Quark Vacuum Polarization and the Lüscher Term

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The Lüscher term found in the QCD string model and in Monte Carlo lattice gauge calculations is significantly modified by vacuum polarization due to light-quark loops. This modification improves agreement with experiment.

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Advances in computing power and development of the Monte Carlo lattice gauge (MCLG) method[1] has made possible direct and fundamental evaluation of nonperturbative QCD. Although the inclusion of light-quark loops is not yet practical, the main results of QCD, confinement and asymptotic freedom, are believed to follow with or without their presence. All high-statistics calculations have therefore been done in the "quenched" approximation and one hopes to obtain results close to experiment with the neglect of quark vacuum polarization.

One of the most impressive MCLG calculations[2] so far is the heavy-quark potential evaluated with use of the California Institute of Technology hypercube array.[3] This potential was calculated on a large lattice with high statistics. To an excellent approximation this potential can be parametrized[2] by

$$V_c(r) \simeq -\alpha_L/r + ar, \quad r > r_0, \tag{1}$$

where α_L has nearly the Lüscher[4] value of $\alpha_L = \pi/12$, the quenched string tension a is a fundamental parameter in the theory, and the constant term has been ignored since it is arbitrary in the MCLG method. The potential $V_c(r)$ of Eq. (1) agrees with the MCLG potential as calculated[2] down to $r_0 \simeq 0.25\ \mathrm{GeV}^{-1}$, a surprisingly small value. In momentum space the corresponding potential to Eq. (1) is

$$\hat{V}_c(Q) \simeq -4\pi\alpha_L/Q^2 - 8\pi a/Q^4, \quad Q < Q_0. \tag{2}$$

The Lüscher term is a characteristic[4] feature of QCD string models and is due to the zero-point transverse oscillations of the stretched string. For large quark separations the internal inconsistencies of string theories[5] in four dimensions are not important and the agreement between string theory and quenched MCLG calculations is satisfying.

Unfortunately, the quenched MCLG potential is not very successful[6] if straightforwardly applied to heavy-quark bound-state data. To see this directly we note that the phenomenologically successful (Cornell) potential of Eichten *et al.*[7] also has the form of Eq. (1) but with $\alpha_L \simeq 0.5$, roughly twice the Lüscher value. If we wish to maintain the correctness of QCD it must turn out that disagreement with data is a result of the quenched approximation. In this paper we will apply a simple model of quark vacuum polarization to the quenched MCLG result and show that the corrections will provide a good phenomenological result.

It is strongly indicated by the basic QCD formalism[8] and by recent MCLG investigations[9] that at large distances the interquark potential is effectively a Lorentz scalar. We will apply a simple model first suggested by Poggio and Schnitzer[10] to investigate quark vacuum-polarization corrections to a quenched scalar potential. In this model light-quark vacuum-polarization bubbles are inserted into the scalar confining potential as shown in Fig. 1. This procedure is motivated partly by the picture of a string between the heavy quarks with virtual-light-quark loops. As we will see, the effect of this polarization is to reduce the string tension and to generate an additional r^{-1} asymptotic term augmenting the Lüscher term. A natural extension of this model would imply string breaking by a cutting of one or more of these bubbles. Although the string-breaking process is of great interest we believe that it does not strongly affect the behavior at intermediate

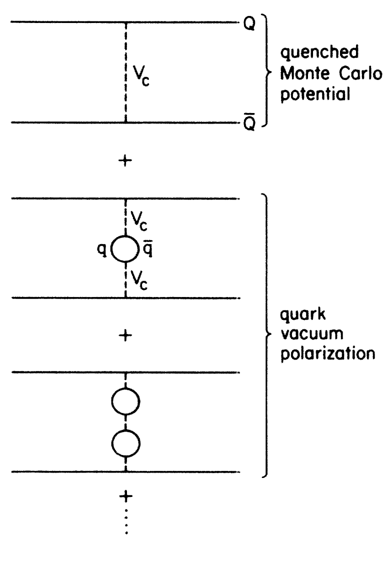

FIG. 1. Diagrammatic representation of the quark-loop corrections in the Monte Carlo lattice gauge potential V_c. The summation is given in Eq. (4).

distance where the low-lying $c\bar{c}$ and $b\bar{b}$ states lie. To see this remember that the difference between open-flavor threshold and the ground-state energy is a universal energy, roughly 0.6 GeV.[11] The string stretch accounting for this energy difference is

$$\Delta r \simeq (0.6\ \text{GeV})/a = 0.6/0.2 = 3\ \text{GeV}^{-1}. \tag{3}$$

Since the ψ ground-state radius is about 3 GeV^{-1}, the string-breaking distance will be about 6 GeV^{-1} and thus well over 1 fm. A supporting argument[12] concerns the MCLG static potential with adjoint charges. In this case ionization behavior was not evident even at 8 GeV^{-1}. We shall assume in this paper that the intermediate-range potential is not significantly modified by string breaking.

Starting with the quenched potential $\hat{V}_c$ in momentum space, the sum of quark polarization bubbles in Fig. 1 gives the renormalized potential $\hat{V}_R$ as[10]

$$\hat{V}_R(Q) = \hat{V}_c(Q)[1 - \bar{\pi}_1(Q) Q^4 \hat{V}_c(Q)]^{-1}, \tag{4}$$

where Q is the magnitude of the quark three-momentum. The quark vacuum polarization $\bar{\pi}_1(Q)$ must have vanishing first and second subtraction constants, so that if $\hat{V}_c$ is confining then $\hat{V}_R$ is also. The polarization then satisfies the subtracted dispersion relation[10]

$$\bar{\pi}_1(Q) = \frac{1}{\pi}\int_{4m^2}^{\infty} \frac{ds\, \text{Im}\bar{\pi}_1(s)}{s^2(s+Q^2)} \underset{Q\to 0}{\longrightarrow} D_0 - D_1 Q^2 + \ldots, \tag{5}$$

where the light-quark mass is m. At large radii the small-Q expansion of $\bar{\pi}_1$ will be useful.

Asymptotic potential.—We wish to examine the effect of light-quark vacuum polarization on the heavy-quark potential for intermediate and large quark separations. In momentum space this corresponds to a small-Q expansion of $\hat{V}(Q)$. For a quenched confining potential $-8\pi a/Q^4$ we see from Eqs. (4) and (5) that the resulting renormalized potential, is for small Q,

$$\hat{V}_R(Q) \underset{Q\to 0}{\longrightarrow} -8\pi K/Q^4 - 4\pi\alpha_R/Q^2, \tag{6}$$

where

$$K = a/(1+8\pi a D_0), \quad \alpha_R = 16\pi D_1 K^2.$$

The effect of vacuum polarization on the linear confinement term is twofold. First, the string tension is reduced from a to K, and, second, a Coulombic term induced which adds to the Lüscher term already present in Eq. (2). In coordinate space, the asymptotic potential is then

$$V(r) \underset{r\to\infty}{\longrightarrow} Kr - (\alpha_L + \alpha_R)/r. \tag{7}$$

We have implicitly assumed that the Lüscher term by itself is not a Lorentz scalar.

Specific model.—The lowest-order expression for $\bar{\pi}_1$ derives from the free-quark loop whose dispersive part is

$$\text{Im}\bar{\pi}_1 = C_A n_f s v^3/8\pi, \tag{8}$$

where $v = (1-4m^2/s)^{1/2}$ is the quark velocity, $\sqrt{s}$ is the c.m. $q\bar{q}$ energy, and C_A and n_f are respectively the number of colors and flavors. From Eq. (5) it follows that (with $C_A = 3 = n_f$)

$$D_0 = 9(80\pi^2 m^2)^{-1}, \quad D_1 = D_0/(14m^2). \tag{9}$$

By Eqs. (6), (7), and (9) the renormalized Lüscher term is

$$\alpha_L + \alpha_R = \pi/12 + 9K^2/70\pi m^4. \tag{10}$$

Using the physical string tension $K = 0.2\ \text{GeV}^2$ and a constituent mass of $m = 0.3$ GeV we find

$$\alpha_L + \alpha_R = 0.46. \tag{11}$$

We observe that renormalization always has the effect of reducing the string tension and increasing the Lüscher term. In the above simple model the scalar confining term by itself gives rise to a Lüscher-type term comparable to the quenched Lüscher term.

An interesting inference of this picture is that the Lüscher term plus renormalized confinement yields an effective Cornell potential[7] although the r^{-1} term has nothing directly to do with perturbative single-gluon exchange. The asymptotic form [see Eq. (1)] is accurate down to radii smaller than the upsilon state. This is still outside the perturbative range and so we conclude that the theoretical basis for the Cornell potential rests more on a string picture with vacuum-polarization corrections than an interpolation between linear confinement and short-range gluon exchange.

We have observed that light-quark vacuum polarization plays an important role in determining the force between two heavy quarks. The effect is particularly significant for $c\bar{c}$ and $b\bar{b}$ states where the magnitude of the effective r^{-1} term is of great importance. The Lüscher term alone is not large enough[6] to properly account for the energy levels and leptonic widths. A simple estimate of the polarization effects on string tension shows that the required doubling[6] of the Lüscher term is easily attainable.

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