Oscillatory Exchange of Atoms between Traps Containing Bose Condensates

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An oscillatory exchange of atoms governed by the phases of the "macroscopic wave functions" between two traps containing Bose condensates, as might be realized with laser cooling and trapping, is predicted. The discussion exploits analogs from lasers and the Josephson junction.

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Experiments on light-pressure cooling[1-5] of atomic beams have recently led to the first demonstrations of magnetic[6] and all-optical[7] traps for neutral atoms. One of the motivations of this effort is to achieve low enough temperatures and high enough atomic densities so that the effects of the statistics of the atoms, e.g., Bose condensation, would eventually be observed.[8] In this Letter I point out that Bose condensation in such traps may lead to a novel macroscopic quantum phenomenon analogous to the Josephson junction[9]: When two traps containing the condensates are brought close to each other, an oscillatory exchange of particles governed by the phases of the "macroscopic wave functions" of the two atomic gases should result.

For the sake of concreteness I concentrate on a noninteracting Bose gas of atoms with mass M and spin 0 in an isotropic three-dimensional harmonic-oscillator potential characterized by the trapping frequency ν. When the energy of the ground state of the trap is chosen to equal zero, the Bose-Einstein statistics gives the occupation number of the state $\mathbf{n}=(n_1,n_2,n_3)$ with energy $\varepsilon_\mathbf{n}=\hbar\nu(n_1+n_2+n_3)$ at temperature T in the form

$$N_\mathbf{n}=[e^{(1/kT)(\varepsilon_\mathbf{n}-\mu)}-1]^{-1}. \tag{1}$$

To ensure positive occupation numbers the chemical potential μ has to remain negative; hence the occupation numbers of the states other than the lowest one ($\mathbf{n}=0$) are bounded. The total number of atoms outside the ground state can be at most

$$N_c=\sum_{n_1,n_2,n_3>0} N_\mathbf{n}(\mu=0). \tag{2}$$

For the quantum-gas limit $kT/\hbar\nu \lesssim 1$ this yields $N_c \lesssim 1$, whereas in the opposite case the critical atomic number is

$$N_c=\zeta(3)[(kT/\hbar\nu)]^3, \tag{3}$$

where $\zeta(3)=\sum_{N>0} n^{-3} \approx 1.202.$[10]

Bose condensation occurs when the number of atoms N exceeds N_c. $N_0=N-N_c$ atoms must then be packed into the ground state. In the high-temperature limit the density of the gas at the trap center is $N(M\nu^2/2\pi kT)^{3/2}$. This correctly[11] suggests that in the trap the maximum density of atoms at the Bose-condensation point scales with the mass and temperature as $(MkT/\hbar^2)^{3/2}$, just as for free atoms.

An anomalously high atomic density at the center of the trap might[11] be employed as a signature of the macroscopic population of the ground state,

$$\langle b_0^\dagger b_0\rangle = N_0. \tag{4a}$$

However, analogously[12] to laser theories[13] which predict the Poissonian photon statistics but not the coherent field commonly used to model an ideal laser, I assume that also the *boson operators* themselves acquire macroscopic expectation values. I therefore write

$$\langle b_0\rangle = e^{-i\phi}\sqrt{N_0},\ \ \langle b_0^\dagger\rangle \approx e^{i\phi}\sqrt{n_0}. \tag{4b}$$

The (random) phase ϕ is attributed to spontaneous symmetry breaking.[14] The expectation value of the boson operator b serves as the order parameter when Bose condensation is viewed as a phase transition, much as in the theory of the λ transition of ^{4}He.[14,15]

The value of the phase ϕ alone apparently has no observable consequences, no more than photon-counting experiments can show that a single laser beam has a phase. But *two* lasers can beat against each other, revealing their *phase difference.* Analogously, I shall consider the case when two traps containing Bose condensates are brought close enough to exchange atoms. When the traps are far apart their ground-state wave functions ψ_l and ψ_r are degenerate; the operators that annihilate bosons with these wave functions are denoted by b_l and b_r. I assume that the interaction between the traps is strong enough to lift the degeneracy, but too weak to mix oscillator states that were not degenerate initially. The relevant Hamiltonian for the ground state thus reads

$$H/\hbar=\kappa(b_l b_r^\dagger + b_r b_l^\dagger). \tag{5}$$

As expected, this Hamiltonian is diagonalized by the boson operators pertaining to the even and odd superpositions of the "left" and "right" wave functions, $\psi_{e,o}=(1/\sqrt{2})(\psi_l \pm \psi_r)$, with corresponding energies $\pm\hbar\kappa$.

The Heisenberg equations of motion for the annihilation operators under the Hamiltonian (5) can be solved immediately. Using the counterparts of Eqs. (4) for both traps and taking the traps to be initially uncorrelated, I obtain for the ground-state population in the "left" trap the expression

$$\langle b_l^\dagger(t) b_l(t)\rangle = N_l \cos^2\kappa t + N_r \sin^2\kappa t + \sqrt{N_l N_r}\sin(\phi_l - \phi_r)\sin 2\kappa t. \quad (6)$$

Apart from trivial oscillations of the population between the traps if they start out with different numbers of atoms, there appears an interference term which is effective even if $N_l = N_r$. Oscillatory transfer of atoms between the traps under such a condition constitutes a novel macroscopic quantum phenomenon.

Another angle to this result is obtained from an elementary textbook treatment of the Josephson junction.[16] Accordingly, I ascribe to the condensate a "macroscopic wave function" ψ such that $|\psi(r)|^2$ gives the particle density, and $\psi(r,t)$ obeys the time-dependent one-particle Scrödinger equation.[17] Assume that at time $t=0$ the same number of atoms is present in both traps, and that the total wave function is a superposition of the isolated-trap ground states, with some phases:

$$\psi(r,t=0) \propto [e^{-i\phi_l}\psi_l(r) + e^{-i\phi_r}\psi_r(r)]. \quad (7)$$

The time-dependent wave equation is solved concisely by utilization of the eigenfunctions $\psi_{e,o}$, which leads to the special case of Eq. (6) with $N_l = N_r$. The interference oscillations are absent when the phase difference $\phi_l - \phi_r$ is a multiple of π, because the initial state then is an eigenstate of the one-particle Hamiltonian and the evolution only affects the unobservable overall phase.

To estimate the time scale of the interference oscillations I calculate the energy difference between the states ψ_e and ψ_o for a double-well potential which sharply switches over between two harmonic oscillators at the midpoint between the wells. In the limit of large trap separation the result implies that

$$\kappa = -(\nu/2\sqrt{\pi})x\exp(-x^2/4), \quad (8)$$

where $x = l/a$ gives the distance between the wells l in the units of the trap length $a = (\hbar/M\nu)^{1/2}$. The form of Eq. (8) mainly reflects the overlap of the wave functions of the two harmonic oscillators.

Although the calculations were carried out for an isotropic harmonic trap, the qualitative conclusions clearly apply to existing[6,7] and proposed[18,19] traps which are not isotropic, some[6] not even harmonic. For a concrete numerical example I employ the parameters of the first all-optical trap.[7] The harmonic-oscillator frequency was of the order of $2\pi\times 100$ kHz, and the temperature was about 240 μK. The criterion (3) gives for these parameters $N_c \sim 10^5$, so that with the present number of atoms, ~ 500, Bose condensation cannot be expected. However, the existing traps are first demonstrations, and improvements towards Bose condensation can be anticipated. The temperature was close to the theoretical[20] "quantum limit" of optical cooling set by the linewidth of the transition, $\sim \hbar\gamma/k$, but methods to circumvent this limit are being contemplated.[21-23] Moreover, the trapping frequency in optical gradient-force traps can in principle be increased at will by an increase of the laser intensity, at least until multiphoton transitions and multiphoton ionization set in.

It should be recognized, though, that spin-polarized hydrogen may be the only atomic substance that remains a gas in thermal equilibrium at microkelvin temperatures.[24] In future experiments it may be worthwhile to notice that hydrogen has an optical pumping cycle similar to sodium[6] which leads to a state with parallel electronic and nuclear spins, thus yielding optimal stabilization of hydrogen against recombination. Unfortunately, because of the short wavelength, optical cooling and trapping of hydrogen pose an overwhelming challenge to today's laser techhnology.

If Bose condensation can be achieved, the next step is to manipulate two traps close to one another. For instance, with $\nu = 2\pi\times 100$ kHz and $M = 1$ amu, the characteristic size of the trap is $a = 0.3$ μm and the period of the population oscillations π/κ is 1 h for $l = 3$ μm. Apart from practical difficulties, the basic physics of electromagnetic traps imposes constraints on the construction of double potential wells which may call for new innovations.[25]

The small (in many-particle standards) number of atoms and the interactions between them constitute additional sources of possible problems, but also of interesting physics. Rigorous phase transitions only emerge when the number of particles is infinite. For trapped atoms the thermodynamic limit can be[26] defined by letting $N \to \infty$, $\nu \to 0$ in such a way that $N\nu^3$ stays constant. The price paid for this procedure is that in an ideal gas the density of the condensate tends to infinity, too. Consequently, even minute atom-atom interactions drastically modify,[11,27,28] or perhaps obliterate,[29] the Bose condensation. If the condensation survives the thermodynamic limit, the question as to how well the macroscopic phase will be defined in a finite sample should still be addressed.

Finally, description of an optical trap as a potential well is a bold simplification. Optical cooling and trapping are complicated dynamical processes which invariably involve transient excitations of the internal states of the atoms, thus distorting their indistinguishability. The implications of the trap dynamics on the condensation and on the macroscopic phase remain to be studied.

In summary, I have speculated about a novel macroscopic quantum phenomenon; oscillation of atoms between two traps containing Bose condensates. It may be that experimental exploration of the effect has to await laser technology capable of handling the hydrogen $1s$-$2p$ transition frequency; but meanwhile, the implications of the atomic interactions and of the finite number of particles on the nature of the Bose condensation and on the macroscopic phase, in simple trap models as well as in realistic dynamic traps, might turn out to be of some theoretical interest.

[1]V. I. Balykin, V. S. Letokhov, and A. I. Sidorov, Zh. Eksp. Teor. Fiz. **86**, 2019 (1984) [Sov. Phys. JETP **59**, 1174 (1984)].

[2]W. D. Phillips and H. Metcalf, Phys. Rev. Lett. **48**, 596 (1982).

[3]W. Ertmer, R. Blatt, J. L. Hall, and M. Zhu, Phys. Rev. Lett. **54**, 996 (1985).

[4]S. Chu, L. Hollberg, J. E. Bjorkholm, A. Cable, and A. Ashkin, Phys. Rev. Lett. **55**, 48 (1985).

[5]R. N. Watts and C. E. Wieman, Opt. Lett. **11**, 291 (1986).

[6]A. Migdall, J. V. Prodan, W. D. Phillips, T. H. Bergeman, and H. J. Metcalf, Phys. Rev. Lett. **54**, 2596 (1985).

[7]S. Chu, J. E. Bjorkholm, A. Ashkin, and A. Cable, Phys. Rev. Lett. **57**, 314 (1986).

[8]The quest toward Bose condensation has also motivated extensive studies of spin-polarized hydrogen. For an overview and references see, e.g., T. J. Greytak and D. Kleppner, in *New Trends in Atomic Physics,* edited by G. Grynberg and R. Stora (North-Holland, Amsterdam, 1984), Vol. 2, p. 1125.

[9]B. D. Josephson, Phys. Lett. **1**, 251 (1962).

[10]Some of these results have already been discussed by V. Bagnato and D. E. Pritchard (to be published).

[11]J. T. M. Walraven and I. F. Silvera, Phys. Rev. Lett. **44**, 168 (1980).

[12]F. W. Cummings and J. R. Johnston, Phys. Rev. **151**, 105 (1966).

[13]M. Sargent, III, M. O. Scully, and W. E. Lamb, Jr., *Laser Physics* (Addison-Wesley, Reading, MA, 1974). See in particular Chap. 17.

[14]D. Forster, *Hydrodynamic Fluctuations, Broken Symmetry and Correlation Functions,* Frontiers in Physics Vol 47 (Benjamin, Reading, MA, 1975).

[15]E. M. Lifschitz and L. P. Pitaevskii, *Statistical Physics* (Pergamon, Oxford, 1980), Pt. 2.

[16]C. Kittel, *Introduction to Solid State Physics* (Wiley, New York, 1986), 6th ed.

[17]The expectation value of the boson field operator in the condensed state provides such a ψ. In optics the electric and magnetic fields correspond to ψ, and Maxwell's equations replace the Schrödinger equation.

[18]R. V. E. Lovelace, C. Mehanian, T. J. Tommila, and D. M. Lee, Nature (London) **318**, 30 (1985).

[19]D. E. Pritchard, E. L. Raab, V. Bagnato, C. E. Wieman, and R. N. Watts, Phys. Rev. Lett. **57**, 310 (1986).

[20]For a comprehensive review of laser-cooling theories consult S. Stenholm, Rev. Mod. Phys. **58**, 699 (1986).

[21]D. E. Pritchard, Phys. Rev. Lett. **51**, 1336 (1983).

[22]S. Chu, J. E. Bjorkholm, A. Ashkin, J. P. Gordon, and L. W. Hollberg, Opt. Lett. **11**, 73 (1986).

[23]M. Lindberg and J. Javanainen, J. Opt. Soc. Am. B **3**, 1008 (1986).

[24]W. C. Stwalley and L. H. Nosanow, Phys. Rev. Lett. **36**, 910 (1976).

[25]The development leading to the present traps is summarized by W. D. Phillips, J. V. Prodan, and H. J. Metcalf, J. Opt. Soc. Am. B **2**, 1751 (1985).

[26]S. R. de Groot, G. J. Hooyman, and C. A. ten Seldam, Proc. Roy. Soc. London, Ser. A **203**, 266 (1950).

[27]V. V. Goldman, I. F. Silvera, and A. J. Leggett, Phys. Rev. B **24**, 2870 (1981).

[28]D. A. Huse and E. D. Siggia, J. Low Temp. Phys. **46**, 137 (1982).

[29]The same argument has been used against two-dimensional Bose condensation by J. J. Rehr and N. D. Mermin, Phys. Rev. B **1**, 3160 (1970). It is not clear to the present author where the dimensionality of space enters into this reasoning.