract from the excitation energy of the breathing mode reliable results for the compression modulus of nuclear matter. On the other hand, it is known that the theoretically determined isotope shifts depend sensitively on the density dependence of the interaction.[3] If we choose f_0^{in} somewhat smaller than zero, then $\delta\langle r^2\rangle$ changes sign and one is no longer able to reproduce even the sign of the experimental isotope shifts.

The results (a) show the same tendency for the isotope shifts but in addition, the theoretical results of the breathing mode now also depend on the compression modulus. The results obtained with set 3 ($K = 230$ MeV) are not in agreement with the experimental data. The agreement clearly improves if we choose a larger value of K. Note that set 2 corresponds to $K = 354$ MeV.

Recently, the charge-distribution differences between ^{208}Pb and some neighboring nuclei have been measured by inelastic electron scattering.[9] As one can see from Fig. 1, the charge-distribution differences between ^{207}Pb and ^{208}Pb and between ^{206}Pb and ^{208}Pb, respectively, are peaked in the inner region between 0 and 2 fm and remain essentially positive up to 5 fm. Beyond 5 fm the charge-distribution differences are negative, which gives rise to negative $\delta\langle r^2\rangle$ in agreement with experiment. The theoretical distributions are calculated within the linear response theory by use of the method described in detail in Refs. 3 and 7. This method enables us to calculate directly the charge-distribution differences including random-phase-approximation correlations.

The pointlike proton-density distributions have been

TABLE I. Compression modulus K, the mean square charge-radius differences between ^{207}Pb and ^{208}Pb ($\delta\langle r^2\rangle^{207}$) and between ^{206}Pb and ^{208}Pb ($\delta\langle r^2\rangle^{206}$), and the excitation energy of the breathing mode E^{0^+}. For cases (a) we used the self-consistency condition Eq. (4); for cases (b) we adjusted f_0^{ex} to E^{0^+}.

Force	K (MeV)	$\delta\langle r^2\rangle^{207}$ (fm^2)	$\delta\langle r^2\rangle^{206}$ (fm^2)	E^{0^+} (MeV)
I (a)	465	−0.056	−0.115	15.44
I (b)		−0.135	−0.269	13.48
II (a)	354	−0.045	−0.099	14.79
II (b)		−0.081	−0.169	13.51
III (a)	230	−0.026	−0.072	13.10
III (b)		−0.012	−0.044	13.55
IV (a)	106	+0.013	−0.017	10.65
IV (b)		+0.048	+0.054	13.50
Expt.		−0.08 ± 0.007[a]	−0.121 ± 0.007[a]	13.50[b]

[a]G. Borchert *et al.*, Nuovo Cimento **73A**, 273 (1983).

[b]J. Speth and A. van der Woude, Rep. Prog. Phys. **44**, 719 (1981).

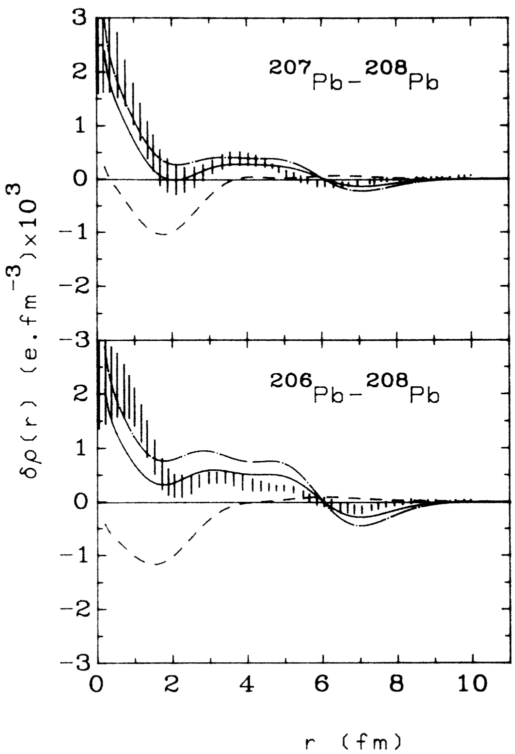

FIG. 1. Charge-density difference between ^{207}Pb and ^{208}Pb and between ^{206}Pb and ^{208}Pb. The full, dashed, and dashed-dotted lines represent the results of our calculations performed respectively with force II (b), force IV (b), and force I (b) of Table I. The experimental data are taken from Ref. 9.

corrected considering the contribution of proton and neutron form factors as discussed by Martorell and Sprung[10] and by Friar and Negele.[11] In addition, we considered the electromagnetic spin-orbit interaction[10,11] that, in the case of lead isotopes, gives a relatively large contribution to the charge-distribution difference between 0 and 2 fm.[10]

In Fig. 1, we compare our theoretical results obtained with three different sets of force parameters with the experimental distributions. The theoretical curves calculated with force II (b) ($K=345$ MeV) nicely reproduce the experimental distribution. The results obtained with force I (b) ($K=465$ MeV) overestimate the experimental data of ^{206}Pb-^{208}Pb. It is obvious from Fig. 1 that the theoretical distribution calculated with force IV (b) ($K=106$ MeV) is qualitatively different from the two previous ones and has no similarity at all to the experimental data. The comparison with the data shows that the Landau parameter $f_0^{\prime\text{in}}$ has to be slightly positive, and it clearly favors a value of K larger than 200 MeV.

In the present approach, we have restricted ourselves to the zero-order Landau parameters. Therefore it is obvious that we do not fulfill the sum rule.[6] If we consider also the (negative) f_1 parameter, which is equivalent to an effective mass $m^*/m<1$, we do not change the isoscalar properties. In particular, the position of the breathing mode will not be changed, because the energy-weighted isoscalar sum rule for the $E0$ states cannot be changed by the introduction of an effective mass <1. The increase of the unperturbed particle-hole energies due to $m^*/m<1$ will be compensated by the additional attractive f_1 contribution in the particle-hole force. It is different in the isovector case. If we lower the effective mass, we need a less repulsive isovector interaction. As $f_0^{\prime\text{in}}$ is given by the symmetry energy, we must reduce $f_0^{\prime\text{ex}}$ slightly. For that reason, our numerical results will be little affected by an effective mass <1. In addition, the compression modulus and the symmetry energy will increase with decreasing effective mass [see Eq. (1)].

In summary, we have shown that the position of the breathing mode does not allow extraction of the compression modulus of nuclear matter in a reliable way. As the breathing mode is surface-peaked, it is sensitive to an average value of the compression moduli (f_0 parameters) inside and outside of the nucleus. We also have shown that the sum-rule approach by Brown and Osnes[5] depends sensitively on the choice of the effective mass. On the other hand, we have demonstrated that the charge-distribution differences between ^{207}Pb and ^{208}Pb and between ^{206}Pb and ^{208}Pb represent the most sensitive experimental quantity to determine the compression modulus inside of the nuclei. Our analysis of these experimental data which have not been considered so far indicates that the compression modulus has to be larger than the presently accepted value[2] of $K=210\ \pm 30$ MeV. This value is also much larger than some theories of supernova explosions require.[1]

We thank Gordon Baym, Gerry Brown, Bernard Frois, Costas Papanicolas, Dan Strottmann, and Jochem Wambach for useful discussions and George Bertsch for reminding us of the self-consistency relation. This work was supported in part by National Science Foundation Grant No. NSF PHY84-15064 and by NATO Grant No. RG 85/0093.

[1]H. A. Bethe and G. E. Brown, Sci. Am. **252**, No. 5, 60 (1985).

[2]J. P. Blaizot, Phys. Rep. **64C**, 171 (1980).

[3]J. Speth, L. Zamick, and P. Ring, Nucl. Phys. **A232**, 7 (1974).

[4]A. M. Migdal, *Theory of Finite Fermi Systems* (Wiley, New York, 1967).

[5]G. E. Brown and E. Osnes, Phys. Lett. **159B**, 222 (1985).

[6]B. L. Friman and A. K. Dhar, Phys. Lett. **85B**, 1 (1979).

[7]J. Speth, E. Werner, and W. Wild, Phys. Rep. **33**, 127

(1977).

[8]G. Rinker and J. Speth, Nucl. Phys. **A306**, 360 (1978).

[9]C. Papanicolas, thesis, Massachusetts Institute of Technology, 1979 (unpublished); B. Frois, in *Nuclear Physics with Electromagnetic Interactions,* edited by H. Arenhoevel and D. Drechsel, Lecture Notes in Physics Vol. 108 (Springer-Verlag, New York, 1979), p. 52; B. Frois and C. Papanicolas, private communication.

[10]J. Martorell and D. W. L. Sprung, Z. Phys. A **298**, 153 (1980).

[11]J. L. Friar and J. W. Negele, Adv. Nucl. Phys. **8**, 219 (1975)