Topological-Exchange-Current Contribution to the Deuteron Magnetic Form Factor

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In the chiral soliton (Skyrme) model all isoscalar electromagnetic currents, including the exchange currents in nuclei, are topological and uniquely determined by the current of the individual nucleons. This allows us to obtain a model-independent isoscalar exchange-current operator, which is used to study the magnetic form factor of the deuteron. The structure of the deuteron is incorporated by our evaluating the expectation value of the exchange-current operator with respect to the wave function obtained from the Paris potential. The magnetic form factor of the deuteron is dominated by the exchange-current contribution and is in agreement with experimental data in the range $0 < q^2 < 33$ fm^2.

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In the Skyrme model for the baryons,[1] in which the baryons are topological solitons of meson fields, the baryon number is carried by an anomalous current operator, which is topological and not a Noether current. In the nuclear case it is therefore not related to the dynamics of the model, but rather to the Wess-Zumino interaction,[2,3] which does not contribution to the energy in the case of SU(2). At the time of writing it is not yet clear exactly what interactions the Skyrme Lagrangean should contain in order to represent the low-energy, large-N_c limit of QCD. Therefore, this current operator, which depends only on the fields of the Lagrangean but not on their interactions, has an exceptional value in the establishment of the validity of the approach.

In this Letter we shall study the magnetic form factor of the deuteron, which in the conventional model represents a difficult problem, requiring mesonic exchange-current mechanisms (perhaps with form factors of their own) that lack a solid theoretical foundation.[4,5] The realization that the exchange current is related to the topological baryon current, on the other hand, makes it possible to calculate the form factors of the deuteron solely from the isoscalar electric structure of the nucleons themselves (by incorporation, of course, of a deuteron wave function). We shall illustrate this by a calculation of the magnetic form factor, which is the only deuteron form factor that is separately known experimentally.

The anomalous baryon current is expressed in terms of an SU(2) field U as[1]

$$B^\mu = \frac{\varepsilon^{\mu\nu\alpha\beta}}{24\pi^2}\mathrm{Tr}\{U^\dagger\partial_\nu U\, U^\dagger\partial_\alpha U\, U^\dagger\partial_\beta U\}. \tag{1}$$

For the single nucleons the topological soliton field has the hedgehog form

$$U(\mathbf{r}) = \exp[i\boldsymbol{\tau}\cdot\hat{\mathbf{r}}\theta(r)], \tag{2}$$

where the function $\theta(r)$ satisfies an equation of motion which, in principle, is derivable from the Lagrangean. As the form of the current (1), on the other hand, does not depend on the Lagrangean, the anomalous current is, in fact, known as soon as θ is, as long as the Lagrangean contains only spinless fields and their first derivatives. We may thus adopt the straightforward approach of the determination of the chiral angle θ from the electromagnetic structure of the individual nucleon and use this function to predict the isoscalar exchange current in the two-nucleon case, i.e., the deuteron.

When quantizing the rotational motion of the solitons, we introduce collective coordinates for rigid rotation in the usual way.[6] Using our earlier results,[7] we may then evaluate the rotational expectational value, which has the effect of replacing the rotational variables by the familiar Pauli σ and τ matrices. The hedgehog *Ansatz* (2) then gives the following isoscalar charge and current densities for the nucleon ($J^\mu = \frac{1}{2}B^\mu$):

$$J_0 = -(1/4\pi^2 r^2)\sin^2\theta(r)\theta'(r), \tag{3a}$$

$$\mathbf{J} = -(1/4\pi^2 r^2)\sin^2\theta(r)\theta'(r)\mathbf{P}/m_N - (1/8\pi^2\lambda r^2)\sin^2\theta(r)\theta'(r)\boldsymbol{\sigma}\times\mathbf{r}. \tag{3b}$$

Using the results of Oka,[8] we have included the convection current, where $\mathbf{P}$ is the momentum operator. Further, λ is the moment of inertia of a single soliton and $\boldsymbol{\sigma}$ is the Pauli spin matrix. (Isospin does not enter, of course). The moment of inertia can be calculated from the chiral angle, provided the Lagrangean is known, or from the N-Δ mass difference [which gives $\lambda = \frac{3}{2}(m_\Delta - m_N)^{-1}$]. As we emphasize electromagnetic interactions in this Letter, we shall choose λ such that the isoscalar magnetic moment of the nucleon is correctly reproduced. This requires $\lambda = 0.0060$ MeV^{-1}, corresponding to $m_\Delta - M_N = 250$ MeV.

The isoscalar electric and magnetic form factors of the nucleon are obtained from a Fourier transform of the current operators (3) and are

$$G_E^S(q) = -(2/\pi)\int_0^\infty dr\, j_0(qr)\sin^2\theta(r)\theta'(r), \tag{4a}$$

$$G_M^S(q) = -(4m_N/3\pi\lambda)\int_0^\infty r^2 dr\,[j_0(qr)+j_2(qr)]\sin^2\theta(r)\theta'(r). \tag{4b}$$

Here, the electric form factor has been normalized to unity at zero momentum transfer, while the magnetic form factor is normalized to the isoscalar magnetic moment. The chiral angle $\theta(r)$ is obtained, e.g., by inversion of the Fourier transform in (4a), with use of the electric form factor as input. The well-known dipole form, $G_E^S(q)=(1+q^2/\Lambda^2)^{-2}$ with $\Lambda^2=0.71$ GeV^2, gives the following equation for $\theta(r)$:

$$\theta(r)-\tfrac{1}{2}\sin 2\theta(r)=\pi e^{-\Lambda r}(1+\Lambda r+\tfrac{1}{2}\Lambda^2 r^2). \tag{5}$$

It is not difficult to modify (5) to represent other, more ambitious fits to the nucleon form factors,[9,10] although in the general case a numerical Fourier transform may be necessary. In Fig. 1 we show two such functions $\theta(r)$ as well as the result of a Lagrangean calculation of the same function.[11] The results are in fair agreement with each other, but as a result of effects which will be discussed below, the curves are not expected to be the same. Because of the large uncertainty in the empirical isoscalar form factor, we shall use that θ which corresponds to the simple dipole fit, given in (5). The large-distance behavior in our case is related to the nearest singularity of the form factor and it is therefore exponentially damped with a mass corresponding to $\frac{1}{3}\Lambda$ (equal to 280 MeV) rather than the pion mass.

It remains to compute the magnetic form factor of the deuteron. We follow the treatment of our earlier work[7] and use the product *Ansatz*[1]

$$U(\mathbf{r}_1,\mathbf{r}_2;\mathbf{r})=U(\mathbf{r}-\mathbf{r}_1)U(\mathbf{r}-\mathbf{r}_2), \tag{6}$$

where $\mathbf{r}_1$ and $\mathbf{r}_2$ are the coordinates of the centers of the two solitons, and $\mathbf{r}$ will be the point of interaction with the electromagnetic field. With this *Ansatz* the baryon current splits into a one-body current for each nucleon and an exchange-current operator:

$$B^\mu(\mathbf{r}_1,\mathbf{r}_2;\mathbf{r}) = B^\mu(\mathbf{r}-\mathbf{r}_1)+B^\mu(\mathbf{r}-\mathbf{r}_2)+B^\mu_{\text{ex}}(\mathbf{r}_1,\mathbf{r}_2;\mathbf{r}). \tag{7}$$

The deuteron magnetic form factor is obtained by a combination of the current (7) with a model for the structure of the deuteron. As will be discussed in more detail below, the structure of the deuteron is not yet understood in the Skyrme model. It is nevertheless clear that any quantitative description of the deuteron must account for the well established fact that it is a loosely bound system of a neutron and a proton, the relative motion of which is describable by a wave function. This provides a justification for the product *Ansatz* and also for a quantization of the rotational motion of the nucleons as free particles, independent of each other. With this approach, the single-nucleon currents in (7) are approximated by the free-space currents.

As we have recently[7] described the steps necessary to compute the expectation values of operators such as (7), we shall only give the resulting formula here for the spatial part of the exchange-current operator:

$$\mathbf{J}_{\text{ex}} = -\frac{i}{72\pi^2\lambda}\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2\exp(\tfrac{1}{2}i\mathbf{q}\cdot\mathbf{r})\mathbf{q}\times\left\{\int d^3R\exp(i\mathbf{q}\cdot\mathbf{R})\sin^2[\theta(|\mathbf{R}+\mathbf{r}|)]\times\left[\frac{\sin 2\theta(R)}{2R}(\boldsymbol{\sigma}^1+\boldsymbol{\sigma}^2)+\left(\theta'(R)-\frac{\sin 2\theta(R)}{2R}\right)(\boldsymbol{\sigma}^1+\boldsymbol{\sigma}^2)\cdot\hat{\mathbf{R}}\hat{\mathbf{R}}\right]\right\}, \tag{8}$$

where $\mathbf{r}$ is the internucleon separation and $\boldsymbol{\tau}_1,\boldsymbol{\tau}_2$ ($\boldsymbol{\sigma}^1,\boldsymbol{\sigma}^2$) represent the isospins (spins) of the nucleons.

In the absence of a realistic description of the deuteron within the Skyrme model we shall be content to evaluate the matrix element of the currents (7) with deuteron wave functions that correspond to the parametrized Paris potential[12] for the nucleon-nucleon interaction. The single-nucleon-current contributions to the magnetic form factor, i.e., the impulse approximation, then lead to the conventional expressions for the magnetic form factor,[13] with the single-nucleon form factors (4a) and (4b). The complete expressions for the exchange contribution to the magnetic form factor will be published separately.

In Fig. 2 we show the calculated magnetic form factor of the deuteron as obtained when the chiral angle is determined by the dipole fit to the isoscalar electric form factor of the nucleon, Eq. (5). At large momentum transfers the exchange-current contribution dominates the form factor. The agreement with the empirical form-factor values[14,15] is good. When the chiral angle of the Skyrme model[11] is used, the magnetic form factor is overpredicted by roughly a factor of 2 at the larger momentum transfers in the figure.

We note that the impulse-approximation value for the magnetic moment of the deuteron is $0.8468\mu_N$ (where μ_N is the nuclear magneton), when the wave function corresponding to Ref. 12 is used. The exchange current contributes an additional $0.04\mu_N$. As the experimental value is $0.857\mu_N$ the exchange current might appear

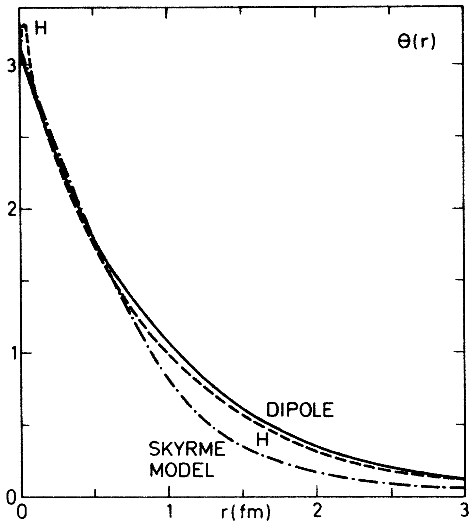

FIG. 1. Chiral angle θ vs distance r, as determined from the isoscalar electric form factor of the nucleon, using the dipole approximation and the fit of Ref. 10 (H). Also shown is the Skyrme model result (Ref. 11).

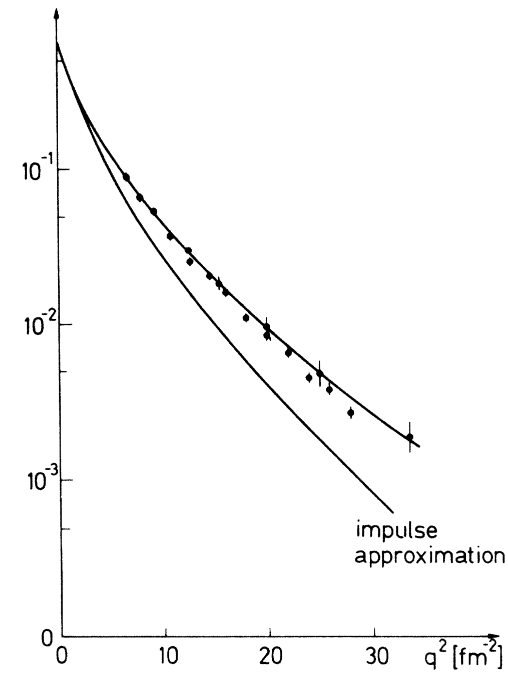

FIG. 2. Magnetic form factor of the deuteron vs momentum transfer. Shown is the contribution from the complete topological current well as that of the impulse approximation, where the exchange current contribution is neglected. The chiral angle has been determined from the isoscalar electric form factor of the single nucleon.

large at small q^2. This is probably in order, however, as the nucleon-nucleon interaction in the Skyrme model is velocity dependent[16,17] and this leads to further exchange currents. While this problem has not yet been studied with use of the Skyrme model, estimates based on the phenomenological interaction[12] give a contribution of approximately[4] $-0.02\mu_N$, which brings the prediction of the present approach very close to the empirical value.

In this Letter we have deliberately avoided all dynamical aspects of the Skyrme model and focused on what we consider the most important aspect, the topological current. Although it would, in principle, have been preferable to treat the deuteron as a $B=2$ soliton of the Skyrme model,[18] such a description is not yet sufficiently accurate for our purposes. A less ambitious approach would be to compute only the nucleon-nucleon interaction in the Skyrme model, and then use it to generate a wave function, but so far nuclear forces obtained by this method have not been sufficiently realistic. We have therefore regarded the problem of the deuteron binding as separate from the evaluation of the form factors.

Within the approach adopted it would be possible to include effects due to changes in shape and size of the nucleons.[19] This would lead to changes in the single-nucleon contributions in (7) in addition to the explicit exchange current (8). As the form factor is dominated by the exchange current, such changes in the single-nucleon contributions are probably not numerically significant for the present purposes.

It is interesting to compare the present calculations of the magnetic form factor to earlier calculations that are based on explicit meson-exchange mechanisms.[4,5,20] Predictions based on such calculations are too large unless vector-meson dominance is incorporated into the electromagnetic coupling.[4,5] The overprediction of the magnetic form factor when we use the Skyrme model, computing the chiral angle θ directly from the Lagrangean with a pion-mass term,[11] may also be understood as a consequence of the neglect of vector-meson dominance. Although the Skyrme model contains vector degrees of freedom, no information regarding the masses of these mesons is present in the model, and predictions will therefore be unrealistic at the corresponding momentum transfers. We have here incorporated the necessary information into the phenomenological single-nucleon chiral angle (5) which is consistent with the experimental data, and are therefore able to account for the deuteron form factor as well. We expect that one would also be able to explain the magnetic form factor by use of a modified Skyrme model,[21] where vector mesons occur explicitly as mediators of the electromagnetic couplings.

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