Implications of the Red-Shift–Number Test for Cosmology

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This Letter discusses the recent measurement of the number of galaxies versus red shift and flux for cosmological models with a nonzero cosmological constant Λ and pressureless matter. The main conclusions are these: (1) If space is flat as the inflationary model predicts, then the density parameter $\Omega = 0.9^{+0.4}_{-0.2}$ and $\Lambda/(3H_0^2) = 0.1^{+0.2}_{-0.4}$. (95%-confidence intervals are used throughout.) (2) For both inflationary and $\Lambda = 0$ models, the present result and primordial nucleosynthesis are consistent only if a large density of nonbaryonic matter exists. (3) Without any assumptions about Λ, the age of the Universe t and Hubble's constant H_0 are constrained by $0.60 < tH_0 < 0.88$.

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Two outstanding questions of cosmology, the values of the mass density and the cosmological constant, may be addressed with a recent measurement[1] of the geometry of the Universe. Using photometrically determined red shifts[2] of 1000 galaxies, Loh and Spillar measured the volume element as a function of red shift and deduced the density parameter Ω assuming a spatially homogeneous and isotropic Universe with pressureless matter and a zero cosmological constant Λ. This Letter discusses the implications of the measurement with relaxed assumptions about Λ. [Here Ω is the present ratio of the mass density to the closure density, and $\lambda = \Lambda/(3H_0^2)$ is the dimensionless form of the cosmological constant.]

Loh and Spillar[1] measure the flux and red shift of every galaxy with a greater than minimum flux in a field. Assuming that the number of galaxies inside a comoving box is unchanging, they deduce the quantity $\phi^* A(z)$. The quantity $A(z)$ is the volume element at the red shift z normalized to $(1+z)^{-3} z^2 dz\, d\omega H_0^{-3}$, and ϕ^* is the Schechter density, which in rough terms is the volume density of bright galaxies. For a homogeneous, isotropic universe dominated by pressureless matter, the metric is

$$H_0^2 ds^2 = d\tau^2 - a^2[dr^2(1-Kr^2)^{-1} + r^2(d\theta^2 + \sin^2\theta\, d\phi^2)],$$

where $a = (1+z)^{-1}$ is the scale factor; the comoving coordinate is

$$r(z) = K^{-1/2} \sin\left[K^{1/2} \int_{(1+z)^{-1}}^{1} E(x)^{-1} x^{-2} dx\right],$$

where $K = \Omega + \lambda - 1$ is the dimensionless curvature on constant-density surfaces;

$$E(x) = (\Omega x^{-3} + \lambda - K x^{-2})^{1/2},$$

$$A(z) = (r/z)^2 E[(1+z)^{-1}]^{-1}.$$

This technique uses galaxies solely as benchmarks in space to measure the change in the expansion rate with distance and the curvature of space. Therefore it is sensitive to any kind of matter that couples to gravity.

The systematic errors[1] of this red-shift–number test are small compared with those of the classical red-shift–magnitude test primarily because the fundamental assumption, that the number of galaxies is constant, is more accurate than the assumption that the luminosity of the brightest galaxy of a cluster is constant. Applying the red-shift–magnitude test, one will confuse an $(\Omega,\lambda) = (0,0)$ universe with a $(1,0)$ universe if the brightest galaxies in rich clusters were brighter at $z = 0.75$ by 46%, an amount which is less than the uncertainties in the evolution.[3,4] But with the red-shift–number test, for one to make the same error, half of the large galaxies must have formed since $z = 0.75$, an unlikely possibility since large galaxies form in about 10^9 yr.[5] Furthermore, evolution in luminosity has almost no effect on the derived value of $\phi^* A$.[1]

The data,[1,6] shown in Fig. 1, are fitted to find ϕ^*, Ω, and λ, and Fig. 2 shows the 67%- and 95%-confidence contours in the Ω-λ plane. The bounds for the independent variables are

$$\Omega - \lambda = 0.9^{+0.7}_{-0.5}, \quad -1.5 < \Omega + \lambda < 7.1. \tag{1}$$

(95%-confidence intervals are used throughout.)

The major axes of the ovals of equal confidence are perpendicular to the lines of constant curvature. (The line $K = 0$ is shown in Fig. 2.) This coincidence occurs because the mean red shift is 0.5; if the mean red shift were small, then the ovals align with the lines of constant $q_0 = \Omega/2 - \lambda$.

Why are the confidence ovals so eccentric? Examining three models with $(\Omega,\lambda) = (1,0), (0.2,0.8)$, and $(4.0,3.2)$ in Fig. 1, one concludes that $\Omega - \lambda$ determines the slope of the curves in Fig. 1, whereas $\Omega + \lambda$ determines the curvature, which is more difficult to measure.

Consider now the implications of Eq. (1).

The inflationary theory[7,8] predicts that $K^{-1/2} H_0^{-1}$ is much larger than the horizon (and H_0^{-1}), and therefore $\Omega + \lambda = 1$. In the inflationary scenario, Eq. (1) implies that $\lambda = 0.1^{+0.2}_{-0.4}$ and $\Omega = 0.9^{+0.4}_{-0.2}$; i.e., that the vacuum

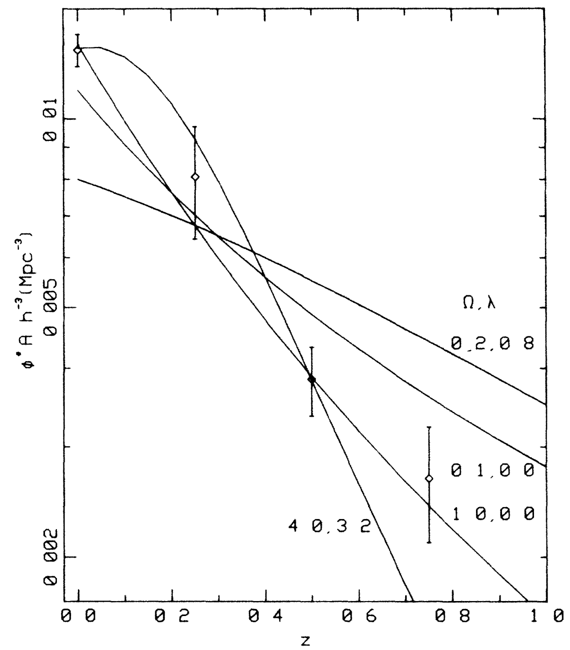

FIG. 1. The normalized number density $\phi^* A$ vs red shift z. The data (Ref. 1 except at $z=0$, which is from Ref. 6) are displayed with 1σ error bars. The curves show several several cosmological models [labeled with (Ω,λ)] with ϕ^* adjusted for minimum χ^2. The model (0.2,0.8), which differs from the Einstein–de Sitter model (1.0,0.0) only in the variable $\Omega-\lambda$, has a different slope. The model (4.0, 3.2), which differs from the Einstein–de Sitter model in the variable $\Omega+\lambda$, has a different curvature. If baryons dominate the matter density, then $(\Omega,\lambda)\approx(0.1,0.0)$ (see Ref. 12), but this model fits poorly: $\chi^2=13.6$ for 3 degrees of freedom.

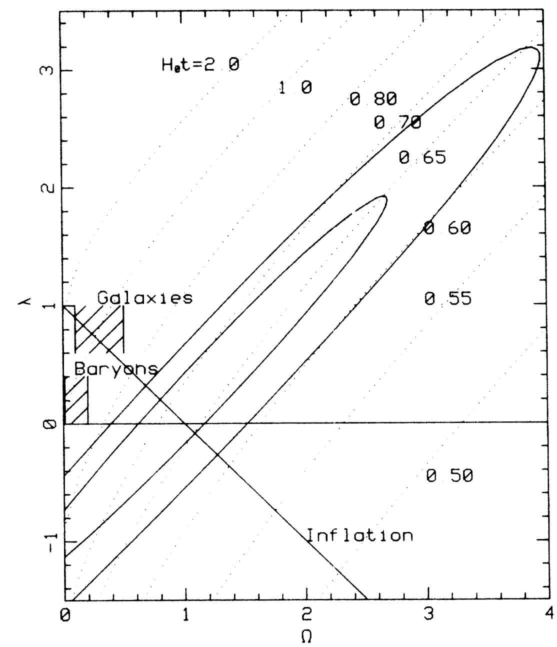

FIG. 2. The 95%- and 67%-confidence regions in the Ω,λ plane. Other measurements (at the 95%-confidence level) of the mass density are shown as hatched regions: $0.01<\Omega_{\mathrm{baryon}}<0.2$ from Ref. 12 and $0.1<\Omega_{\mathrm{gal}}<0.5$ from Ref. 9. The prediction of the inflationary theory is shown as the diagonal line. Loci (dotted lines) of models with equal $H_0 t$ are shown.

energy density is small compared with the energy density in matter.

The notion that space is flat and the matter density is 0.2, which is the most straightforward interpretation of the dynamical measurements of galaxies,[9] is strongly rejected. ($\chi^2=60$ for three degrees of freedom.) If the model $(\Omega,\lambda)=(0.2,0.8)$ is true [draw the (0.2,0.8) model through the point at $z=0$ in Fig. 1], either 50% of the bright galaxies at $z=0.5$ have been lost in these data, or 50% of the bright galaxies have formed since $z=0.5$. Neither possibility is tenable. No large number of unidentified objects exists in the data, and if galaxy formation were so common at $z=0.5$, a large number of extremely bright galaxies with anomalous colors[5] would be present in these data.

The mass density inferred from Eq. (1) may be compared with two other measurements. The abundance of the light elements and the theory of primordial nucleosynthesis[10,11] constrain the mass density in baryons by $0.011<\Omega_{\mathrm{baryon}}<0.19$.[12] Equation (1) is inconsistent with the limits on Ω_{baryon} for both an inflationary universe ($\Omega>0.7$) and a $\lambda=0$ universe ($\Omega>0.4$) unless a large density of nonbaryonic matter exists.

Dynamical measurements, sensitive only to the mass that is associated with galaxies, yield $0.1<\Omega_{\mathrm{galaxy}}<0.5$.[9] The present data are inconsistent with this limit for an inflationary universe unless the matter is not clustered with galaxies. For a $\lambda=0$ universe, the present data are marginally consistent with Ω_{galaxy}.

The age of the Universe t and Hubble's constant are related by $H_0 t=\int_0^1 E(x)^{-1}x^{-1}dx$. From Fig. 2 one concludes that $0.60<H_0 t<0.88$. These limits are stronger than those currently available from measurements of t and H_0 separately.[13]

What is the correct cosmological model of the Universe? The Hubble recession, the existence of the 3-K radiation, the explanation of the abundances of the light elements, and the isotropy of the 3-K radiation are the bases of the big-bang cosmology. Parenthetically, these data provide another proof of the existence of the big bang.[14] As for the parameters Ω and λ, one experiment and four theoretical arguments are pertinent. (i) This measurement constrains $\Omega-\lambda$. (ii) The inflationary scenario argues for $\Omega+\lambda=1$. Unless the Universe began in a finely tuned state, the existence of the big bang implies that (iii) $\Omega=1$[15] and (iv) $\lambda=0$.[9] (v) Since the natural value of the cosmological constant, the square of the Planck mass, is 10^{121} times larger than the limits implied by Fig. 2, some principle or mechanism makes λ identically zero.[16] Accepting the experiment (i) and any of the theoretical arguments leads one to conclude that $\Omega \approx 1$ and $\lambda \approx 0$. One attractive combination of the arguments is this: If one accepts the experimental verdict and the inflationary scenario because it solves the horizon problem,[15] then the natural [in the sense of arguments (iii), (iv), and (v)] values $\Omega = 1$ and $\lambda = 0$ are preferred.

This experiment and this technique for measuring the volume element have not been checked independently, and undiscovered systematic effects may change the conclusions. Mentioned here are some possible problems and some studies that may help to resolve them.

The values in Eq. (1) depend on the value of ϕ^* at $z \approx 0$, and there are questions whether the study by Kirshner, Oemler, Schechter, and Shectman (KOSS)[6] has sampled an average or an extraordinary point in space. Reference 1 shows that when the Harvard-Smithsonian Center for Astrophysics[17] sample and the KOSS sample are corrected to have the same shape for the luminosity function, the values of ϕ^* agree; this is strong evidence that ϕ^* is known adequately since the depths of the two samples differ by a factor of 10 in distance. The ongoing red-shift survey at the Harvard-Smithsonian Center for Astrophysics, if augmented with photometry, and other deeper ones similar to the KOSS survey will be very important in settling this point.

The photometric method for measuring red shifts has been checked against spectroscopic red shifts mainly at $z = 0.4$ with both red and blue cluster galaxies.[2] My conclusion is that $\phi^* A$ at $z = 0.5$ should be secure and the errors at other red shifts have been estimated correctly, but there are doubts. A further check using the spectroscopic survey of ~ 300 field galaxies at $0.1 < z < 0.5$ of Koo and Kron is in progress. Other samples of spectroscopic red shifts in small fields at higher red shifts would be extremely useful.

The Loh-Spillar data show that the characteristic luminosity L^* of galaxies has decreased by $\sim 20\%$ since $z = 0.5$. It is possible that the luminosity of galaxies has evolved in such a way that ϕ^* has increased by 60% (to make an $\Omega = 0$ universe alias as $\Omega = 1$) while keeping L^* almost constant? Theoretical studies of luminosity evolution should be made to see whether the tests of evolution[1] that have already been performed are adequate.

More data are being collected so that more detailed intrinsic checks will be possible. In the near term four times as many galaxies will be available, and the goal is to enlarge the sample a hundredfold in several years. Finally, one should remember that only a substantial error, one that lowers $\phi^* A$ at $z = 0.5$ by 37%, can make an $\Omega = 0$ universe appear to be an $\Omega = 1$ universe.

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