Range of Feeble Forces from Higher Dimensions

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The beautiful scheme of Bars and Visser by which feeble intermediate-range forces arise from higher dimensions is investigated. It is found that the desirable relation between the coupling and mass of the feeble-force vector, $m_V \approx g_V \Lambda_{\text{weak}}$, is difficult to achieve in simple schemes of symmetry breaking, though it is possible if new scales are introduced. An analogy is made with the paraphoton picture of Holdom.

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Recently, a very clever scheme has been proposed by Bars and Visser[1] by means of which very weak, Abelian gauge forces can arise in the effective low-energy four-dimensional limit of Kaluza-Klein theories. The effective coupling of this kind of force to ordinary particles of mass $m \ll M_{\text{Pl}}$, where M_{Pl} is the Planck mass, is of order $m/M_{\text{Pl}} \approx 10^{-20}$; hence Bars and Visser dub these forces "feeble." The value of $g_V \approx 10^{-20}$ for the coupling is roughly what would be needed to explain the "fifth force" of Fischbach *et al.*[2] Indeed, this is one of the considerations that motivated the authors of Ref. 1, although they emphasize that the possibility of such feeble forces is phenomenologically interesting quite apart from the validity of the claims in Ref. 2. They point out the fascinating phenomenological relation between the mass and coupling of the feeble-force vector required to account for the claims of Ref. 2 that $m_V \approx g_V \Lambda_{\text{weak}}$. One might suppose that such a relation would naturally emerge in a simple fashion in a theory in which symmetry breaking occurs at a scale Λ_{weak}. Actually this relation is difficult to achieve. Indeed, simply breaking the gauge symmetries at the scale Λ_{weak} in the normal way leads either to infinite-range forces that decouple exactly from light matter or to forces with the range of the weak interactions. It is possible to break symmetries in such a way as to achieve the above relation but this appears to require introducing some further complication such as additional breaking scales.

We will demonstrate the truth of these assertions in the toy model of Ref. 1, though they apply more generally to all models of that type. We adhere to the notational conventions of Ref. 1.

The model consists of the following elements. There are five space-time dimensions of which one is compactified on a circle. The *fünfbein* has the usual Kaluza-Klein structure:

$$E_\mu^a = e_\mu^a(x,y) = \sum_{-\infty}^{\infty} e_{\mu n}^a(x)\exp(inyM); \quad E_\mu^5 = (16\pi G)^{1/2} V_\mu(x); \quad E_5^5 = 1; \quad E_5^a = 0.$$

μ is a four-dimensional world index and takes values 0, 1, 2, 3. a is a four-dimensional tangent-space index. N and A will be used to denote five-dimensional world and tangent-space indices taking values 0, 1, 2, 3, and 5. We have set the classically massless scalar dilatation mode, denoted by $\sigma(x)$ in Ref. 1, equal to zero as we are only interested in long-range vector, not scalar, interactions. This simplifies our formulas without affecting the conclusions in any way. $V_\mu(x)$ is the Kaluza-Klein gauge field. The fifth coordinate is $y \equiv x^5$ with $0 \le y \le 2\pi/M$. The matter fields are contained in a single Dirac fermion $\chi(x,y)$. This can be expanded in modes on the circle in the usual way:

$$\chi(x,y) = \left(\frac{M}{2\pi}\right)^{1/2} \sum_{-\infty}^{\infty} \exp\left(\frac{i\gamma_5\pi}{4}\right) \psi_n(x)\exp(inyM).$$

The chiral rotation $\exp(i\gamma_5\pi/4)$ is introduced to bring the four-dimensional mass terms to the usual form. The fields that correspond to ordinary light matter, such as quarks and leptons, are the zero modes of the Dirac operator on the circle, in this case just $\psi_0(x)$. It is well known that, if the model consisted only of the foregoing, the light matter ψ_0 would be exactly neutral under the long-range Kaluza-Klein force mediated by $V_\mu(x)$, for the Kaluza-Klein charge operator, which we will denote by Q_{KK}^0, is just $i\partial_5/M$. Bars and Visser noticed that by the introduction of an explicit five-dimensional gauge field, denoted by $A_N(x,y)$, it is possible to shift the Kaluza-Klein charges by a small amount. Let the gauge coupling and charge operator of this explicit U(1) be called g and Y. Then the kinetic energy and mass terms for ψ_0 in the effective $d=4$ theory will have the form

$$\bar{\psi}_0\gamma^\mu[i\partial_\mu+gYA_\mu^{(0)}(x)-(16\pi G)^{1/2}V_\mu(x)gYA_5^{(0)}(x)]\psi_0-\bar{\psi}_0 gYA_5^{(0)}(x)\psi_0. \tag{1}$$

We have expanded

$$A^N(x,Y)=\sum_{-\infty}^{\infty}A^{(n)N}(x)\exp(inyM),$$

and used gauge freedom to bring $A^5(x,y)$ to the form $A^{(0)5}(x)\equiv\phi(x)$. Notice now that if $\langle A_5^{(0)}\rangle\equiv\langle\phi\rangle\neq0$, and if ψ has nonzero charge Y, the fermion ψ_0 will acquire a coupling to V_μ of order $(16\pi G)^{1/2}\langle\phi\rangle$, and a mass m_0 of order $\langle\phi\rangle$. So the Kaluza-Klein charge of the light particles is of order

$$Q_{\rm KK}=(16\pi G)^{1/2}m_0=m_0/M_{\rm Pl},$$

as promised. (We have departed from the notation of Ref. 1 by explicitly introducing the charge operator Y.) Let us make several observations. First, the Kaluza-Klein charge operator $Q_{\rm KK}$ is shifted from the naive one as follows:

$$Q_{\rm KK}=Q_{\rm KK}^0+gYA_5^{(0)}/M=(i\partial_5+gYA_5^{(0)})/M=iD_5/M. \tag{2}$$

Excited modes, ψ_n, of the fermion with $n\neq0$ will still have substantial Kaluza-Klein charges $Q_{\rm KK}\cong n$. But ψ_0 which had heretofore had no coupling to the boson V_μ now has a "feeble" coupling. We have only written the terms in Eq. (1) that are relevant for our considerations. We can write the four-dimensional covariant derivative, in general, as

$$iD_\mu=i\partial_\mu+gYA_\mu^{(0)}(x)-(16\pi G)^{1/2}V_\mu(x)[i\partial_5+gY\langle A_5^{(0)}\rangle]. \tag{3}$$

So far we have simply been reiterating the conclusions of Ref. 1. Now, we have reached the significant point where we can talk of giving mass to $V_\mu(x)$. Suppose we do this by having some Higgs field

$$H(x,y)=\sum_{-\infty}^{\infty}H^n(x)\exp(inyM)$$

acquire a vacuum expectation value (VEV). The crucial point to notice is that there are two U(1) gauge groups in four dimensions as emphasized by Bars and Visser, $\mathrm{U}(1)_{\text{Kaluza-Klein}}$ and $\mathrm{U}(1)_{\text{explicit}}$, or $\mathrm{U}(1)_V$ and $\mathrm{U}(1)_A$ as we will denote them. Let us suppose that $H(x,y)$ has a charge under $\mathrm{U}(1)_A$ of $Y=y$. Then $H^0(x)$ will have

$$Q_{\rm KK}(H^0)=(i\partial_5+gY\langle A_5^{(0)}\rangle)/M=gy\langle A_5^{(0)}\rangle/M=m_0/M. \tag{4}$$

One might expect that when $\langle H^0(x)\rangle$ acquires a VEV of Λ then $V_\mu(x)$, which couples to $(16\pi G)^{1/2}MQ_{\rm KK}$, will acquire a mass of

$$m_V=(16\pi G)^{1/2}m_0\Lambda=\left(\frac{m_0}{M_{\rm Pl}}\right)\Lambda\approx10^{-20}\Lambda. \tag{5}$$

However, this is mistaken. There are, before symmetry breaking, *two* massless vectors: A_μ and V_μ. If H^0 acquires a VEV only one linear combination becomes massive, and that is mostly A_μ. The orthogonal linear combination, which is mostly V_μ, remains exactly massless. Let us rewrite Eq. (3) as

$$iD_\mu=i\partial_\mu+gY[A_\mu^{(0)}(x)-\{(16\pi G)^{1/2}\langle A_5^{(0)}\rangle\}V_\mu(x)]-(16\pi G)^{1/2}(i\partial_5)V_\mu(x). \tag{6}$$

It is clear that any VEV of a Higgs-field zero mode $(H^{n=0})$ will give mass only to the linear combination $A_\mu^{(0)}(x)-\{(16\pi G)^{1/2}\langle A_5^0\rangle\}V_\mu(x)\equiv A_\mu'$. The factor in the curly brackets is $\sim10^{-20}$. So essentially it is $A_\mu^{(0)}$ that acquires mass, and it is a mass of $gy\langle H^0\rangle$ which is not small but of order Λ. The (orthogonal) massless vector is $V_\mu(x)+\{(16\pi G)^{1/2}\langle A_5^0\rangle\}A_\mu^{(0)}\equiv V_\mu'(x)$. Rewriting the covariant derivative (3) in terms of A_μ' and V_μ', we find

$$iD_\mu=i\partial_\mu-(16\pi G)^{1/2}(i\partial_5)V_\mu'(x)+[gY+16\pi G\langle A_5^0\rangle i\partial_5+gY(16\pi G)\langle A_5^0\rangle^2]A_\mu'. \tag{7}$$

Note that the massless mode $V_\mu'(x)$ couples to the naive Kaluza-Klein charge $Q_{\rm KK}^0=i\partial_5/M$, not to the shifted charge $Q_{\rm KK}$. So $V_\mu'(x)$ decouples exactly from the sector of light matter fields. The feeble force is thus of infinite range and of zero strength on ordinary quarks, leptons, etc. It is perhaps worth pointing out that since A_μ' couples to ∂_5 as well as Y, in the presence of any possible Kaluza-Klein type superheavy dark matter this theory would generate new weak forces on ordinary light matter.

Can we perhaps break the gauge symmetries in another way to recover the desired feeble force? Not easily. This can be seen from Eq. (6). We can imagine the symmetry breaking being done by Higgs-field vacuum expectation values as far as the essential group-theoretical aspects are concerned. There are four kinds of Higgs field (we normalize g so that Y=integer):

$$H^{0,0},\ Q^0_{\rm KK}=Y=0,$$

$$H^{0,y},\ Q^0_{\rm KK}=0,\ Y=y,$$

$$H^{n,0},\ Q^0_{\rm KK}=n,\ Y=0,$$

$$H^{n,y},\ Q^0_{\rm KK}=n,\ Y=y.$$

We have been looking at the case where only $\langle H^{0,y}\rangle \approx \Lambda_{\rm weak}$ contribute. Clearly, $\langle H^{0,0}\rangle$ is irrelevant. If only $\langle H^{n,0}\rangle \approx \Lambda_{\rm weak}$ contribute with various n, then only $V_\mu(x)$ gets a mass of order $\Lambda_{\rm weak}$ and $A_\mu^{(0)}$ remains exactly massless (and $A_\mu^{(0)}$ couples to gY so is not a feeble force). If a single $\langle H^{n,y}\rangle \approx \Lambda_{\rm weak}$ contributes then one linear combination of V_μ and $A_\mu^{(0)}$, $gyA_\mu^{(0)} - (16\pi G)^{1/2}MnV_\mu$, gets mass of order $\Lambda_{\rm weak}$ and the orthogonal combination remains exactly massless and mediates a nonfeeble force. Finally, if a combination of nonorthogonal VEV's (in $Q^0_{\rm KK}$-Y space) of $\langle H^{0,y}\rangle$, $\langle H^{n,0}\rangle$, and $\langle H^{n,y}\rangle$ contribute, then both vectors acquire masses of order $\Lambda_{\rm weak}$ and there are only short-range (i.e., weak-range) forces. It is possible to give the feeble-force vector the desired range and coupling if we introduce new supersmall scales. There are three interesting cases. (We are indebted to Bars and Visser for pointing out to us these possibilities which we had not considered.)

1. Have $\langle H^{n,0}\rangle \approx \Lambda_{\rm weak} m_0/M_{\rm Pl}$ [where m_0 is a typical quark mass $O(g_V)M_{\rm Pl} \approx 10^{-20}M_{\rm Pl}$]; and keep A_μ massless. Then $m_V \approx \langle H^{n,0}\rangle \approx g_V\Lambda_{\rm weak}$ as desired, but Y must be identified with electric charge and hence V_μ couples proportionally to charge, and hence not to neutral matter.

2. Have $\langle H^{n,0}\rangle \approx \Lambda_{\rm weak} m_0/M_{\rm Pl}$, and $\langle H^{0,y}\rangle \approx \Lambda_{\rm weak}$ to make A_μ have mass $O(\Lambda_{\rm weak})$. In that case Y need not be electric charge but the problem noted in Eq. (7) recurs: The feeble force decouples from light matter.

3. One can have a feeble force with $g_V \approx m_0/M_{\rm Pl} \approx 10^{-20}$ and $m_V \approx g_V\Lambda_{\rm weak}$ by having $\langle H^{n,y}\rangle \approx \Lambda_{\rm weak} \times m_0/M_{\rm Pl}$ and $\langle H^{0,y'}\rangle \approx \Lambda_{\rm weak} m_0^{1/2}/M_{\rm Pl}^{1/2}$. An interesting side effect is that A_μ mediates a short-range force of range $M_{\rm Pl}^{1/2}/\Lambda_{\rm weak} m_0^{1/2} \approx 1$ Å. The price we have paid is to introduce two new supersmall mass scales into the problem. This appears presently to be the simplest way to obtain the desired result, however. [A serious objection to this third case is that it really makes no use of the Kaluza-Klein charge-shifting mechanism. Any two U(1) gauge interactions—whether from higher dimensions or not—could be used to get a feeble force (at least at the classical level) with $g_V \approx m_0/M_{\rm Pl}$ and $m_V \approx g_V\Lambda_{\rm weak}$ by having Higgs-field VEV's of the sort in case 3. One then might as well dispense with the extra dimensions altogether unless they can be used to explain the supersmall scales of the VEV's in some natural way. A second more fundamental problem is that at the quantum level particles like $H^{n,y}$ running around loops will give corrections to the classical $O(m_0/M_{\rm Pl})$ charge shifts of $O(\alpha)$ via the Holdom mechanism described below.]

We have been assuming that the kinetic-energy terms of $A_\mu^{(0)}$ and V_μ have canonical orthogonal form as indeed they do. However, it is easy to see that even if the $A_\mu^{(0)}$ and V_μ were not orthogonal, it would not change the conclusions of our analysis.

The phenomenon we have been describing should seem familiar to those who have studied the "paraphoton" scheme of Holdom,[3] for in some respects the two schemes are isomorphic to each other. The kinetic-energy terms of V and A are to the relevant order

$$E_{\rm kin} = -\tfrac{1}{4}(F_{\mu\nu}^{(A)2} + F_{\mu\nu}^{(V)2}),$$

where $F_{\mu\nu}^{(V)} = \partial_\mu V_\nu - \partial_\nu V_\mu$ and similarly for $F_{\mu\nu}^{(A)}$. Let us make the nonunitary transformation from A and V to A' and V:

$$\begin{pmatrix} V_\mu \\ A_\mu^{(0)} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} V_\mu \\ A'_\mu \end{pmatrix}, \qquad (8)$$

where

$$\tan\theta = (16\pi G)^{1/2}\langle A_5^{(0)}\rangle.$$

Then the covariant derivative in Eq. (6) takes the simple form

$$iD_\mu = i\partial_\mu + gYA'_\mu - [(16\pi G)^{1/2}M]Q^0_{\rm KK}V_\mu. \qquad (9)$$

The kinetic-energy terms now become nonorthogonal,

$$-\frac{1}{4}(F_V\ F_A)\begin{pmatrix} F_V \\ F_A \end{pmatrix} = -\frac{1}{4\cos^2\theta}(F_V\ F_{A'})\begin{pmatrix} 1 & \sin\theta \\ \sin\theta & 1 \end{pmatrix}\begin{pmatrix} F_V \\ F_{A'} \end{pmatrix}. \qquad (10)$$

This maps into the Holdom picture with the following identifications:

$V_\mu \leftrightarrow$ ordinary photon, $Q^0_{\rm KK} \leftrightarrow$ ordinary electric charge, $(16\pi G)^{1/2}M \leftrightarrow e$,

$A'_\mu \leftrightarrow$ paraphoton, $Y \leftrightarrow$ paracharge, $g \leftrightarrow$ paraphoton gauge coupling.

The excited Kaluza-Klein modes (with $Q^0_{KK}=n\neq 0$) of fields with $Y=0$ correspond to ordinary matter in the Holdom scheme, while ordinary light matter with $Y\neq 0$ and $Q^0_{KK}=0$ in the Bars-Visser scheme corresponds to paramatter. So the statement that ordinary light matter in the Bars-Visser scheme has tiny charge under $U(1)_V$ corresponds to the fact that paramatter in the Holdom scheme has tiny electric charge. The fact that VEV's of such ordinary-matter Higgs fields do not give a tiny mass to V_μ but leave a new V'_μ massless, which couples to naive Q^0_{KK} and thus not to ordinary matter, corresponds to the fact in the Holdom scheme that VEV's of paramatter Higgs fields do not give a tiny mass to the photon but leave a slightly rotated photon massless that couples to naive electric charge and thus not to paramatter. Indeed, an explicit statement of this last point (which is the same as ours in disguised language) can be found already in Ref. 3.

In conclusion, Kaluza-Klein gauge forces coupling to charges shifted by gauge-field VEV's can lead to feeble forces of the type advocated by Fischbach *et al.*; however, the desirable relation $m_V \approx g_V \Lambda_{\rm weak}$ does not emerge in the simple schemes of breaking but requires something more elaborate. In that sense the relation $g_V \approx m_0/M_{\rm Pl}$ has a natural explanation (in a nontechnical sense) in the scheme of Ref. 1, while the mass relation does not appear to be particularly natural at present. We believe that this question deserves further investigation, and hope that other symmetry-breaking mechanisms will emerge from those efforts. However, in the presence of the superheavy Kaluza-Klein matter, ordinary light matter may experience a new force in addition to its normal interactions.

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