Final-State–Interaction Analysis of Inelastic Electron Scattering on ^{3}He

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The coincidence cross section of electrodisintegration of ^{3}He is studied in various kinematic configurations. In solution of the Faddeev equations for the scattering states of the trinucleon system, the effect of final-state interactions is taken exactly into account. It is not negligible and becomes substantial when the process of direct proton knockout is suppressed.

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Electrodisintegration experiments are expected to provide detailed and interesting information about the dynamical processes occurring in the nucleus.[1] In the theoretical analysis of the three-nucleon system it is customary to neglect final-state-interaction (FSI) effects. A nucleon is knocked out directly, leaving the remaining two nucleons as the bound deuteron or in an unbound, but correlated, state.[2-4] In the plane-wave impulse approximation (PWIA) the results can be interpreted in a simple way in terms of the properties of the trinucleon bound-state wave function. The results based on such calculation have been used in the past as evidence for the failure of the conventional description in terms of only hadronic degrees of freedom. However, before one is able to draw any definite conclusions from these experiments, the effect of FSI has to be accounted for. Recent approximate calculations[5] indicate that in some kinematic regions it may play an important role. In particular, this may have consequences on the scaling analysis of inclusive reactions.[6,7]

In this Letter we report on a calculation of the electrodisintegration process of the trinucleon system where the FSI between the outgoing three nucleons is taken exactly into account by solution of the Faddeev equations for the continuum states. In particular we present results for the two-body breakup reaction $^3\mathrm{He}(e,e'p)d$ in kinematic regions which have been studied previously by various groups. The calculation of the disintegration process can be divided into three parts: (i) the PWIA contribution, (ii) the remaining Born-type diagrams, and (iii) the connected three-body diagrams in the final state describing the FSI between all three free particles. In calculating the FSI we assume that the dynamics of the three-nucleon system can essentially be treated nonrelativistically. Relativistic kinematics is only used to relate the four-momentum of the photon to the three-momentum and energy of the three-nucleon system.

For the case of the two-body breakup the various contributions to the electrodisintegration process of ^{3}He are shown in Fig. 1. The corresponding coincidence cross section is given by

$$d^5\sigma/d\Omega_{e'}dE_{e'}d\Omega_{p'} = \sum_{s_t}\sum_{s_n,s_d} |{}^{(-)}\langle \mathbf{p}'s_n;\phi_d s_d | H_{\mathrm{em}} | \psi_t s_t\rangle|^2, \qquad (1)$$

where $|\psi_t s_t\rangle$ is the trinucleon bound-state wave function with total azimuthal spin s_t. The ingoing scattering state ${}^{(-)}\langle \mathbf{p}'s_n;\phi_d s_d|$ describes the off-shell scattering of three free nucleons into the final state in which particle 1 is a nucleon with spin s_n and momentum $\mathbf{p}'$, and particles 2 and 3 form a deuteron with spin s_d and momentum $-\mathbf{p}$ with $\mathbf{p}=\mathbf{p}'-\mathbf{Q}$, $\mathbf{Q}$ being the photon momentum. In the PWIA approximation Eq. (1) factorizes into the

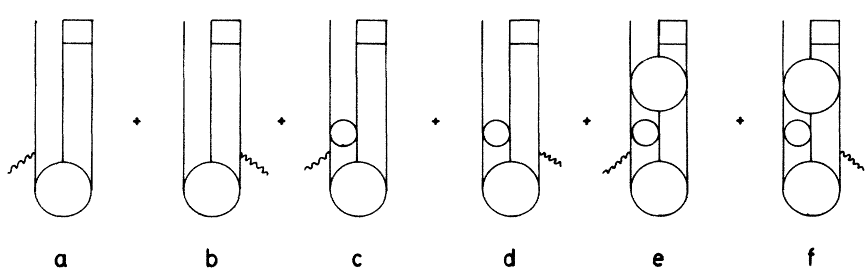

FIG. 1. Diagrammatic representation of the various contributions to the electrodisintegration process. (a) is the PWIA approximation, while (b) is the deuteron knockout process. The connected diagrams, describing the FSI, consist of the multiple-scattering series, the first order being given by (c) and (d) and all the higher-order diagrams by (e) and (f), which can be determined by solution of the Faddeev equations.

half-off-shell electron-nucleon cross section $(d\sigma/d\Omega)_{ep}$ and the nucleon momentum distribution function ρ_2:

$$d^5\sigma/d\Omega_{e'}dE_{e'}d\Omega_{p'}=k(d\sigma/d\Omega)_{ep}\rho_2(\mathbf{p}) \tag{2}$$

with k a kinematic factor and

$$\rho_2(\mathbf{p})=\frac{3}{2}\sum_{s_t}\sum_{s_n,s_d}|{}_1\langle\mathbf{p}'s_n;\phi_d s_d|\psi_t s_t\rangle|^2. \tag{3}$$

If we go beyond the PWIA the simple factorization of the electron nucleon cross section does not hold any longer, but the one-nucleon current operator occurs explicitly in the integration over the intermediate nucleon momenta. We assume that it has the form

$$J_\mu=i[\gamma_\mu(F_1+\kappa F_2)+i(p+p')_\mu\kappa F_2/2m_N], \tag{4}$$

where the electromagnetic form factors F_n are taken to be given by those of Höhler *et al.*[8] In the evaluation of the nucleon-current matrix elements an off-shell extrapolation is needed. Recently the sensitivity to the different choices used in the literature has been investigated in detail by de Forest.[9] Here we adopt the choice used by Dieperink *et al.*[2] Current conservation is explicitly made use of to eliminate the longitudinal part of the current.

To calculate the connected three-body diagrams we first solve the Faddeev equations for the required positive energies. The solutions have to be determined half off shell. The resulting amplitudes are then used in calculation of the electromagnetic-current matrix elements. On evaluating Eq. (1) we have to deal with a five-dimensional integral over the Jacobi variables of the three-nucleon system. The various logarithmic and pole singularities which are present due to the free Green's function and the complex three-body amplitude are accounted for by use of subtraction techniques, similarly as has been done by Kloet and Tjon.[10] Gaussian quadrature is employed to do the various integrals.

The actual computations are performed with the local s-wave spin-dependent interactions of Malfliet and Tjon (set I-III) as input.[11] These interactions are known to give a good description of neutron-deuteron scattering.[10] To facilitate the calculations we use the unitary pole expansion (UPE).[12] The rate of convergence is fast so that keeping only the first term in the expansion already gives the most dominant contribution. For the triton binding energy we get in this approximation $E_{^3\mathrm{H}}=-8.63$ MeV, whereas the final result is given by $E_{^3\mathrm{H}}=-8.58$ MeV.[13] The results of the calculations presented here have been obtained with the use of this approximation. The partial-wave decomposed equations which form a coupled set of one-dimensional integral equations are solved by calculation of the multiple scattering series of these equations and reconstruction of the Faddeev amplitudes with use of the Padé approximant method.[10] To get a convergent result for the full amplitude not more than fifteen partial waves are necessary in the energy regions we have considered.

We now turn to discuss the various kinematic situations. Figure 2 presents the ^{3}He$(e,e'p)d$ coincidence data as measured and analyzed by Johansson[14] and reanalyzed by Gibson and West[15] together with the calculated Born + FSI cross sections. Also the PWIA result is shown. This kinematic situation has also been studied by Lehman and his co-workers,[16] who have carried out an exact calculation using Yamaguchi separable potentials. We have also examined this case taking the potential parameters from Ref. 10 and find qualitatively the same results. The small differences found may be partially due to the use of nonrelativistic electromagnetic operators by Heimbach *et al.*[16] as compared to Eq. (4). For the Yamaguchi form factors we find that the cross section is reduced by about 4%–7% by FSI. The small influence of FSI is not surprising, because the kinematics in this experiment were such that rather low nucleon momenta ($p<100$ MeV/c) were probed with a rather high three-momentum transfer ($Q=443$ MeV/c). In this situation the process of direct proton knockout dominates [Fig. 1(a)]. As is clear from the figure the UPE calculation gives a better reproduction of the data than does the

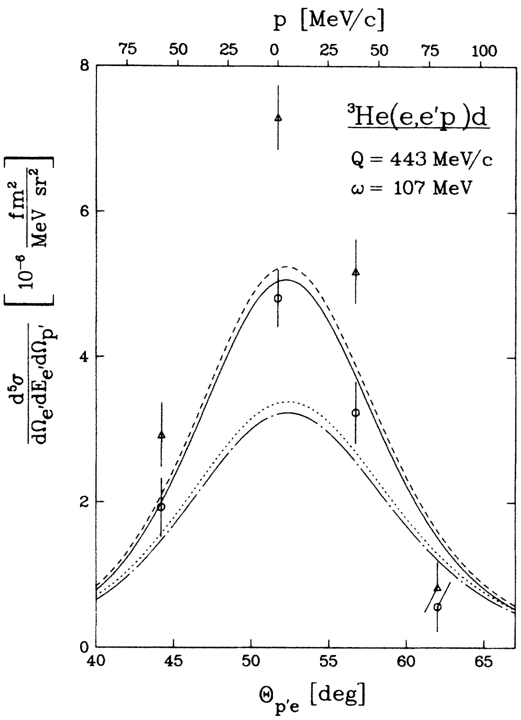

FIG. 2. Coincidence cross section at fixed momentum and energy transfer. The upper curves correspond to a PWIA (dashed) and PWIA + FSI (solid) calculation using a one-term UPE separable potential. The lower curves correspond to a PWIA (dotted) and PWIA + FSI (dash-dotted) calculation with form factors of the Yamaguchi type. Original data are from Ref. 14 (triangles) and reanalyzed data are from Ref. 15 (circles).

calculation with Yamaguchi form factors.

Figure 3 shows the Saclay data taken at a momentum transfer $Q=428$ MeV/c and $\omega=100$ MeV.[17] Our results are plotted together with the results obtained by Laget,[5] who calculated the cross section from the first few terms of a diagrammatic expansion of the full transition amplitude. Meson-exchange currents are also included in his calculations. The relative proton-deuteron energy is about 60 MeV, and ten partial waves are used to include FSI. From Fig. 3 we see that for small initial momenta p the agreement with experiment is good. At larger momenta the calculated results are too high, indicating that the high-momentum components of our wave functions contain too little correlation. The Reid soft-core results of Laget in the PWIA indeed show a steeper falloff.

The importance of the contributions from the PWIA and FSI amplitudes can be very different, depending on the kinematic configuration. The PWIA amplitude, completely dominating at low initial momentum of the probed nucleon, falls off rapidly with increasing momenta. For the FSI amplitude [Figs. 1(c)–1(f)] we find a much slower falloff as a function of the same momentum p. Since the PWIA amplitude and the real part of the FSI amplitude have opposite signs, the real part of the full amplitude which results from a coherent sum of all diagrams in Fig. 1 is considerably reduced at high nucleon momenta. As a consequence the imaginary part of the FSI amplitude, which has the same order of magnitude as its real part, gives rise to a significant contribution of the cross section. It should be mentioned, however, that this subtle interplay between the different amplitudes is not fully reflected in the cross section, because spin-dependent terms contribute incoherently.

We have also studied the Amsterdam kinematical setup,[18] at which direct deuteron knockout dominates [Fig. 1(b)]. The results are shown in Fig. 4. This experiment is taken at parallel kinematics, which eliminates the contribution to the cross section from coupling to the component perpendicular to the transverse nucleon current. A pure PWIA calculation underestimates the experimental cross section by three orders of magnitude. Direct deuteron knockout indeed is the dominating process, but now the influence of FSI is much more pronounced than it is in kinematic configurations in which direct proton knockout is favored. One possible approximation to the connected three-body amplitude is to keep only the lowest-order connected diagrams, given by Figs. 1(c) and 1(d). The result is also shown in Fig. 4. It is clear that this approximation is not correct in this kinematic

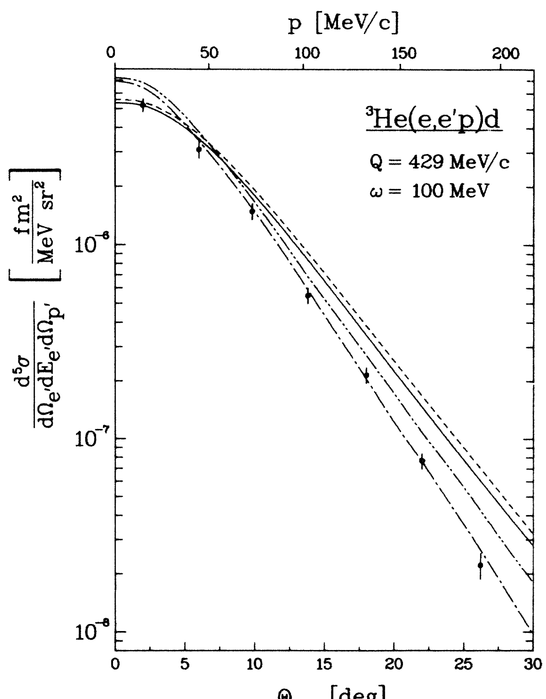

FIG. 3. Coincidence cross section at fixed momentum and energy transfer. The dashed and solid curves correspond respectively to a Born and Born + FSI calculation. The other curves are results from Laget (Ref. 5), representing a PWIA (dash-dot-dotted) and full (dash-dotted) calculation. The full calculation of Laget includes effects of meson-exchange currents. Experimental data points are from Saclay (Ref. 17).

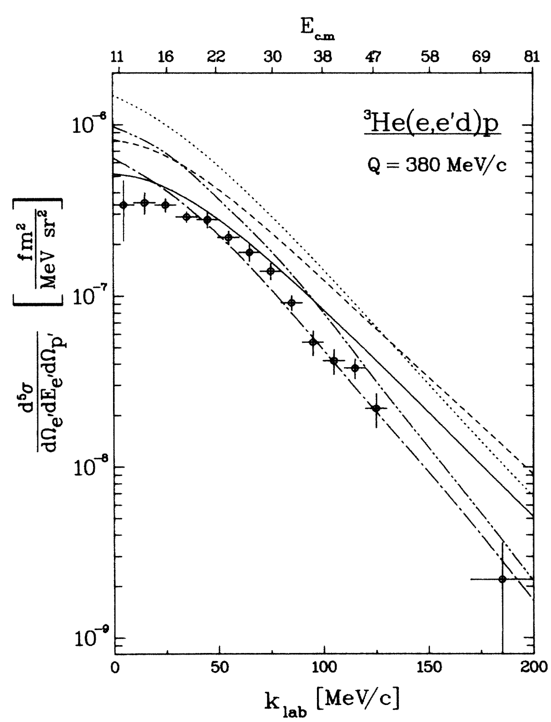

FIG. 4. Coincidence cross section at fixed momentum transfer. The dotted curve represents a calculation involving only the Born-type and lowest-order connected diagrams [Figs. 1(a)–1(d)]. The various other curves are the same as in Fig. 3. Experimental data points are from Ref. 18.

situation. Therefore it is necessary to determine the complete multiple scattering series solution.

To summarize, a full Faddeev three-body calculation has been performed to describe inelastic electron scattering, based on a local *s*-wave spin-depedent Yukawa-type potential with a soft repulsive core. FSI have a significant effect on the cross sections resulting from PWIA calculations, especially in those regions where the process of direct proton knockout is suppressed. A detailed account of this work will be presented in a forthcoming paper.

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