Intranuclear *N-N* Collision Model for the Production of High-Energy Gamma Rays in Heavy-Ion Collisions

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High-energy γ rays observed by Stevenson *et al.* in the reaction of 20-, 30-, and 40-MeV/nucleon ^{14}N+C and Pb are calculated from the Boltzmann master equation. The production of γ rays is included via a semiclassical *np* bremsstrahlung calculation. Good agreement is found in shape and magnitude between calculated and experimental results.

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Considerable interest has been generated by the measurement and interpretation of high-energy ($E_\gamma > 20$ MeV) γ rays resulting from collisions of energetic heavy ions.[1–4] If these γ rays are produced in the early moments of the nuclear collision, as seems likely, then they provide an excellent probe of the initial reaction dynamics. In addition, unlike subthreshold pion measurements, the detected yields of high-energy γ rays are not complicated by reabsorption within the surrounding nuclear matter. Stevenson *et al.* recently reported spectra of γ rays of up to ~ 110 MeV in the bombardment of Pb and C targets with ^{14}N projectiles at energies of 20, 30, and 40 MeV/nucleon.[1] They found that the γ-ray angular distributions were consistent with production in the nucleon-nucleon (*N-N*) center-of-mass sytem. Similar results were reported in observations of high-energy γ rays by Grosse *et al.* for $^{12}C + ^{12}C$ and $^{12}C + ^{238}U$ collisions at energies ranging from 48 MeV/nucleon to 84 MeV/nucleon[3] and by Kwato Njock *et al.* for the $^{40}Ar + ^{197}Au$ system at 30 MeV/nucleon.[4] However, it remains to be shown that the experimental slopes of the observed energy spectra are consistent with a realistic cascade calculation in which the *np* bremsstrahlung cross section provides the microscopic link between the intranuclear cascade and the resulting γ-ray spectrum[5]; this is the goal of the present work.

Preequilibrium nucleon spectra following central collisions of heavy ions have been reproduced with great success[6] by use of the Boltzmann master equation (BME) as encoded by Harp, Miller, and Berne[7] and as modified by Blann, Mignerey, and Scobel for treating heavy-ion collisions.[8] This established the validity of the nucleon energy distributions used in the intranuclear cascade process. The BME approach has also been used to reproduce successfully subthreshold pion cross sections.[9] In this work, we will add a channel to the BME for inelastic *np* bremsstrahlung reactions (the *pp* contributions being negligible compared with *np*[10]), and compare the resulting γ-ray spectra with the results of Stevenson *et al.*[1] The input parameters assumed are those used in the past[8] which successfully reproduced precompound neutron spectra[6] and subthreshold pion cross sections.[9] No additional parameters are introduced by addition of the γ-ray channel. We first present the BME and discuss the assumptions made in distributing the energies of the coalescing projectile nucleons; we next present the equations used to evaluate the inelastic *np* channel, and then compare the calculated spectra with the results of Stevenson *et al.*

The BME is defined by the set of coupled differential equations for the time-dependent change in the number of nucleons of type x in a bin at energy i (above the bottom of the nuclear potential well):

$$dN_i^x/dt = g_i^x \sum_y \left\{ \sum_{jkl}' \left[\omega_{kl\to ij}^{xy} g_k^x g_l^y g_j^y n_k^x n_l^y (1-n_i^x)(1-n_j^y) - \omega_{ij\to kl}^{xy} g_j^y g_k^x g_l^y n_i^x n_j^y (1-n_k^x)(1-n_l^y) \right] - n_i^x \omega_{i\to i'}^x \right\} + f_i(p,n). \quad (1)$$

Here the superscripts x and y designate the type of nucleon (neutron or proton), g_m^x represents the number of type-x single-particle states (in the 1-MeV-wide bin used in our code) at energy m, n_m^x represents the fraction of g_m^x which is occupied, and ε_F^x is the Fermi energy for a nucleon of type x in the target nuclear potential well (ε_F^x is assumed fixed throughout the reaction). The sum over states is restricted by energy conservation, denoted by a prime on the sum. The $\omega_{ij\to kl}^{xy}$ represent the rates for a single nucleon of type x at energy i to scatter with a single nucleon of type y at energy j to give final nucleon energies k and l. These rates are based on free N-N scattering cross sections (calculated from the equations of Chen *et al.*[11]) for pp, nn, or np scattering as appropriate. The Pauli exclusion principle is taken into account via the terms $1-n_m^x$ in Eq. (1). The next to the last term in Eq. (1) represents the rate of emission of nucleons at energy i into the continuum at energy i'. The $\omega_{i\to i'}^x$ is calculated on the basis of the usual statistical inverse-cross-section method[7] from the rate of capture of particle x at laboratory energy i' into the nucleus at energy i. The final term, $f_i(p,n)$, is the rate of insertion of nucleons at energy i from the coalescing projectile. For this term, we assume a constant velocity of approach based on the projectile c.m. energy reduced by the Coulomb-barrier height.

We calculate the energy distribution of the coalescing nucleons as the distribution of n particles (where n is taken to be the smaller of the projectile and the target mass numbers) sharing U units of excitation energy randomly, but with the constraint that no exciton may have more than Z units of energy in the initial partition. If we assume a sharp cutoff at the projectile Fermi energy ε_F, then $Z=[\varepsilon_F^{1/2}+(E/A)^{1/2}]$, where E/A is the laboratory bombarding energy per nucleon. To illustrate the effect of high-momentum components in the initial exciton distribution, we also did calculations with the sharp cutoff at ε_F removed, the only constraint being conservation of energy. The excitation U is taken to be the excitation energy which a compound nucleus would have if formed. The nuclear collision is followed via the BME in time steps ($\Delta t=2\times10^{-23}$ sec) which are shorter than the average N-N collision period.

We calculate a rate of γ-ray production when a neutron of energy i collides with a proton of energy j to give final nucleon energies of k and l and a γ ray of energy m:

$$\omega_{ij\to klm}^{pn\gamma}=\frac{\sigma_{\gamma pn}[(2/M)(\varepsilon_i^n+\varepsilon_j^p)]^{1/2}}{V\sum'_{nop}g_ng_og_p}. \quad (2)$$

Here V is the nuclear volume of the composite system, which is taken the same as in the calculation of the scattering rates, namely a sphere of radius parameter $r_0=1.5$ fm. The $\sigma_{\gamma pn}$ is the energy-differential cross section for production of a γ ray at energy m by $np\gamma$ bremsstrahlung. Our model for $\sigma_{\gamma pn}$ is discussed further below. The rate given by Eq. (2) is entered into the master equation (1) and all np collision energies are summed over to give the rate of production of γ rays of energy m,

$$d^2N_m^\gamma/dE\,dt=\sum_{ijkl}{}'\omega_{ij\to klm}^{pn\gamma}g_i^ng_j^pg_k^ng_l^pn_i^nn_j^p(1-n_k^n)(1-n_l^p). \quad (3)$$

The γ-ray yield per np collision was calculated from the following expression, which is close to the classical formula[12]:

$$\frac{d^2N}{dE_\gamma d\Omega_\gamma}=\frac{1}{E_\gamma}\frac{\alpha}{(2\pi)^2}\sum_{k=1}^{2}\left|\frac{\hat{\boldsymbol{\varepsilon}}_k\cdot\boldsymbol{\beta}_i}{1-\hat{\mathbf{q}}\cdot\boldsymbol{\beta}_i}-\frac{\hat{\boldsymbol{\varepsilon}}_k\cdot\boldsymbol{\beta}_f}{1-\hat{\mathbf{q}}\cdot\boldsymbol{\beta}_f}\right|^2P_{\text{fac}}(1+X). \quad (4)$$

Here $\alpha=\frac{1}{137}$ is the fine-structure constant, $\boldsymbol{\beta}_i$ and $\boldsymbol{\beta}_f$ are the proton initial and final velocity (in units of c), and $\hat{\boldsymbol{\varepsilon}}_1$, $\hat{\boldsymbol{\varepsilon}}_2$, and $\hat{\mathbf{q}}$ are unit vectors designating the two directions of polarization and the direction of propagation of the γ ray. The last two factors modify the classical formula by quantum effects. The P_{fac} is a correction due to final-state phase space,[13] $P_{\text{fac}}=\beta_f\gamma_f/\beta_i\gamma_i$, where γ is the relativistic contraction factor. The last factor is a crude simulation of meson-exchange effects.[14] Without this correction, the formula systematically underpredicts the measured np bremsstrahlung. In this work we use a value $X=1$, consistent with the findings of Ref. 14, effectively doubling the classical rate.[15] Equation (4) was evaluated for each np collision energy $i+j$. Before insertion into the master equation, the results of Eq. (4) were averaged over all initial np collision configurations and over all final proton directions. Integration of this result over γ-ray angle and multiplication by the np elastic scattering cross section then gives $\sigma_{\gamma pn}$ in Eq. (2).

The BME bremsstrahlung calculation determines directly the γ-ray yield per reaction. This yield, when multiplied by the reaction cross section, then gives the bremsstrahlung cross section to be compared with experimental results. We have assumed a simple geometric reaction cross section, viz., $\sigma_R=\pi r_0^2A_T^{2/3}$, with $r_0=1.2$ fm to simulate a central collision.

We compare our calculated spectra with the results of Stevenson *et al.*[1] for ^{14}N$+^{12}$C and Pb$\to\gamma X$ in Fig. 1. The beam energies were 20, 30, and 40 MeV/nucleon. We show the 90° data and assume that angle-integrated results are reasonably given by an isotropic angular distribution about the 90° results. Angle integrations of the experimental results show that no more than 20% uncertainty is introduced by this assumption. In Fig. 1 we present calculated results for the two initial exciton distributions mentioned above, namely, with (solid lines) and without (dashed lines) a sharp cutoff at the Fermi energy.

The calculated spectra for ^{14}N+Pb with the sharp-cutoff exciton distribution are in excellent agreement with experimental results over the entire energy range. When the sharp-cutoff constraint is removed, the γ-ray cross section increases slightly for $E_\gamma\gtrsim60$ MeV, but the calculation still agrees quite well with the experimental data.

For the ^{14}N$+^{12}$C system, the calculated spectra are insensitive to the presence (or lack) of a sharp cutoff in

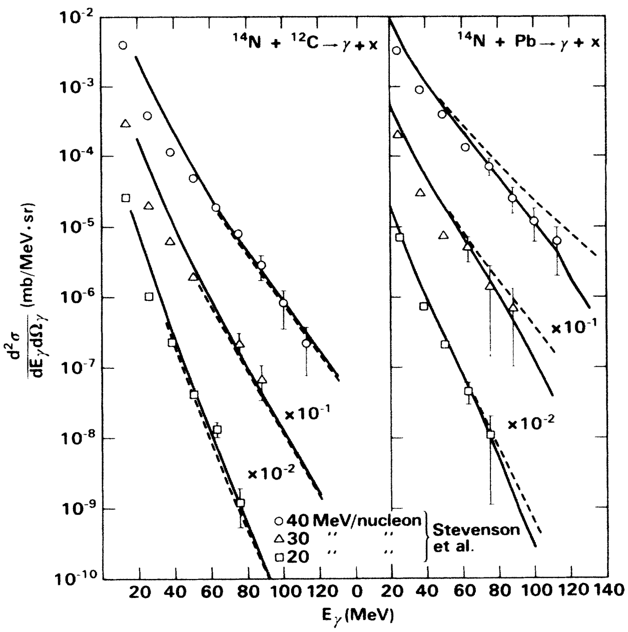

FIG. 1. Calculations of high-energy γ rays via nucleon-nucleon bremsstrahlung with the Boltzmann master equation are compared with the data of Stevenson *et al.* (Ref. 1) for ^{14}N+Pb and ^{14}N+C at 20 (squares), 30 (triangles), and 40 (circles) MeV/nucleon. The spectra are artificially offset from one another by factors of 10 for clarity of display, with the spectra at the top having the scale indicated. The calculations were carried out for initial exciton distributions with (solid lines) and without (dashed lines) a sharp cutoff at the projectile Fermi energy.

the initial exciton distribution. Again good agreement is obtained between the calculated and the experimental results, especially for $E_\gamma \gtrsim 50$ MeV. For the lower γ-ray energies, the calculation somewhat overpredicts the experimental results. Overall, within the uncertainties of the calculated semiclassical *np* bremsstrahlung cross sections, the BME gives a quite satisfactory interpretation of the absolute yields of the high-energy γ rays observed in these reactions. We emphasize that in predicting the γ-ray spectra of Stevenson *et al.*,[1] no parameters have been changed from those used in successfully predicting preequilibrium nucleon spectra.[8]

We conclude that the *np* bremsstrahlung mechanism can (within reasonable uncertainties) quantitatively reproduce the shape and magnitudes of the high-energy γ rays observed by Stevenson *et al.*[1] We shall shortly apply the same calculation to results of other experiments.[2-4] We hasten to point out that agreement of these model calculations with the data in Ref. 1 in no way invalidates alternative suggestions for the reaction mechanisms. More extensive experimental measurements are necessary for eventual discrimination between the various reaction models being proposed to explain the recent observation of high-energy γ rays in heavy-ion collisions.

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