Is There a Breakdown of Quantum Electrodynamics?

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Through use of a previous calculation of the light-by-light scattering contribution to the electron anomaly in sixth order, it is shown that theory and experiment disagree by 4 standard deviations. Our result is $[a \equiv (g-2)/2]$ $a_e(\text{theor}) - a_e(\text{expt}) = 604(149)\times 10^{-12}$.

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About ten years ago it was noticed that the light-by-light scattering contribution to the anomalous magnetic moment of the muon $a_\mu(\gamma\gamma)$ (see Fig. 1) as calculated by several authors[1-4] disagreed with each other well outside of their assigned 91% confidence levels:

$$\frac{a_\mu(\gamma\gamma)}{(\alpha/\pi)^3} = \begin{cases} 18.4(1.1) & (\text{Ref. 1}), \\ 20.77(43) & (\text{Ref. 2}), \\ 19.76(16) & (\text{Ref. 3}), \\ 19.79(16) & (\text{Ref. 4}). \end{cases} \quad (1)$$

(Throughout this paper the number in parentheses represents the error in the last significant figures.) It was shown[5] that the previous difficulty was due to a singularity in a four-dimensional subspace which the seven-dimensional Feynman parametric integrand possesses.

The integration was treated in a careful and systematic way by introduction of a cutoff ε and extrapolation to $\varepsilon = 0$. The integrations were performed with several sets of variables. The results were consistent with each other and in each case with the extrapolation, for example, $I(\varepsilon) = I_0 - B\sqrt{\varepsilon}$ with $B = (\pi^2/3)\ln(m_\mu/m_e) = 17.5$. The result was significantly higher than those in Eq. (1):

$$a_\mu(\gamma\gamma) = 21.32(5)(\alpha/\pi)^3. \quad (2)$$

A more recent calculation[6] by Kinoshita, Nizic, and Okamoto yields a result significantly larger than Eq. (1) and closer to the result given in Eq. (2):

$$a_\mu(\gamma\gamma) = 20.952(11)(\alpha/\pi)^3. \quad (3)$$

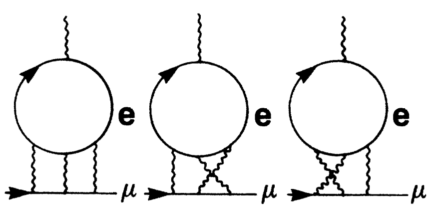

FIG. 1. Feynman diagrams of the photon-photon scattering type contributing to the sixth-order muon anomalous magnetic moment. There are three more diagrams obtained by reversal of the direction of the electron loop.

With use of the result in Eq. (2), the latest hadronic contribution[6]

$$a_\mu(\gamma\gamma) = 702(19)\times 10^{-10}, \quad (4)$$

and the ac-Josephson-effect value of the fine-structure constant[7]

$$\alpha^{-1} = 137.035963(15) \quad (5)$$

(this value of α is used throughout this paper), I obtain

$$a_\mu(\text{theor}) = 1\,165\,925(3)\times 10^{-9}, \quad (6)$$

in excellent agreement with the experimental result from the last CERN $g-2$ experiment[8]

$$a_\mu(\text{expt}) = 1\,165\,922(9)\times 10^{-9}. \quad (7)$$

The earlier CERN result[9] was

$$a_\mu(\text{expt}) = 1\,165\,895(27)\times 10^{-9}. \quad (8)$$

The result in Eq. (2) was obtained before the latest CERN result Eq. (7) was obtained.

The light-by-light scattering contribution to the muon anomaly can be written as

$$a_\mu(\gamma\gamma) = [A\ln(m_\mu/m_e) + O(1)](\alpha/\pi)^3 \quad \text{for } m_\mu/m_e \gg 1. \quad (9)$$

The coefficient A had been evaluated twice[10,11]:

$$A = \begin{cases} 6.38(8) & (\text{Ref. 10}), \\ 6.29(6) & (\text{Ref. 11}). \end{cases} \quad (10)$$

We subsequently evaluated A with the ε-cutoff method. A is given by a five-dimensional integral with the same singularity structure as in $a_\mu(\gamma\gamma)$. Our result was again significantly higher than the previous calculations given in Eq. (10). We obtained[12]

$$A = 6.58(2). \quad (11)$$

On the basis of this result we guessed that the answer was $2\pi^2/3$. A subsequent analytical calculation[13] showed that the result was indeed

$$A = 2\pi^2/3. \quad (12)$$

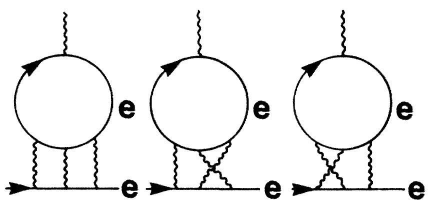

FIG. 2. Feynman diagrams of the photon-photon scattering type contributing to the sixth-order electron anomalous magnetic moment. There are three more diagrams obtained by reversal of the direction of the electron loop.

This gave us great confidence in our technique (the ε-cutoff method). Based on this confidence, I decided to apply this method in the case of the electron (see Fig. 2), since the integrand in this case has the same singularity structure. Here again I found a result significantly higher than others have found. Their value is[14,15]

$$a_e(\gamma\gamma)=0.370986(20)(\alpha/\pi)^3. \quad (13)$$

My published value[16] is

$$a_e(\gamma\gamma)=0.400(6). \quad (14)$$

More recent computations that we have done are consistent with Eq. (14) and lead to a slightly better value,

$$a_e(\gamma\gamma)=0.398(5). \quad (15)$$

This is the value which I shall use here.

Here again the integrations were performed with several sets of variables. The results were consistent with each other and, in each case with the extrapolation, for example, $D(\varepsilon)\sim I_0-B\sqrt{\varepsilon}$ with $B=2.89$ separately evaluated. I have tried to reconcile the result (15) with that of Eq. (13). Unfortunately it is difficult to compare the two calculations.

The results for the anomalous magnetic moment of the electron are given in Table I.[17] It can be seen that there is a significant discrepancy between theory and experiment[18]:

$$a_e(\text{theor})-a_e(\text{expt})=604(4)(128)(76)\times10^{-12}. \quad (16)$$

The numbers in parentheses represent the estimated uncertainties in the experimental result, the fine-structure constant, and the theoretical result, respectively. (The theoretical error is really a 91% confidence level.) If I am conservative and combine the errors quadratically I obtain

$$a_e(\text{theor})-a_e(\text{expt})=604(149)\times10^{-12}. \quad (17)$$

If one takes this discrepancy seriously and treats the electron as a composite particle with composite mass M and assumes[19] that $\delta a_e\sim m_e/M$, then $M\sim10^3$ TeV.

TABLE I. a_e in units of 10^{-12}.

$\alpha/2\pi$	1 161 410 039(128)
$-0.328\,478\,966(\alpha/\pi)^2$	−1 772 306
$1.204(5)(\alpha/\pi)^3$	15 083(65)
$-0.8(1.4)(\alpha/\pi)^4$	−23(40)
Muon vacuum polarization	2.8
Hadronic vacuum polarization	1.6(2)
Weak interaction	0.05
a_e(theor)[a]	1 159 652 797(128)(76)
a_e(expt)	1 159 652 193(4)
a_e(theor) − a_e(expt)[b]	604(4)(128)(76) = 604(149)

[a]Numbers in parentheses represent the estimated uncertainties in the fine-structure constant and the theoretical result, respectively.

[b]Numbers in parentheses represent the estimated uncertainties in the experimental result, the fine-structure constant, and the theoretical result, respectively.

However, if the composite model has chiral invariance, then $\delta a_e\sim(m_e/M)^2$ and $M\sim20$ GeV. This is already ruled out by high-energy tests such as $e^+e^-\to\mu^+\mu^-$ at the SLAC and DESY storage rings PEP and PETRA.

There are some interesting theoretical possibilities to account for this discrepancy. For example, an elementary pseudoscalar axion[20] enters a_e with a negative sign. Also, in an E_6 model with exotic leptons, the contribution to a_e is negative and of the right magnitude to account for this discrepancy.[21]

In summary, the purpose of this paper is to point out that there is a serious discrepancy between theory and the latest experimental measurement of a_e, provided that Eq. (15) is used for $a_e(\gamma\gamma)$. In addition, I wish to emphasize the necessity of a completely analytical calculation of $a_e(\gamma\gamma)$.

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