Purely Cubic Action for String Field Theory

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We show that Witten's open–bosonic-string field-theory action and a closed-string analog can be written as a purely cubic interaction term. The conventional form of the action arises by expansion around particular solutions of the classical equations of motion. The explicit background dependence of the convential action via the Becchi-Rouet-Stora-Tyutin operator is eliminated in the cubic formulation. A closed-form expression is found for the full nonlinear gauge-transformation law.

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It has been conjectured by Friedan,[1] Witten,[2] Yoneya,[3] and possibly others that the interacting-string field-theory action might be expressed in the purely cubic form

$$S \sim \int \phi^3, \tag{1}$$

where ϕ is a suitably defined string field. The more familiar form of the action discussed in the recent literature,[2–7]

$$S \sim \int (\phi Q \phi + \phi^3), \tag{2}$$

where Q is the Becchi-Rouet-Stora-Tyutin (BRST) operator, might then arise by expansion around some appropriate ground state. Since the construction of Q involves background fields, such a formulation might have the important advantage of being background independent.

In an inspiring paper,[8] Hata *et al.* have come very close to an explicit realization of this conjecture within the context of their closed-string field theory.[4] The key idea is to find a string field ϕ_0 with the property

$$\phi_0 * \phi = Q\phi$$

for any string field ϕ, where the product $*$ is defined in Ref. 4. They indeed construct such a ϕ_0, but it does not quite seem to satisfy the classical equations of motion of (1), so that the action does not in any obvious way reduce to the conventional form (2).

In this paper we realize this conjecture in the context of Witten's open–bosonic-string field theory[6] and a closed-string analog developed by Lykken and Raby.[7] That is, we begin with a purely cubic action of the type (1). We regain the conventional form (2) by expanding around a particular solution of the classical equations of motion. The nonlinear gauge invariance of the cubic action, which is homogeneous and linear in the string field, and the important issue of background independence of the resulting formalism are discussed. The arguments rely mainly on formal properties of the theory.

We begin, in the notation of Witten,[6] with the cubic action

$$S = \int A * A * A, \tag{3}$$

where A is a Grassmann-odd, ghost-number $-\frac{1}{2}$ string field obeying a certain reality condition. Recall that although the product $*$ is noncommutative, it does satisfy

$$\int A * B = (-1)^{AB} \int B * A, \tag{4}$$

where $(-1)^{AB} = -1$ if A and B are both odd and $+1$ otherwise. It follows immediately that S is invariant under the homogeneous infinitesimal gauge transformations

$$\delta A = A * \Lambda - \Lambda * A, \tag{5}$$

where Λ is Grassmann even and has ghost number $-\frac{3}{2}$. The classical equation of motion of (3) is

$$A * A = 0. \tag{6}$$

For any solution A_0 one can define an operator D_{A_0} by

$$D_{A_0}B = A_0 * B - (-1)^B B * A_0, \quad (7)$$

where $(-1)^B = -1$ if B is Grassmann odd and $+1$ if it is even. It then follows simply from (4), (6), and (7) that D_{A_0} obeys

$$(D_{A_0})^2 = 0, \quad (8a)$$

$$\int D_{A_0}B = 0 \ \forall \ B, \quad (8b)$$

$$D_{A_0}(A*B) = (D_{A_0}A)*B + (-1)^A A * D_{A_0}B. \quad (8c)$$

Thus D_{A_0} is a derivation. The last two properties, in fact, hold for any string field A_0. It is the crucial first property which requires A_0 to be a solution to (6). Note that since $*$ is a nonlocal operation on the space of string configurations, D_{A_0} is in general a nonlocal operator.

If we now expand

$$A = A_0 + \tilde{A},$$

then the action (3) becomes

$$S = \int (\tfrac{3}{2}\tilde{A} * D_{A_0}\tilde{A} + \tilde{A} * \tilde{A} * \tilde{A}), \quad (9)$$

and the gauge invariance (5) becomes

$$\delta\tilde{A} = D_{A_0}\Lambda + \tilde{A} * \Lambda - \Lambda * \tilde{A}. \quad (10)$$

To recover Witten's form of the string action we must therefore find a field A_0 satisfying (6) and

$$D_{A_0} = Q,$$

where Q is the BRST operator associated with some background. We will show that the unique solution with this property is given by

$$A_0 = Q_L \mathcal{I}, \quad (11)$$

where Q_L (Q_R) is the BRST charge density integrated over the left (right) half of the string ($Q = Q_L + Q_R$) and I is the identity operator obeying

$$\mathcal{I} * B = B * \mathcal{I} = B \ \forall \ B.$$

$\mathcal{I}$ is 1 if the left and right halves of the string coincide (including ghost variables), and zero otherwise. It also involves a midpoint insertion giving it ghost number $-\tfrac{3}{2}$.

To show that (11) is the unique solution, we first establish three properties of Q_L and Q_R:

$$Q_R \mathcal{I} = -Q_L \mathcal{I}, \quad (12a)$$

$$(Q_R A) * B = -(-1)^A A * Q_L B, \quad (12b)$$

$$\{Q, Q_L\} = 0. \quad (12c)$$

Equation (12a) follows immediately from the properties of $\mathcal{I}$:

$$QB = Q(\mathcal{I} * B) = Q\mathcal{I} * B + QB \ \forall \ B,$$

which implies that $Q\mathcal{I} = 0$. Equation (12b) follows from conservation of BRST charge (on a curved world sheet). It is simply the vanishing of the integral of the BRST charge density around a closed curve sandwiched between A and B. Equation (12c) can be derived beginning with the operator product expansion for the BRST current $\mathcal{J}_+(z)$ multiplied by $\mathcal{J}_+(w)$ in the critical dimension as computed by Banks, Nemeschansky, and Sen.[9] If we do a contour integral over z and use standard formulas[10] relating (anti)commutators to the singular part of the operator product expansion, we find

$$\{Q, \mathcal{J}_+(w)\} = 0.$$

Integrating $\mathcal{J}_+(w)$ over half the string we find

$$\{Q, Q_L\} = 0.$$

We can now easily show that $Q_L \mathcal{I}$ has the desired properties. By use of (12b) we have

$$Q_L \mathcal{I} * Q_L \mathcal{I} = Q_R Q_L \mathcal{I}.$$

On the other hand, if we use (12a) followed by (12b) we have

$$Q_L \mathcal{I} * Q_L \mathcal{I} = -Q_R \mathcal{I} * Q_L \mathcal{I} = Q_L^2 \mathcal{I},$$

so that

$$Q_L \mathcal{I} * Q_L \mathcal{I} = \tfrac{1}{2} Q Q_L \mathcal{I} = \tfrac{1}{2}\{Q, Q_L\}\mathcal{I} = 0,$$

i.e., $Q_L I$ obeys the equations of motion.[11]

Similarly, for any string field B

$$D_{Q_L \mathcal{I}} B = (Q_L \mathcal{I}) * B - (-1)^B B * Q_L \mathcal{I} = QB$$

by use of (12a) and (12b). Therefore $D_{Q_L \mathcal{I}} = Q$.

Finally, to see uniqueness, suppose that $\hat{A}_0$ is another solution to (6) with $D_{\hat{A}_0} = Q$. Then $C = Q_L \mathcal{I} - \hat{A}_0$ must satisfy

$$D_C B = C * B - (-1)^B B * C = 0$$

for all B. This implies $C = 0$.

To summarize, if we expand around the solution $A_0 = Q_L \mathcal{I}$,

$$A = Q_L \mathcal{I} + \tilde{A},$$

we find the action

$$S = \int (\tfrac{3}{2}\tilde{A} * Q\tilde{A} + \tilde{A} * \tilde{A} * \tilde{A})$$

with the inhomogeneous infinitesmal gauge invariance

$$\delta\tilde{A} = Q\Lambda + \tilde{A} * \Lambda - \Lambda * \tilde{A}.$$

In other words, we have regained Witten's open-string field theory.

In Ref. 7, it was shown that a gauge-invariant closed–bosonic-string field theory could be formulated by the generalization of Witten's construction to closed strings with two preferred points on each string. (However, it is not known if the closed-string Veneziano amplitudes are reproduced.) We have shown that, if we mimic our construction for the open string, this action can also be expressed in the purely cubic form. A similar result may hold for the closed-string field theory of De Alwis and Ohta.[5]

One of the major drawbacks of the usual formulations of string field theories (open or closed) is that they involve background fields. For the closed string, e.g., a fixed background space-time metric appears explicitly in the BRST charge.[12] Indeed, Q does not satisfy $Q^2=0$ unless the background corresponds to a conformally invariant sigma model.[9] This is clearly unsatisfactory because the sigma-model backgrounds become dynamical variables in string field theory. Specific backgrounds should arise only in association with specific states, not in the fundamental formulation of the dynamics. It is certainly one of the central problems of string theory to find a formulation that does not involve reference backgrounds.

In Witten's open-string field theory, Q represents a reference background and $\tilde{A}$ represents the second-quantized fluctuation field around that background. What we have shown is that by shifting $\tilde{A}$ one can eliminate this specific reference to a background. This is precisely the manner in which one expects the explicit background dependence to disappear in the transition from a first-quantized to a second-quantized formalism. In our second-quantized formulation, the backgrounds arise as solutions to the equations of motion.

While our cubic action is certainly a step in the right direction, because of the formal nature of our arguments we cannot claim with certainty that all background dependence has been eliminated. For example, one might worry about the precise definition of the star operation. Its abstract definition does not involve any background. However, when one attempts to give an explicit representation, some background dependence may appear. The star operation $A(X_1)*B(X_2)$ equates the right half of the string with embedding X_1 with the left half of the string with embedding X_2 and then integrates over half-string embeddings (and similarly for the ghosts). The notion of two half strings coinciding is independent of a metric on either space-time or half-string space. However, in order to integrate ordinary functions we need a metric, or at least a measure, on half-string space. The half-string space metric implicit in the star operations of (6) and (7) is the metric induced from flat space-time. Thus the star operation, as usually defined, refers to a space-time metric, which is undesirable for a closed-string field theory. However, this background dependence can be at least formally removed by considering the string field to be a density of weight $\frac{1}{2}$ on string space. The product of two such fields is a density of weight 1 which can be integrated in a well-defined manner without a background metric. The suggestion that the string field should be a density of weight $\frac{1}{2}$ has in fact been made previously from other considerations by Friedan.[1]

For the open string, the issue of background dependence is even more subtle. It appears that the domain of integration may depend implicitly on a choice of background gauge field via string–end-point boundary conditions. Resolution of these issues will require more detailed investigation.

We now discuss the gauge invariance of the cubic action

$$\delta A = A*\Lambda - \Lambda*A. \tag{5}$$

Witten's formulation divides string space into left and right parts which we denote $\mathcal{S}_L$ and $\mathcal{S}_R$. The string field is a function on $\mathcal{S}_L \times \mathcal{S}_R$ which is Hermitian in the sense that

$$A(X_L,X_R) = A^*(\bar{X}_R,\bar{X}_L),$$

where the bar denotes changing the orientation along the string so that left and right and interchanged. The generators of the gauge transformation are anti-Hermitian:

$$\Lambda(X_L,X_R) = -\Lambda^*(\bar{X}_R,\bar{X}_L).$$

The present formulation allows one to exponentiate the infinitesimal gauge transformation (5) to find the finite gauge transformations explicitly[13]:

$$A' = U^{-1}*A*U, \tag{13}$$

where

$$U = e^{\Lambda} \tag{14}$$

and the right-hand side of (14) is defined by its power series expansion ($e^{\Lambda} = \mathcal{I} + \Lambda + \frac{1}{2}\Lambda*\Lambda + \dots$). The gauge group is thus an infinite-dimensional generalization of $U(N)$. Note that U is Grassmann even with ghost number $-\frac{3}{2}$.

After shifting the field by $A_0 = Q_L\mathcal{I}$ we have

$$A' = U^{-1}*(Q_L\mathcal{I})*U + U^{-1}*\tilde{A}*U.$$

Expanding the gauge-transformed field about the same background $A' = Q_L\mathcal{I} + \tilde{A}'$ and using identities (12) we obtain

$$\tilde{A}' = U^{-1}*QU + U^{-1}*\tilde{A}*U,$$

which is the finite gauge transformation for Witten's string-field theory.

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[11]There exist other solutions of Eqs. (6). For example, (6) is satisfied by any string field of the form $Q_L\psi$, where ψ is annihilated by Q.

[12]The background gauge-field dependence of the open string Q arises in a more subtle manner from boundary conditions at the string end points.

[13]This nonlinear invariance was noticed independently by S. Raby (private communication).