Nuclear Collective Flow as a Function of Projectile Energy and Mass

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The dependence of collective nuclear flow on multiplicity and beam energy for Ca+Ca, Nb+Nb, and Au+Au collisions has been measured with the Plastic Ball detector at the Bevalac. Event by event the data are analyzed with the transverse-momentum method and a new quantitative measure of the flow effect is extracted. It is expected that comparison of the present systematic results with model calculations will lead to a more precise determination of the nuclear-matter equation of state.

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Recently, collective flow of nuclear matter has been established in high-energy nuclear collisions.[1] Both the flow of participant nucleons and the bounceoff of spectator products were observed. Hydrodynamics predicted such collective effects[2] and is in qualitative agreement with the data.[3] In addition, the study of entropy production by light-fragment formation suggests the need for the inclusion of compressional energy.[4] In the meantime several theoretical papers have been published that describe this collective effect from a semiclassical microscopic viewpoint emphasizing the importance of the short-range nature of the nuclear force[5] and the density-dependent mean-field aspect,[6] respectively. Flow also has been observed more recently in collisions of asymmetric mass systems.[7,8] Here we present new data for collisions of Ca+Ca, Nb+Nb, and Au+Au at several beam energies between 150 and 1000 MeV per nucleon measured with the Plastic Ball spectrometer[9] at the Bevalac with a minimum-bias trigger. Charged particles up to ^{4}He emitted in the nuclear reaction are identified by the Plastic Ball and each event can be analyzed in terms of global variables. A new quantitative measure of the flow in the framework of the transverse-momentum analysis[10] is devised. This systematic study of the dependence of the flow on the multiplicity of charged particles, target-projectile mass, and beam energy represents a comprehensive body of data that should enable theoretical model calculations to obtain further information on the nuclear-matter equation of state.

Until recently, the data from 4π detectors have been analyzed with the sphericity method, which yields the flow angle relative to the beam axis of the major axis of the best-fit kinetic energy ellipsoid and also gives the ellipsoid aspect ratios. The aspect ratios, and to a lesser degree the flow angles, are influenced and distorted by fluctuations.[11] Since all the experimental biases and inefficiencies are folded into this observable it is extremely difficult to compare the experimental results with theoretical predictions. However, the reaction plane can also be determined from the collective transverse-momentum transfer[10,12] and recently Danielewicz and Odyniec have proposed a better, more exclusive way to analyze the momentum contained in directed sidewards emission.[10] They also propose presenting the data in terms of the mean transverse momentum per nucleon in the reaction plane $\langle p_x/A\rangle$ as a function of the center-of-mass rapidity. By removing autocorrelations this method is sensitive to the true dynamic correlations and has led to indications for collective flow effects in cases where the kinetic energy flow analysis was not sensitive enough.[10,13] Studying the momentum transfer as a function of rapidity permits one to distinguish between participant and spectator contributions and to exclude regions with large detector bias.

In the transverse-momentum analysis[10] the reaction plane is determined by the vector Q calculated for each event from the transverse momentum components p_t of all the particles observed in the forward and backward hemispheres in the center of mass:

$$Q = \sum_i p_{ti}^{\text{forw}} - \sum_i p_{ti}^{\text{back}},$$

where pions are not included. It should be noted that if the sign of the second sum were positive, only transverse-momentum conservation of the observed particles would be tested. Also in this work particles near midrapidity were not excluded as they were in the original paper.[10] Each event can be rotated around the beam axis (z axis) so that Q defines the x axis of a new coordinate system. Autocorrelations are removed by our calculating Q individually for each particle without including that particle. Evidently Q is only an estimate for the true reaction plane and the projections into the estimated plane are too small by a factor $1/\langle\cos\phi\rangle$, where ϕ is the angle between the estimated and the true planes. The quantity $\langle\cos\phi\rangle$ can be estimated[10] by our randomly dividing the events into two subevents and averaging the cosine of one-half the angle between the Q vectors of the subevents.

Since the charged-particle multiplicity is related to the impact parameter, we classify the events according to the participant proton multiplicity (N_p), defined to include protons bound in clusters but to exclude all pions, and particles in the target and projectile spectator regions.[4] (N_p differs from the previously used[1] multiplicity of charged particles, M_c.) The average multiplicity depends on the target-projectile mass and on the bombarding energy. In order to make meaningful comparisons between these different cases the multiplicity bins chosen should correspond to approximately the same range in normalized impact parameter. To this end the multiplicity distributions were subdivided into bins of constant fractions of the maximum multiplicity. The multiplicity distributions have a similar shape for all systems and energies: a monotonic decrease with increasing multiplicity to a plateau before the steep decrease at the highest multiplicities. Therefore the maximum multiplicity (N_p^{max}) can be defined at the point where the distribution drops to one-half the plateau height. Table I contains the value of $N_p^{max}/2Z$ for all systems reported here. The data accumulated with a minimum-bias trigger are then divided into five bins. Four bins are of equal width between zero and maximum multiplicity, each containing 25% of N_p^{max}, and one bin has multiplicities larger than N_p^{max} and contains the most central collisions. Spectator particles which are not included in the participant proton multiplicity are also excluded from the analysis presented here.

Figure 1 shows an example of the mean transverse momentum per nucleon projected into the reaction plane, $\langle p_x/A \rangle$, as a function of the normalized center-of-mass rapidity y/y_{proj}. Only statistical errors are shown. The data points are already corrected for the deviation from the true reaction plane: The value of $\langle \cos\phi \rangle$ varied between 0.66 and 0.9 and was 0.82 for this particular case. The data exhibit the typical s-shape behavior known from Ref. 10 which demonstrates the collective transverse-momentum transfer between the forward and backward hemispheres.

TABLE I. Maximum participant proton multiplicities N_p^{max} divided by the sum of the projectile and target nuclear charges for all measured systems and beam energies.

E/A (MeV/nucleon)	Au+Au	Nb+Nb	Ca+Ca
150	0.41	0.46	
250	0.58	0.63	
400	0.71	0.78	0.75
650	0.81	0.88	
800	0.85	0.90	
1050		0.95	0.90

It is the aim of this paper to extract quantitative information with as little detector bias as possible from the type of data presented in Fig. 1, thus allowing us to compare different mass systems at different energies with each other and with theoretical model calculations. The maximum transverse-momentum transfer occurs close to the target and projectile rapidities, where there is great sensitivity to the exclusion of spectator particles and where the experimental biases are most disturbing. For this reason the maximum value is not a good choice. However, to a good approximation all curves are straight lines near midrapidity. If the data are plotted as a function of the normalized rapidity the slope at midrapidity, which we call flow, has the dimensions of MeV/c per nucleon and is a measure of the amount of collective transverse-momentum transfer in the reaction. Since the flow is determined at midrapidity it is a characteristic of the participants. Technically it is obtained by fitting a polynomial with first- and third-order terms (and also a constant) to the s-shaped curve. The fit was done for y/y_{proj} between -1 and 1. Because of detector biases the curve is not completely symmetric about the origin: Therefore a second-order term has been included in the fit in cases where χ^2 can be improved considerably, as is the case for the higher energies and the heavier-mass systems. The coefficient of the first-order term, which is the slope of the fitted curve $y/y_{proj}=0$, is the flow. In Fig. 1 it is the slope of the solid line through the origin.

In Fig. 2 the flow is plotted as a function of the participant proton multiplicity for the three systems Ca+Ca, Nb+Nb, and Au+Au, all at a beam energy

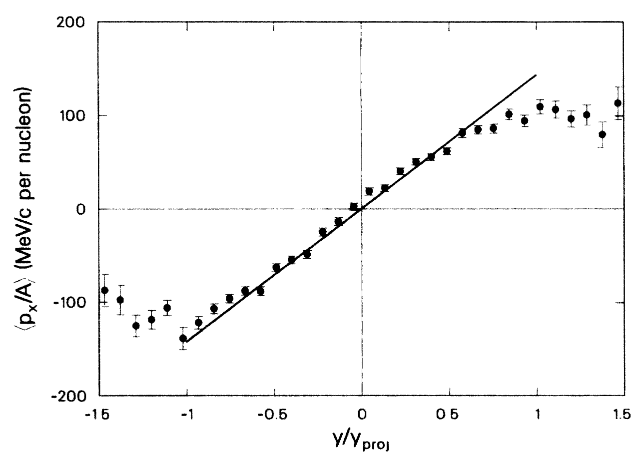

FIG. 1. Mean transverse momentum per nucleon projected into the reaction plane as a function of the normalized center-of-mass rapidity for 400-MeV per nucleon Nb+Nb in the third multiplicity bin, between 50% and 75% of N_p^{max}. The slope of the solid line represents the flow obtained from fitting the data.

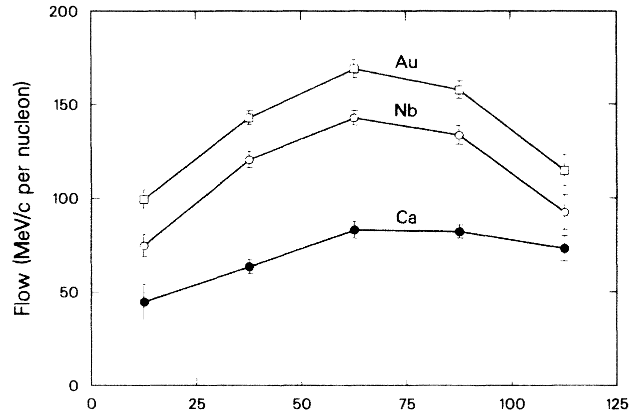

FIG. 2. Flow as a function of the participant proton multiplicity (N_p/N_p^{max}) for the three systems measured at a beam energy of 400 MeV per nucleon.

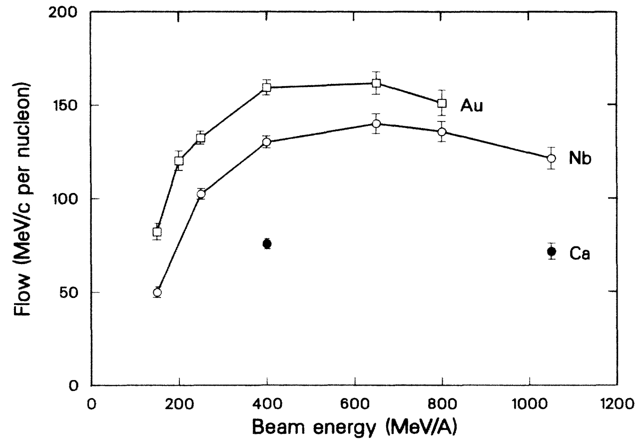

FIG. 3. Flow for minimum-bias events as a function of beam energy.

of 400 MeV per nucleon. As already seen previously from the distributions of the flow angle[14] the amount of flow increases with increasing target-projectile mass. The multiplicity dependence, however, shows the flow peaking at intermediate multiplicity, while the mean flow angle increased monotonically with multiplicity.[1] This is because the present flow quantity goes to zero at the highest multiplicities (for zero impact parameter) while the previously obtained flow angles were affected considerably by the spectators at the lower multiplicities. It should be noted that the transverse-momentum method is not able to distinguish between prolate and oblate shapes.

The dependence of the flow on the beam energy is shown in Fig. 3. The values are obtained from minimum-bias events without any multiplicity cuts and by averaging over particles, not over events. The values are only 10 MeV/c (20 MeV/c for the 1050-MeV per nucleon Nb case) lower than the maximum values at medium multiplicities (see Fig. 2) because there are not many particles in low-multiplicity events and not many events at high multiplicity. The flow increases with increasing beam energy and reaches a maximum at about 650 MeV per nucleon, followed by a slight falloff towards higher energies. A flat curve from 400 MeV per nucleon up is almost consistent with the data (especially if the maximum flow values at medium multiplicity were plotted). The energy dependence of the flow differs considerably from the behavior of the mean flow angles[14] since the flow is a measure for the transverse-momentum transfer while the flow angle measures the ratio between mean transverse and mean longitudinal momentum.

The errors plotted in Figs. 2 and 3 are statistical errors only as obtained from the fit procedure multiplied by $\sqrt{\chi^2}$. The choice of the degree of the fitting polynomial and of the fit interval introduces a systematic error of less than 10 MeV/c per nucleon. The spectator cut has a similar effect. Although the detector bias influences the flow less than the flow angle, its effect is still difficult to estimate and is energy and multiplicity dependent. Therefore all theoretical predictions should be subjected to the appropriate Plastic Ball acceptance filter (a FORTRAN subroutine is available from the authors) before being compared to the experimental results. The importance of this correction was underlined by a study of the 400-MeV per nucleon Nb data with a statistical-model code[15] extended to include the flow effect[1] which showed that the apparent flow was 20% lower at the highest value when the detector response was properly taken into account. It is well possible that the apparent decrease of the flow at the highest energies seen in Fig. 3 is influenced by the detector response.

The observation of collective flow indicates that a pressure buildup develops during the collision. This new method to describe the flow should allow for a more quantitative comparison of the data to theoretical model predictions. Cascade calculations simulating a purely thermal equation of state show some flow, but three detailed comparisons[14,16,17] with experimental data show that "there is too little intrinsic pressure built up in the cascade model."[16] Vlasov-Uehling-Uhlenbeck calculations on the other hand, show that the magnitude of the flow effect strongly depends on the nuclear-matter equation of state.[18] With use of a stiff equation of state those calculations are in qualitative agreement with the excitation function of flow for the Nb+Nb data. Composite-particle yields[4] appear to be also sensitive to the equation of state and low pion yields[19] already have given evidence for a stiff equation of state. It is expected that the present comprehensive set of data on the multiplicity, beam energy, and mass dependence of the flow will allow for a more systematic comparison with several model calculations so as to reliably extract the nuclear-matter equations of state.

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