Effective Mass of Neutrons Diffracting in Crystals

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Neutrons propagating in a crystal under diffraction conditions exhibit an effective inertial mass which is lower by 5–6 orders of magnitude than their vacuum rest mass and of both positive and negative sign. This is verified experimentally by measurement of the enormously enhanced deflection of neutrons subjected to a magnetic force while passing through a silicon crystal.

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Effects of external forces on an *electron* in a crystal are often described via the concept of an effective inertial mass, m^*, defined by the tensor[1]

$$(1/m^*)_{\mu\nu}=\partial^2\omega(\mathbf{K})/\hbar\,\partial K_\mu \partial K_\nu. \qquad (1)$$

Here μ and ν are Cartesian coordinates, $\mathbf{K}$ is the wave vector of the in-crystal electron, and $\omega(\mathbf{K})$ is the unperturbed dispersion relation. The value of m^* is typically within one or two orders of the normal electron mass and is often observed through the attendant effect on the cyclotron frequency. The present Letter applies the effective-mass concept to a Bragg-diffracting *neutron.* First, we calculate from Eq. (1) that the effective inertial mass of the diffracting neutron is *five orders smaller* than normal and, hence, in the presence of an external force, the neutron's trajectory must exhibit angular and spatial deflection five orders larger than normal. Second, we report experimental confirmation of the predicted deflections and, hence, of the mass values.

We note that the diffraction literature does not use the effective-mass concept. It has, however, long been known that the propagation directions are orthogonal to the dispersion surfaces[2] and, since the dispersion surfaces are strongly curved near the Bragg condition, the propagation directions are very sensitive to the relative angle between wave vector and lattice planes. Reorientation of the lattice planes by bending the crystal continuously alters this relative angle which results in large curvatures of the trajectories.[3] Certainly, one can equally well alter this relative angle by tilting the wave vector with an external force.[4] In both cases, the resulting trajectory curvatures are large and of both signs because of the shape and the dual nature of the dispersion surface. The trajectories for the case of a constant external force, which are analogous to those for a homogeneous elastic deformation,[5] have been calculated from first principles by Werner[6] who starts from Schrödinger's equation for neutrons experiencing both the periodic potential of the crystal and the linear potential of the force. He considers the trajectory slope parameter $\Gamma=\tan\Omega/\tan\theta_B$, where θ_B is the Bragg angle and Ω is the angle between the trajectory and the lattice planes, and by deriving the spatial differential equation governing Γ (p in his notation) he obtains the trajectory. His result agrees with our effective-mass result.

The effective-mass explanation[7] of the trajectory follows. The dispersion relation[8] characterizing neutrons near Bragg reflection from lattice planes with reciprocal-lattice vector $\mathbf{G}$ is

$$\omega(\mathbf{K})=\omega_0(\mathbf{K})+\hbar^{-1}V_0 + \hbar^{-1}V_G[f\pm(f^2+1)^{1/2}]. \qquad (2)$$

Here $\hbar\omega_0(\mathbf{K})=\hbar^2K^2/2m$ is the kinetic energy of the neutron (of rest mass m) in the crystal, V_0 is the mean value and V_G the Gth Fourier coefficient of the crystal potential, and the function $f(\mathbf{K})=\hbar^2(2\mathbf{K}\cdot\mathbf{G}+G^2)/4mV_G$ is introduced.[9] In the tensor generalization of Newton's second law[1] the acceleration $\mathbf{a}$ due to a force $\mathbf{F}$ is

$$a_\mu=(1/m^*)_{\mu\nu}F_\nu. \qquad (3)$$

From Eqs. (1)–(3), the instantaneous acceleration of an in-crystal neutron is

$$\mathbf{a}=\frac{\mathbf{F}}{m}\pm(1-\Gamma^2)^{3/2}(\mathbf{G}\cdot\mathbf{F})\frac{\hbar^2}{4m^2V_G}\mathbf{G}, \qquad (4)$$

where the relation[10] $\Gamma=\pm f/(f^2+1)^{1/2}$ has been used to eliminate $\mathbf{K}$. The first term in Eq. (4) is the conventional Newton's second law in vacuum. The second term arises from the interaction with the crystal medium. For a constant force, it is equivalent to Werner's spatial differential equation.

The second-term acceleration has several remarkable features: (1) It depends only on the component of the force parallel to $\mathbf{G}$; (2) it is directed along $\mathbf{G}$; (3) it is independent of neutron wavelength for a given Γ; (4) it varies with the trajectory slope parameter Γ; (5) it may be *orders of magnitude larger* than the first term[11]; and (6) it may be of either *positive or negative sign,* i.e., *the acceleration may be opposite to the force.* For neutrons propagating along the lattice planes ($\Gamma=0$) and for the force directed along $\mathbf{G}$, the first term may be neglected and we obtain the effective mass $m^*=\pm 2mV_G/E_G$, where $E_G\equiv\hbar^2G^2/2m$ is the kinetic energy of a neutron with a wavelength equal to the interplanar spacing. For

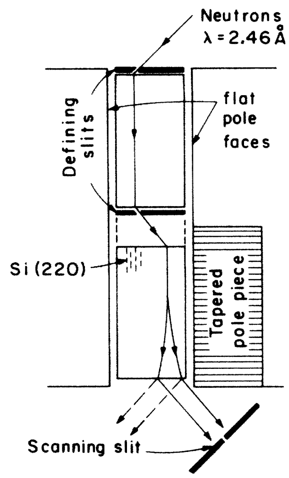

FIG. 1. The experimental arrangement used in the study of the reduced inertial mass of neutrons in crystals.

the Si(220) reflection we obtain therefore $m^*/m = \pm 4.72\times 10^{-6}$.

In the experiment, we studied the deflection effects of an external force on the trajectory of 2.46-Å-wavelength neutrons diffracted by (220) planes in a silicon crystal.[12] The necessary high collimation of the incident beam was provided by a precursor crystal, a crystal collimator as shown in Fig. 1, which for stability reasons was monolithic with the second crystal on a common base. Each of these crystals had a thickness of 52.2 mm. As in a conventional Stern-Gerlach experiment, the force on the neutron inside the second crystal was provided by an inhomogeneous magnetic field created between flat and tapered electromagnet pole pieces; semiclassically, $\mathbf{F} = -\nabla(\pm \mu B)$. In order to minimize the effect of inhomogeneous fields in the first crystal and in the crystal gap, a homogeneous magnetic field (of magnitude equal to that at the neutron entrance point in the second crystal) was applied to the first crystal. The magnetic field was mapped in detail and this was used to calculate deflections in the second crystal by means of Eq. (4), where the small deflections permitted $\Gamma = 0$.

Figure 2 shows characteristic results for the measured intensity distribution in the forward diffracted beam leaving the back face of the second crystal. The field gradient indicated on the figure is the gradient at the entrance point of the neutrons into the second crystal. The magnetic field gradient varies somewhat along the neutron trajectories, increasing towards the tapered electromagnet pole (positive position values in Fig. 2). As many be seen from Fig. 2 and as also indicated by the trajectory calculations, this results in a focusing action on the left-hand-side peak and a defocusing or broadening of the other peak.

Figure 3 shows deflections of the focused peak (measured with both field directions) as a function of electromagnet current, i.e., field gradient, for comparison with the calculated distribution. The agreement is within experiment uncertainty. We note that the vacuum rest mass of the neutron would result in minute, unmeasurable deflections; e.g., a 5-T/m field gradient acting on the neutrons in free space for 41.5 μs, which was

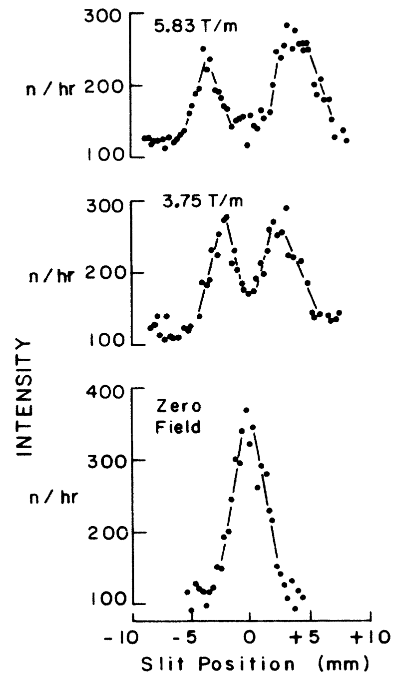

FIG. 2. Characteristic examples of the spatial intensity distribution of the neutrons leaving the crystal for various magnetic field gradients.

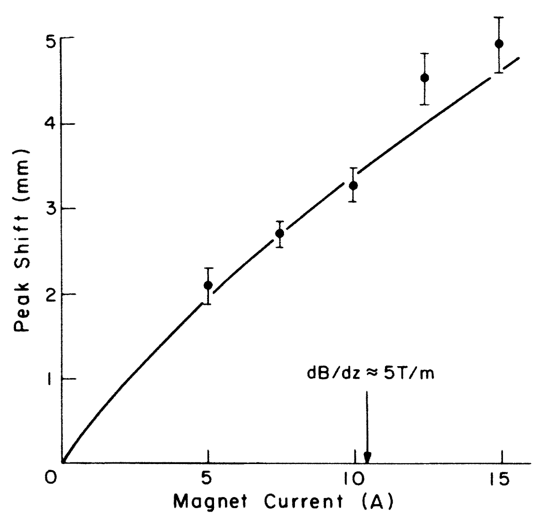

FIG. 3. Comparison between measured deflections of the focused peak and theoretical values using the effective-mass concept. These deflections are 2.1×10^5 larger than those in the same fields in free space.

their flight time through the crystal, would result in a deflection of only 250 Å.

An important feature of the experiment is the use of unpolarized incident neutrons. Because of the two different signs of the effective mass it is expected that each of the two spin states splits into two rays deflected towards and away from the tapered magnet pole. Since the force exerted by the field gradient is opposite for the two spin states, this causes each of the two peaks in Fig. 2 to contain both spin states and both positive- and negative-mass states.

In order to separate the positive- and the negative-effective-mass states and, hence, to demonstrate explicitly that in-crystal neutrons can be deflected opposite to a force acting on them independently of the spin state, the beam was bent on its way from the first to the second crystal by aluminum-prism refraction (Fig. 4). This results in two beams propagating in symmetrical directions in the second crystal, each of well-defined effective-mass sign, yet still unpolarized. Application of the inhomogeneous magnetic field then leads to a Stern-Gerlach splitting of each of these. It is to be expected that the positive-effective-mass state splits in the same manner as in a free-space Stern-Gerlach experiment, while for the negative-effective-mass state the splitting should be inverted, i.e., the neutrons of either polarization state in that beam should be deflected opposite to the forces acting on them.

This was verified experimentally by measuring the *polarization* of the emerging split beams by transmission through a magnetized polycrystalline Fe plate of thickness 9.5 mm. For intensity reasons the aluminum prism was chosen such that its bending effect canceled the field-deflection effect for a given combination of effective mass and polarization. In order to achieve this cancelation, the apex angle of the deflecting prism had to be different for the two prism orientations (see Fig. 4), indicating some slight intrinsic imperfection of the crystal arrangement (e.g., a misorientation of the two crystals by 0.053 μrad). It is seen that the right-hand-side satellite peaks in Fig. 4 are significantly higher than the left-hand-side peaks and this results from the highly asymmetric spatial intensity distribution in a forward diffracting beam due to dynamical diffraction effects. The polarization measurements summarized in Table I demonstrate explicitly that the effective mass of neutrons can be of both positive and negative sign.

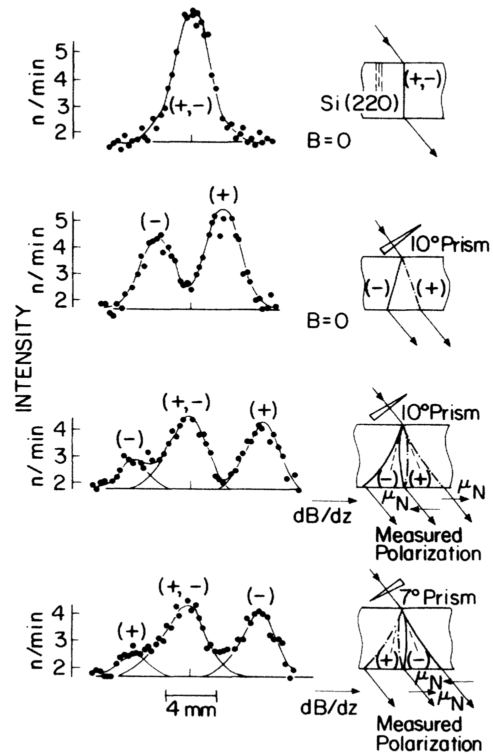

FIG. 4. Separation of the positive- (+) and negative- (−) effective-mass states. The negative-effective-mass neutrons are deflected opposite to the force acting on them.

The effects presented here suggest applications of magnetic focusing in crystals with modest fields and in the search for previously unobserved neutron interactions.[13] Also, in a gravitational deflection experiment,[6] for which Earth's **g** is equivalent to a magnetic field gradient of 1.7 T/m, the neutrons with negative effective inertial mass will fall upwards![14] The neutron effects are

TABLE I. The sign of the effective mass as determined by neutron polarization measurements. For the orientation directions of the magnetic moment, of the force, and of the trajectory curvature, the sign refers to the positive Z axis pointing towards the tapered magnet pole piece. The last line of the table exhibits a pure negative-mass state.

Peak observed (Fig. 4)	Transmission Measured	Transmission Expected	Magnetic moment orientation	Force direction	Trajectory curvature	Mass sign
Prism apex towards right:						
Center peak	0.298 ± 0.045	0.302	−	−	+, −	+, −
Right peak	0.453 ± 0.032	0.501	+	+	+	+
Prism apex towards left:						
Center peak	0.474 ± 0.027	0.501	+	+	+, −	+, −
Right peak	0.286 ± 0.026	0.302	−	−	+	−

more spectacular and more directly observable than those from the effective mass of electrons. This can be traced to the different rest masses and interactions of the two particles and to the availability of a beam that can propagate over large distances in crystals. However, the physical origin is the same: Any particle, massive or massless, in a suitably periodic medium, will exhibit an abnormal inertia.

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[1]C. Kittel, *Introduction to Solid State Physics* (Wiley, New York, 1976), 5th ed., p. 219.

[2]N. Kato, Acta Crystallogr. **11**, 885 (1958); P. P. Ewald, Acta Crystallogr. **11**, 888 (1958).

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[4]For example, for the case of a force provided by an inhomogeneous magnetic field this has been pointed out explicitly by A. Zeilinger, Nukleonika **25**, 871 (1980).

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[6]S. A. Werner, Phys. Rev. B **21**, 1774 (1980).

[7]M. A. Horne, unpublished.

[8]This relation, a consequence of Schrödinger's equation in a periodic medium, is not exhibited in the neutron diffraction literature, which traditionally treats ω and $\mathbf{K}$ as independent and dependent variables, respectively. Of course, it follows from equations in this literature. For example, in Ref. 6, set the determinant of coefficients in Eq. (12) to zero, solve the resulting quadratic *exactly* for Q^2 (linear with our ω), and evaluate using definition (13) with $\mathbf{g}=0$ to obtain the unperturbed $\omega(\mathbf{K})$.

[9]The same function appears in the literature as $y(\mathbf{k})$, where $\mathbf{k}$ denotes a vacuum wave vector incident on the crystal instead of the in-crystal $\mathbf{K}$ of our discussion. See, e.g., Eqs. (9.16) and (9.23) of H. Rauch and D. Petrascheck, in *Neutron Diffraction,* edited by H. Dachs (Springer-Verlag, Berlin, 1978), p. 303. Where the crystal entrance face is perpendicular to the lattice planes, as in our experiment, boundary conditions require $\mathbf{K}\cdot\mathbf{G}=\mathbf{k}\cdot\mathbf{G}$ and thus $f=y$.

[10]The two signs show that, for a given $\mathbf{K}$, there are two trajectories. This relation is in, e.g., Rauch and Petrascheck, Ref. 9, Eq. (9.42), with $y=f$ or it may be obtained more directly from the group velocity $\mathbf{v}_{\text{group}}=\nabla_K\omega(\mathbf{K})$ which is tangent to the trajectory.

[11]The magnitude may be increased by reducing V_G, through variation of crystal composition or selection of crystal structure factor, at the expense of Bragg intensity.

[12]Preliminary results were presented at the Thirteenth Congress of the International Union of Crystallography, Hamburg, 1984: A. Zeilinger, C. G. Shull, M. A. Horne, S. A. Werner, Acta Crystallogr., Suppl. **A40**, C-345 (1984).

[13]This has already been used to reduce by five orders of magnitude the upper limit of the neutron's magnetic charge [K. D. Finkelstein, C. G. Shull, and A. Zeilinger, Physica (Amsterdam) **136B**, 131 (1986)].

[14]Of course, neutrons in the positive-effective-mass state will deflect downwards and by Eq. (4) the average of the upward and downward acceleration is $\mathbf{g}$.