Resonant Microwave Absorption in Superconductor-Normal-Superconductor Junctions

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Absorption of microwave power by superconductor-normal-superconductor junctions with square-well pair potentials at low temperatures is calculated to occur in sharp steps whenever the microwave energy is an integer multiple of the minimum energy required to excite quasiparticles from the ground state into bound Andreev states.

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Oscillations and step structures in the electromagnetic response of weakly coupled superconductors have been traced back previously to electron↔hole (Andreev) scattering by spatial variations of the pair potential. Examples are the Tomasch effect in superconducting tunnel junctions,[1,2] the quantized resistances in superconductor-normal-superconductor (SNS) film structures,[3,4] and the subharmonic energy-gap structures in microbridges.[5] The purpose of this paper is to show that similar, resonancelike phenomena are to be expected when photons lift quasiparticles into the size-quantized energy states formed by Andreev scattering in SNS junctions.

We consider an SNS junction with alternating potential difference $V(t)=V_0\cos\omega t$ applied across the N region of thickness $2a$. We assume that $2a$ is less than the critical thickness $2a_c$ above which phase coherence between the S layers is destroyed.[6] Then the vector potential corresponding to $V(t)$ must satisfy the gauge-invariant Josephson equation. In the N layer it is given by

$$\mathbf{A}=-\mathbf{e}_z c(V_0/2a\omega)\sin\omega t, \tag{1}$$

where a gauge is chosen which makes the superconducting pair potential $\Delta(z)$ real.[7] In the S layers $\mathbf{A}=0$.

The Bogoliubov–de Gennes equations for the two-component quasiparticle wave function in spinor notation, $\hat{\psi}(\mathbf{r},t)$, are given by

$$i\hbar(\partial/\partial t)\hat{\psi}=\hat{H}^0\hat{\psi}+\hat{H}^I\hat{\psi}, \tag{2a}$$

with

$$\hat{H}^0=(1/2m)(\mathbf{p}^2-\hbar^2k_F^2)\hat{\sigma}_z+\Delta(z)\hat{\sigma}_x \tag{2b}$$

describing the SNS junctions without fields, and

$$\hat{H}^I=-(e/2mc)(\mathbf{p}\cdot\mathbf{A}+\mathbf{A}\cdot\mathbf{p})\hat{I} \tag{2c}$$

being the interaction between quasiparticles and field; only terms linear in $\mathbf{A}$ are kept; $\hat{\sigma}_x$, $\hat{\sigma}_z$ are Pauli matrices, and $\hat{I}$ is the unit matrix.

In SNS junctions with N layers so thick that the pair potential $\Delta(z)$ can be approximated by a square-well potential,[8-12] which vanishes in $-a\le z\le +a$ and has the constant value Δ in the S regions, the energy eigenvalue equation for the bound states $E<\Delta$ of the unperturbed junction ($\hat{H}^I=0$) is

$$E_n(k_{Fz},\pm)=\hbar^2k_{Fz}(n\pi+\eta)/2ma, \quad n=0,1,2.\dots \tag{3}$$

These are the well-known Andreev levels,[8,9] localized in the N region; $\eta=\arccos E_n/\Delta$, $k_{Fz}\equiv[k_F^2-k_x^2-k_y^2]^{1/2}$. After linearizing $\arccos(E/\Delta)$ in such a way that the areas under the exact and the linearized curve are the same, i.e., $\eta\approx\pi/2-\pi^2E_n/8\Delta$, one has $\pi/2$ instead of η and an additional term $\hbar^2k_{Fz}\pi^2/8\Delta$ in the denominator on the right-hand side of Eq. (3).[13] The quasiparticle momenta in x and y directions are $\hbar k_x$ and $\hbar k_y$; + and − refer respectively to running waves with momenta in $+z$ and $-z$ directions,[6,8,9] or, alternatively, to standing waves of even and odd parity.[13] Within the (Andreev) approximation where continuity of the wave amplitudes implies continuity of the derivatives because of $(\Delta m/\hbar^2k_{Fz}^2)\ll 1$,[8,9] the + and − states are degenerate, as shown by Eq. (3).

Andreev reflection and the formation of the Andreev levels occurs only if $k_{Fz}>k_0\equiv 2(m\Delta)^{1/2}/\hbar$.[4] We assume that the superconducting regions extend over several coherence lengths so that the bound-state spectrum consists of Andreev levels (3) for $k_{Fz}\ge k_0$, and of the few ordinary potential-well states of a particle in a box for $k_{Fz}<k_0$.[4,14] Then the density of states is a piecewise linear function of E for $E\ge E_0(k_0)$ with jumps at each $E_n(k_0)$. These jumps should manifest themselves in the absorption of microwave power.

We treat $\hat{H}^I$ as a small perturbation of the eigenfunctions of $\hat{H}^0$ and do first-order time-dependent perturbation theory along the lines indicated in Refs. 6 and 13. We assume low temperatures $T\ll T_c$ and frequencies $\omega\le\Delta/\hbar$. Then the principal transitions in power absorption occur between the ground state and the excited bound states in the N region (and show the hybrid nature of the N layer of a junction): A photon knocks an electron out of the full Fermi sphere, a ground-state configuration of the N region, thus creating an "electron" (of energy E_p) above and a "hole" (of energy E_q) below the Fermi surface. When the excitations penetrate the superconducting banks, Andreev scattering[8] converts "electrons" into "holes" of the same energy, and vice versa, so that very shortly after the absorption of one quantum $\hbar\omega$ two Andreev levels of generally different energies are populated, and, as in a superconductor, *each* state has an electron and a hole component. The total momentum of each pair of excitations is practically zero and the total energy is

$$E_p(k_{Fz},\pm)+E_q(k_{Fz},\mp)=\hbar\omega. \quad (4)$$

In the formalism of the Bogoliubov-de Gennes equations, transitions between the ground state and pairs of excited states are described as transitions between states of negative and positive energy which together make up the complete set of solutions of the Bogoliubov-de Gennes equations.[6,7,15] Since the interaction $\hat{H}^I$ of Eq. (2) changes the parity of the quasiparticle wave functions and leaves the total momentum of the system unchanged, the energies in Eq. (4) refer to excitations of either opposite parities or opposite momenta.

The energy eigenvalues obtained from Eq. (3) with the linearized $\arccos(E/\Delta)$ turn the absorption condition (4) into

$$\hbar\omega=\frac{(p+q+1)(k_{Fz}\varepsilon_0/k_0)}{1+\pi k_{Fz}\varepsilon_0/8k_0\Delta},\quad p,q\in N_0, \quad (5)$$

where $\varepsilon_0\equiv\hbar^2k_0\pi/2ma$. The right-hand side of Eq. (5) increases monotonically with $k_{Fz}\geq k_0$. Thus, for a given $\hbar\omega$, the integer $p+q$ is maximum when k_{Fz} is minimum, and a new value of $p+q\equiv R$ becomes possible whenever $\hbar\omega$ passes through one of the values

$$\hbar\omega_R=(R+1)\varepsilon_0[1+\pi\varepsilon_0/8\Delta]^{-1}\leq\Delta. \quad (6)$$

Of course, at $\omega=\omega_R$, according to Eq. (5), there are also transitions with $p+q<R$ and $k_{Fz}>k_0$, $E_0(k_{Fz})\leq\Delta$. Thus, the total number N_R of transitions allowed by energy and momentum conservation at ω_R is found by counting all different combinations of p and q for which $0\leq p+q\leq R$:

$$N_R=\sum_{p+q=0}^{R}(p+q+1)=(R+1)(R+2)/2. \quad (7)$$

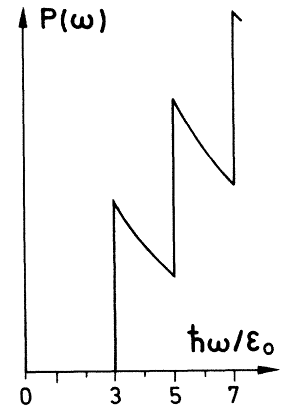

FIG. 1. Microwave power absorption (arbitrary units) at $T\ll T_c$ by an SNS junction of normal-layer thickness $2a=4000$ Å; the pair potential in the superconducting banks is $\Delta=1$ meV; $k_F=1$ Å^{-1}; $\omega/2\pi\leq 242$ GHz. In the energy spectrum (3) the approximation $\eta=\pi/2$ is being used so that because of $\beta_n=(\eta+\frac{1}{2})\pi/2a$ there are only contributions from odd $p+q+1$ to the sum in Eq. (9). In Eq. (6), $\pi\varepsilon_0/8\Delta=\pi^2\hbar(64ma^2\Delta)^{-1/2}=0.05$ is neglected. Without these approximations, i.e., with quasiparticle energies $E_n=\hbar^2k_{Fz}\pi(n+\frac{1}{2})/[2ma+\hbar^2k_{Fz}\pi^2/8\Delta]$, resonances occur also at photon energies given by Eq. (6) with even $p+q+1\equiv R+1$. If linewidth broadening by scattering from the impurities and phonons becomes comparable to $\hbar\omega$, the sharp steps in $P(\omega)$ will be rounded off.

This number stays constant for $\omega_R\leq\omega<\omega_{R+1}$ and jumps at ω_{R+1} by the amount

$$\Delta N_{R+1}\equiv N_R+1-N_R=R+2=\hbar\omega_{R+1}(1+\pi\varepsilon_0/8\Delta)/\varepsilon_0, \quad (8)$$

according to Eq. (6).

Steps in the microwave absorption spectrum are associated with the jumps ΔN_R in the number of transition channels. Power absorption calculated in first-order time-dependent perturbation theory results as

$$P(\omega)=\left(\frac{V_0e\hbar}{4ma}\right)^2\left(\frac{h}{\delta}\right)\frac{1}{\hbar\omega}\sum_{k_{Fz}}\sum_{p,q}(L_x^2+L_y^2)^{1/2}(k_F^2-k_{Fz}^2)^{1/2}(2a+\xi_p)^{-1}(2a+\xi_q)^{-1}\delta_{E_p+E_q,\hbar\omega}\times\left(\frac{\sin(\beta_p+\beta_q)a}{\beta_p+\beta_q}\{[1-\cos(p-q)\pi]2k_{Fz}-[1+\cos(p-q)\pi](\beta_p-\beta_q)\}\right)^2. \quad (9)$$

Here $\beta_n\equiv E_n(k_{Fz})m/\hbar^2k_{Fz}$ and $\xi_n=\hbar^2k_{Fz}/m(\Delta^2-E_n^2)^{1/2}$. Energy conservation is expressed by the Kronecker delta which picks out all discrete values of k_{Fz}, $k_0\leq k_{Fz}\leq k_F$, that satisfy Eq. (4) or its approximation (5) for a given $\hbar\omega$. The factor involving the squares $(L_x^2+L_y^2)$ of the junction dimensions in x and y directions results from integration over all k_x and k_y which lie in a small ring of radius $(k_F^2-k_{Fz}^2)^{1/2}$ and thickness $[(2\pi/L_x)^2+(2\pi/L_y)^2]^{1/2}$. The average linewidth δ of a bound state is assumed to be broader than the perturbation-theoretic energy resonance. Consistent with the Andreev approximation, terms of relative order of $(\Delta m/\hbar^2 k_{Fz}^2)^2$ are neglected. The terms with $p=q$ are zero. This is a consequence of the fact that in the transition matrix elements for $p=q$, the contributions from the surface charges ($\sim$divA) in $z=\pm a$ cancel against the contributions from the field in the N region.

Equation (9) has been evaluated numerically for various a; Fig. 1 shows one result. Although the transition-matrix elements give different weights to the transition channels, the jumps in power absorption are nearly the same at all ω_R, so that it looks as if they were proportional to the ω-independent product of the main ω-dependent factors ΔN_R, $\hbar\omega_R$, and $\mathbf{A}^2$ that determine the change in power absorption at ω_R.

There are three reasons for the steep, resonancelike increases of power absorption at equidistant frequencies ω_R: (1) the pairwise excitation of quasiparticles from the ground state and the numerous possible ways of distributing them at given total energy over equidistant Andreev levels; (2) the square-well shape of the pair potential which, in conjunction with Andreev scattering, is responsible for the constant spacing of the energy levels at fixed momenta parallel to the NS interfaces; and (3) the lower cutoff of the Andreev spectrum at $k_{Fz}=k_0 \equiv 2(m\Delta)^{1/2}/\hbar$. Nonequidistant energy levels, in general, will lead to irregular, smaller jumps in the number of transition channels and absorbed power. Thus, microwave absorption can serve as a test of the pair potential's spatial variation in an SNS junction. Suppression of the proximity effect in the N region by a magnetic field may help tailor the square well[3] without destroying the Andreev levels.[13] The absorption spectrum is expected to have a small background resulting from transitions into the few ordinary potential-well states with $k_{Fz} < k_0$. Generation of the high-frequency electric field normal to the phase boundaries in the N layer is a problem for which strip-line techniques may offer the solution.

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