Energy Dependence of Source Sizes in Nuclear Interferometry

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Particle emission from a spatially static nucleus at finite temperature is studied by means of a computer simulation. In particular, the two-particle correlation function at small relative momentum is constructed from the analysis of 20000 emission events. The apparent size of the emitting region is found to decrease with increasing energy of the emitted particles, as has been observed experimentally. In the simulations, the large apparent source size associated with slow particles arises because of the longer average emission time of these particles, which tends to wash out the correlations.

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The technique of using two-particle correlations for estimation of the spatial and temporal dimensions of reaction regions has been used extensively in both nuclear- and particle-physics experiments for more than a decade. In nuclear reactions, a variety of ejected particles—from pions to nuclear fragments as heavy as lithium—have been used for the measurements of correlation functions, in both identical- and nonidentical-particle combinations.[1-5] Koonin[6] had provided a method for analysis of the experimental data by characterization of the source region from which particles emerged as having a Gaussian form in space and time, with two associated parameters r_0 and τ. In the analysis of the first generation of the experiments, it was often assumed that the correlation function measured the spatial distribution of particles at a particular freeze-out density. Finite-lifetime effects were generally not included in the analyses because of the large data sets required to obtain reasonably accurate constraints on both the spatial and temporal scales simultaneously.

More recent experiments have extended the analysis to include the ejectile-energy dependence of the correlation function. At intermediate bombarding energies, the values of r_0 obtained with $\tau=0$ are observed[3-5] to decrease with increasing ejectile energy. If one assumes that particles in a given energy range all freeze out at the same density, this implies that fast particles measure the early stages of the reaction when the reaction region is relatively dense, and slow particles measure the later stages after the system has expanded. The possibility emerged that study of two-particle correlations would allow the reaction trajectory to be mapped out since the coincidence data could also be used to obtain the relative populations of excited states, from which a temperature could be extracted.[7] However, not all systems investigated[4,5] have shown a dependence of the temperature on the ejectile energy. Difficulties in the interpretation of the energy dependence of the source size have been raised by Pratt,[8] who found energy-dependent correlation functions for sources consisting of expanding spherically symmetric shells.

What we wish to investigate here is whether the energy dependence simply reflects the different time scales associated with particle emission. Suppose one considers a spatially static source region at finite temperature. Fast particles leave the system rapidly and their correlation should reflect a source size close to the true one. The cooler nucleus left after fast-particle emission can reequilibrate to produce more unbound particles, albeit at a lower average energy. Hence, slow particles are characterized by longer emission times and their correlations will be weaker, indicating a larger apparent source size.

Because the time evolution of the reaction region in a nuclear collision is potentially exceedingly complicated, computer simulations are one of the obvious tools to be used in its study. One requires a model in the simulation which executes very rapidly, since a large number of individual events are needed to construct a correlation function. The execution time of the model used here was slightly better than two events per minute on an IBM 3081; a total of more than 20000 events was used to obtain the results reported.

The system which we wish to simulate consists of forty nucleons ($Z=N=20$) at normal nuclear matter density with a temperature of 5 MeV. This choice of size and temperature for the reaction region is typical of what is thought to be produced in the intermediate-energy reaction measurements.[4,5] The nucleons are initially distributed uniformly in a sphere of radius $R_u=4.0$ fm. The nucleons in this reaction region are given momenta chosen from a finite-temperature Fermi-Dirac distribution. Because the Coulomb interaction between protons is included, the Fermi energy of the protons in the initialization is raised too high above that of the neutrons if the ideal-Fermi-gas density of states is used. As was detailed in Beauvais, Boal, and Wong,[9] the proton density of states in the initialization is changed to

$$dn=[\gamma g/(2\pi)^3]d^3p,$$

with the parameter $\gamma=1.3$ required to achieve roughly the same number of protons as neutrons above the Fermi sea at finite temperature. Indeed, one can verify from Fig. 1, which shows the energy spectrum of particles emitted from this source after an elapsed time of 300 fm/c, that the normalization of the high-energy tails is very similar for both protons and neutrons.

Because the system being simulated emits only a few particles after 300-fm/c elapsed time, the simplification is made that the nuclear potential in which the nucleons move is a step-function potential fixed in space. The nucleons are allowed to collide (by use of the same cross sections as are used in Ref. 9) and any collisions which result in either particle having an energy less than the Fermi energy is considered blocked. (As a technical point the Fermi energy is defined here as the chemical potential which is associated with normal nuclear matter at finite temperature. The difference between the zero-temperature value and the $T=15$ MeV value is small but important for the initialization.)

The nucleon trajectories are followed classically. If a nucleon does not have sufficient energy to escape then it is scattered elastically from the potential well wall. Each proton's charge is spread out uniformly about its position in a sperical volume with the same rms radius as is observed experimentally (0.8 fm). This procedure avoids the difficulties associated with integration of the equations of motion near the singularity associated with point charges. The equations of motion themselves are integrated by use of the leapfrog method. Because of the small velocities involved compared to the projectile velocity in intermediate energy collisions, a rather large time step is chosen for propagation: 1.0 fm. This results in the occasional time step where a proton is propagated improperly in the presence of a number of other nearby protons. Events in which conservation of energy is violated by more than 5% are discarded.

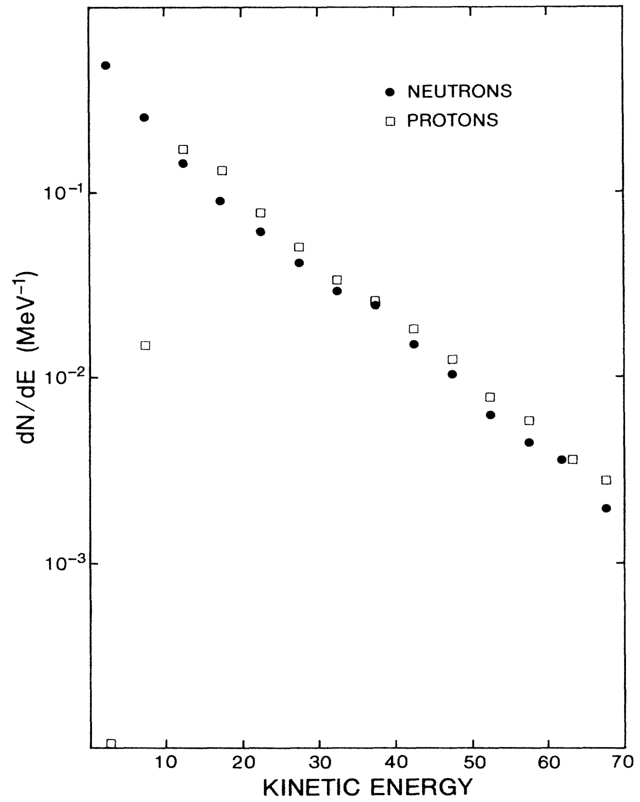

FIG. 1. Free-particle energy spectra obtained in the simulation for particles emitted from a mass 40 nucleus at $T=15$ MeV. Total elapsed time for each event was 300 fm/c. A total of 20 100 events were generated in the simulation.

The main difference between this simulation and that of Ref. 9 (where an attempt was made to construct the correlation function generated in a heavy-ion collision, including impact-parameter averaging, etc.) is the replacement of the self-generated potential with a spatially fixed one. This allows a substantial decrease in execution time per event which is required to generate the data sample (201 100 events) used in construction of the correlation function.

The total elapsed time for each event is set at 300 fm/c, which leaves a fairly cold residual system at the end of the event. The energy spectra of the emitted particles show the expected form, with the temperature determined from the slope in the high-energy region being 15 MeV. In the low-energy region, the effect of the Coulomb barrier is evident in the comparison of the proton and neutron spectra. However, is should be stressed that the spectra are not necessarily the same as what one would obtain from a strict sequential-evaporation picture: Both rapid and slow emission are included here, and the particles are free to rescatter from each other once they are emitted.

The correlation function for the events is obtained by use of the same method as in Ref. 9: (i) Coincidences are selected event by event and sorted into relative momentum $[=(p_1-p_2)/2]$ bins 10 MeV/c in size. (ii) A large number of "random" coincidences is generated by mixing of the events. (iii) The correlation function is constructed by comparison of the (appropriately normalized) true- to random-coincidence rates.

The results are shown in Fig. 2 for several different cuts on the summed kinetic energies of the observed protons. One can check that the negative correlation at small relative momenta corresponds to the Coulomb interaction between the protons by construction of the neutron correlation function. Although we do not show it for lack of space, the neutron correlation function $R(p_1,p_2)$ is consistent with zero over the entire momentum range. For momenta larger than 50 MeV/c, $R(p_1,p_2)$ for protons is also consistent with zero with a typical uncertainty of less than 0.05.

The results show that the correlation function becomes more negative with increasing ejectile energy. To make a quantitative estimate of how the apparent source size changes, we apply the same method of analysis as originally proposed by Koonin only dropping the s-wave nu-

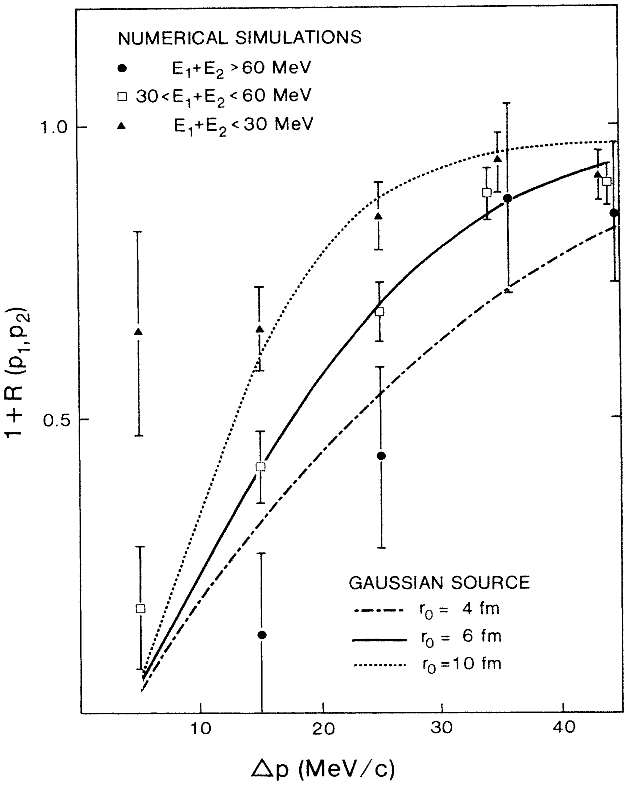

FIG. 2. Two-proton correlation functions obtained for the same conditions as in Fig. 1. Three summed kinetic energy cuts are shown. The smooth curves are the predictions of the model given in Ref. 6 but with the nuclear term omitted. The curves were generated with use of an integration method outlined in Ref. 10.

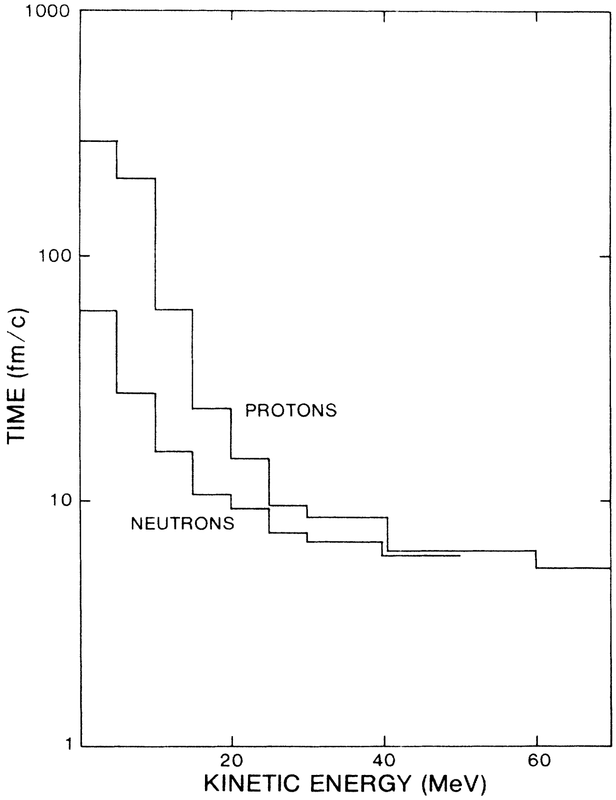

FIG. 3. Average time at which the free nucleons first crossed the nuclear boundary. Same conditions as in Fig. 1.

clear term in the interaction. (A larger number of partial waves are also used in the calculation for better convergence at large r_0; see Ref. 10.) Three representative curves are shown for $r_0=4$, 6, and 10 fm. One can see that the magnitude in the change of r_0 with ejectile energy is very similar to what is observed experimentally. For example, in a study of *pp* correlations in the reaction $^{14}\mathrm{N}+{}^{197}\mathrm{Au}$ at 35*A* MeV, it was observed that the parameter r_0 (in a $\tau=0$ analysis) has the values 5.2 ± 0.5, 4.0 ± 0.3, and 3.6 ± 0.2 fm for the summed-energy cuts of 24–50, 50–75, and 75–100 MeV. Although the calculations here should not be compared directly (ours is an idealized isolated system: no impact-parameter dependence, etc.) the variation of r_0 with summed-energy cut is in the same range.

The reason for the change in the apparent source size in the simulations lies in the average emission time of the particles. Because the Coulomb force is long ranged, the particles will continue to interact, if only weakly, long after they have crossed the boundary of the emitting nucleus. While this makes it difficult to define a precise emission time, the time at which the particle crosses the boundary should be accurate enough for our needs. The average emission time, so defined, is shown as a function of ejectile energy in Fig. 3. For both protons and neutrons, the emission times are constant at about 5–6 fm/*c* for the highest-energy particles, and then substantially increase for lower-energy particles. This just reflects the intuitive picture we advanced above: High-energy particles leave the reaction region rapidly but the continuous reequilibration of the residual system results in later emission of particles with lower average energy. As one would expect, the emission times for protons exceed those of neutrons at low energies because of the presence of the Coulomb barrier.

The following simple calculation demonstrates qualitatively that the lifetime is important. In the 20000 events analyzed the average center-of-mass velocities of the two-proton pairs was 0.11*c*, 0.14*c*, and 0.19*c*, while the average boundary crossing times was 68.4, 28.1, and 16.2 fm/*c* for the coincident protons observed in the three summed-energy ranges 0–30, 30–60, and 60 MeV. We use these values as an estimate of $v'\tau$ (where Koonin's v' is just the two-particle c.m. velocity here) and use $r_0=\sqrt{\frac{2}{5}}R_u$ as an estimate of r_0 (this latter condition arises by equating the rms radii of the Gaussian and uniform distributions). The parameter $\rho_0=[r_0^2+(v'\tau)^2]^{1/2}$ in Koonin's formulation then has the values 4.0, 4.8, and 7.9 fm for the three energy cuts, respectively. These values agree with the results shown in Fig. 2.

To summarize, we have developed a simplified model for the computer simulation of particle emission from a static nucleus at finite temperature. The simulations show an ejectile-energy dependence of the apparent source size (obtained by ignoring of lifetime effects) which is very similar to that observed experimentally. In the simulations, the variation in the effective source size arises from the different time scales associated with particle emission. This implies that one cannot deduce that reaction systems are expanding simply on the basis of the change in r_0 determined by neglect of lifetime effects. A better experimental signature of lifetime effects would be the variation of the correlation function as the angle between the relative velocity and the two-particle c.m. velocity is changed.

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