Model for Thermal Transport in Tokamaks

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The cause for anomalous electron-energy confinement in tokamaks in an active area of research. In this Letter we present a model for the anomalous electron-energy transport and compare it with experimental results.

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Experiments on tokamaks for the last two decades have shown that the electron-energy confinement time is anomalously small. The cause for this anomaly has been attributed to plasma instabilities driven by various sources like density and temperature gradients. The models for anomalous electron-energy transport recently used by Perkins[1] and Tang[2] are those due to drift waves whose typical scale lengths are greater than the ion gyroradius ρ_i. While they appear to yield gross energy-confinement scalings with plasma parameters consistent with regression fits of experimental data, the drift-wave models are inadequate. A drawback of these models is that they predict electron- and ion-energy transport to be the same, whereas most experimental observations indicate the electron-energy channel to be the dominant energy loss in tokamaks. An exception is that of Doublet III with high-power neutral-beam injection in which the ion transport is reported dominant.[3] Another feature of drift-wave models which is not consistent with experimental data is the radial dependence of the thermal conductivity $\chi_e(r)$. The anomalous conductivity from drift-wave models has a strong electron temperature dependence, $\sim T_e^{7/2}$. As a result $\chi_e(r)$ decreases as a function of r contrary to experimentally inferred χ_e profiles.

An alternative mechanism for electron thermal transport is magnetic fluctuations. Many authors[4-8] have invoked stochastic magnetic fields to explain the anomalous transport. But the source of these magnetic fluctuations has not been specified.

The most natural free-energy source for instabilities responsible for electron thermal transport is the electron temperature gradient. We wish to report here a short-wavelength collisionless microinstability driven by electron temperature gradient which also generates magnetic fluctuations with scale lengths extending to c/ω_{pe}, the collisionless skin depth. Here c is the velocity of light and ω_{pe} the plasma frequency. Magnetic fluctuations have been correlated with energy transport in the TCA tokamak.[9]

A quasilinear analysis for these modes yields a modified Ohkawa[4] formula for electron thermal transport which displays the experimentally derived $\chi_e(r)$ radial dependence quite reasonably and predicts well the temperature profiles recently observed on the Princeton tokamak fusion test reactor (TFTR) by Boyd and Stauffer.[10] Also, the central temperature scaling with plasma parameters is in good agreement with regression fit of experimental data.

Because the wavelength λ of the unstable modes is such that $\rho_e < \lambda \ll \rho_i$, where ρ_i and ρ_e are the ion and electron gyroradii, respectively, the ions respond adiabatically and are hardly affected by the mode. The local theory of this instability in the electrostatic approximation is well documented in literature.[11-13] We have recently studied the properties of the mode in a sheared magnetic field together with full gyrokinetic and finite-beta effects.[14] The mode is found to have a threshold temperature gradient $\eta_e = (d\ln T_e/dr)(d\ln n/dr)^{-1} \simeq 1$ for instability. The mode frequency ω ranges from 0.05 to 0.40 times the electron diamagnetic frequency, $\omega_{ne}^* = k_y c T_e/eB_0 L_n$, where T_e is the electron temperature, e the electron charge, B_0 the toroidal magnetic field, L_n the density scale length, and k_y the azimuthal wave number, for $\eta_e \simeq 2-4$. The growth rates are a factor of 4 smaller than the real frequency. The mode is unstable for $\rho_e < k_y^{-1} < \rho_i$.

Following quasilinear theory[15] the anomalous electron-energy transport coefficient is given by

$$\chi_e = \sum_{k_x,k_y,\omega} \frac{\pi^{1/2}c^2k_y^2}{B_0} \times \int \frac{dv_\parallel}{v_e} e^{-v_\parallel^2/v_e^2} \left| \tilde{\phi} - \frac{v_\parallel}{c}\tilde{A}_\parallel \right|^2 \delta(\omega - k_\parallel v_\parallel), \quad (1)$$

where $\tilde{\phi}$ is the perturbed electrostatic potential, $\tilde{A}_\parallel$ the perturbed parallel vector potential, ω the wave frequency, v_e the electron thermal velocity, $k_\parallel$ the parallel wave number, and $v_\parallel$ the parallel electron velocity. The parallel vector potential when expressed in terms of $\tilde{\phi}$ by Ampère's law gives

$$\chi_e \simeq \sum_{k,\omega} \frac{\pi^{1/2} e^{-\omega^2/k_\parallel^2 v_e^2}}{k_\parallel v_e} \left| \frac{ck_y\tilde{\phi}}{B_0} \right|^2 \left| \frac{k_\perp^2}{k_\perp^2 + 4\pi i \sigma_\parallel \omega/c^2} \right|^2. \quad (2)$$

Here $\sigma_\parallel$ is the parallel electron conductivity. The saturation magnitude of $\tilde{\phi}$ is given by the wave-breaking criterion $\tilde{\phi} = 2\pi\omega B_0/ck_yk_\perp$. χ_e maximizes for $k_\perp^2 = 4\pi i\sigma_\parallel\omega/c^2$. Using the dispersion relation

$$\frac{4\pi i\sigma_\parallel\omega}{c^2} \simeq \frac{2\omega_{pe}^2}{c^2}\frac{\omega^2}{k_\parallel^2 v_e^2}\frac{\omega_{ne}^*}{\omega}, \tag{3}$$

we get

$$\chi_e \simeq \sum \frac{\pi^{5/2}}{2}\frac{c^2}{\omega_{pe}^2}k_\parallel v_e \frac{\omega}{\omega_{ne}^*}, \tag{4}$$

since $k_y \sim k_x$, $k_\parallel \simeq \hat{s}/qR$.

Finally, to get an upper-bound estimate of χ_e we use numerical values of the eigenmode frequency,[14]

$$\omega_{\max}/\omega_{ne}^* \simeq 0.0145\eta_e(1+\eta_e). \tag{5}$$

The electron thermal conductivity is therefore

$$\chi_e(r) = 0.1(c^2\hat{s}/\omega_{pe}^2)(v_e/qR)\eta_e(1+\eta_e), \tag{6}$$

where $q(r)$ is the plasma safety factor $q(r) = rB_0/RB_p(r)$ where $B_p(r)$ is the poloidal field, and $\hat{s} = r(dq/dr)/q$. We will set $\hat{s} \simeq 1$ and drop the unity term compared to η_e. This is a simplified version of the thermal conduction coefficient. It allows for an analytical treatment which agrees well with numerical results for the $\chi_e(r)$ given by Eq. (6).

We will focus on the consequences of using this transport coefficient in a simple energy transport model. The steady-state equations for electron-energy transport in an Ohmically heated tokamak are

$$r^{-1}(d/dr)(r\chi_e n\, dT_e/dr) + jE = 0, \tag{7}$$

$$r^{-1}d(rB_p)/dr = 4\pi j/c. \tag{8}$$

The first term in Eq. (7) is the thermal conduction loss and the second term is the Ohmic heating. Equation (8) is Ampère's law, and $j = \sigma E$, where $\sigma = 1.96ne^2/\tau_{ei}\gamma(Z_{\text{eff}})$,

$$\gamma(Z_{\text{eff}}) = 1.96Z_{\text{eff}}[0.29 + 0.46/(1.08 + Z_{\text{eff}})],$$

τ_{ei} the electron-ion collision time, and E the toroidal electric field. If we combine Eqs. (7) and (8) and use a Gaussian density profile $n = n_0e^{-ar^2/a^2}$ the following equation for the temperature can be derived:

$$T^{-1/2}dT/dr = -\lambda r, \tag{9}$$

where $\lambda = (cEB_0a^2/c_1\pi a^4)^{1/3}$ and $c_1 = 0.1c^2m_e^{1/2}/(2\sqrt{2}\pi \times e^2)$. This equation can be readily solved to give

$$T(r) = T(0)[a(a^2-r^2)/(a^2-r_1^2)]^2, \quad r > r_1. \tag{10}$$

Here we have introduced a radius r_1 such that $q(r) = 1$ and $T(r) = T(0)$ for $r < r_1$. This essentially models the sawtooth behavior that occurs inside the $q = 1$ surface which leads to a flattening of the temperature profile for $r < r_1$. Significant progress on the understanding of this behavior has been recently achieved by Denton *et al.*[16]

With use of Ohm's law and the fact that $q(0) = 1 = cB_0/2\pi\sigma(0)ER$, Eq. (8) can be integrated between r_1 and a to yield

$$1/q_L = \tfrac{1}{4} + \tfrac{3}{4}r_1^2/a^2. \tag{11}$$

Thus the inversion radius r_1 is determined in terms of the limiter safety factor $q_L = ca^2B_0/2RI$, where I is the plasma current. We observe that for $r_1 = 0$, $q_L \simeq 4$, i.e., all discharges will stop sawtoothing for $q_L > 4$. For the more exact thermal conduction model [Eq. (6)] a numerical study yields $q_L \simeq 5.6$. Experimentally, the typical value for TFTR and Alcator C tokamaks is around 7.[10,17] In Fig. 1 we plot $1/q_L$ vs r_1/a. Curve 1 corresponds to Eq. (11); curve 2 is obtained by solution of Eqs. (7) and (8) with the χ_e given by Eq. (6). The circled dots correspond to experimental points for TFTR (see Fig. 5 of Taylor *et al.*[18]).

In Fig. 2 we plot the radial profile of $\chi_e(r)$ from our numerical model for $q_L = 4.0$ and $\alpha = 3.0$. χ_e has been normalized to $c^2v_e(0)/\omega_{pe}^2(0)R$, where 0 denotes the origin. Inside the $q = 1$ $(r/a = 0.25)$ surface, χ is infinite and leads to the flattened temperature. Outside the $q = 1$ surface, χ_e increases as a function of r. This increase is consistent with experimentally inferred thermal conductivity profiles.

The next point of comparison is the scaling of the central electron temperature. From Eq. (6)

$$T^{1/2}(0) = \tfrac{1}{4}\lambda(a^2 - r_1^2). \tag{12}$$

Using the definition of λ and $q(0)$ we get

$$T(0) \propto B_T^{2/3}R^{-1/3}a^{2/3}f(q_L)\gamma(Z_{\text{eff}})^{1/3}, \tag{13}$$

where $f(q_L) = \tfrac{4}{3}(1 - 1/q_L)$ for the analytic model. For

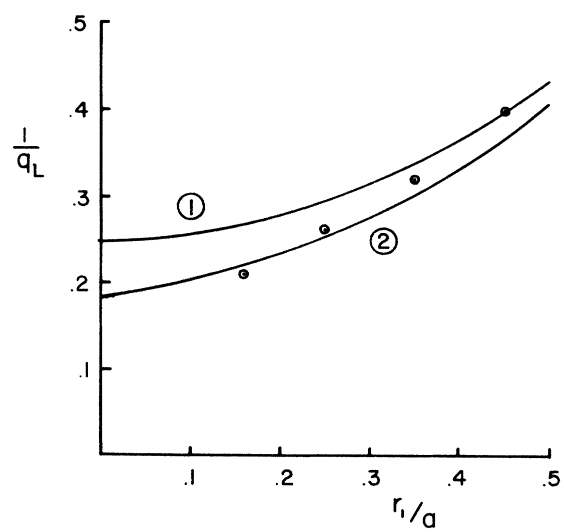

FIG. 1. $1/q_L$ vs r_1/a. Curve 1, $1/q_L = \frac{1}{4} + 3r_1^2/4a^2$; curve 2, numerical model for χ_e given by Eq. (6); and circles, experimental points for TFTR (Ref. 18, Fig. 5).

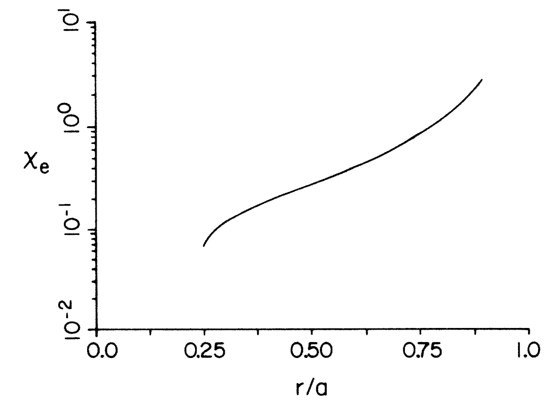

FIG. 2. $\chi_e(r)$ vs r for $q_L = 4.0$ and $\alpha = 3.0$. χ_e is normalized to $\chi_e(0) = c^2 v_e(0)/\omega_{pe}^2(0)R$.

the numerical model $f(q_L) = K q_L^{0.19}$, where K is a constant. This is in good agreement with the TFTR scaling [Eq. (8), Ref. 18]

$$T(0) \sim B_T^{0.6} R^{-0.1} a^{0.7} Z_{\mathrm{eff}}^{0.5} q_L^{0.2}. \tag{14}$$

Finally the most encouraging evidence in favor of the proposed model is the fit of Eq. (10) to experimental temperature profiles. In Fig. 3 the experimental data are presented with T normalized to $T(0)$ and r to a.[10] The dashed portion is the instantaneous temperature inside the sawtooth regime. The dots correspond to the results from Eq. (10). The experimental data are from the electron-cyclotron emission measurements by Stauffer and Boyd.[10]

We have concentrated basically on data from TFTR. Since this machine has a large minor radius (~ 80 cm), the region between the $q = 1$ surface and the outer region where radiation physics dominates is sufficiently large. So the genuine cause for anomalous transport is not masked by other loss effects and hence the dominant balance between Ohmic heating and thermal conduction is a good model.

Thus, based on our thermal conduction model we have shown that for Ohmically heated tokamaks the model predicts temperature profiles and central temperature scalings consistent with experimental observations. However, it is also observed that for low-q_L discharges, the temperature profile remains nearly invariant, independent of the heating profile due to auxiliary sources.[10,17] This is referred to as profile consistency. We wish to outline briefly here why the present form of the thermal conduction displays profile consistency. Let us consider the following simple model:

$$\frac{1}{r}\frac{d}{dr} r k_\perp \frac{dT}{dr} + \frac{P}{2\pi^2 R a^2 N} = 0, \tag{15}$$

where P is a fraction of the auxiliary power source for the electrons, and $N = 2\int_0^a P(r) r\, dr/a^2$. Use of the sim-

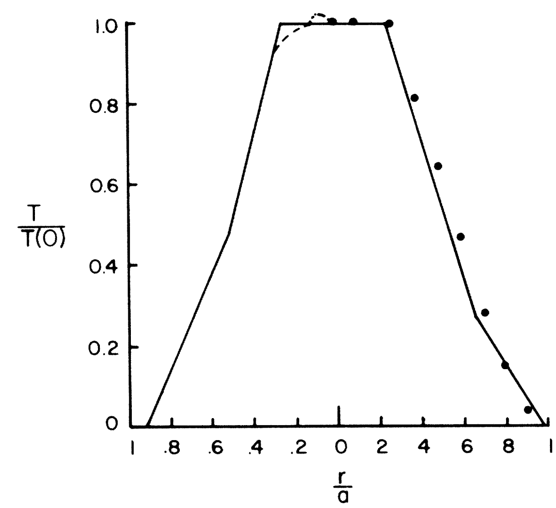

FIG. 3. Solid curve: Experimental result of normalized temperature profile for TFTR for $R = 2.79$ m, $a = 0.55$ m, $q_L = 3.3$, $B_T = 3.04$ T. Dots: Theory points from Eq. (10).

plified version of Eq. (6) leads to

$$\frac{1}{T^{1/2}}\frac{dT}{dr} + (a^2 \bar{P}/\pi^2 a^6 c_1)^{1/3} q^{1/3} r = 0, \tag{16}$$

where $\bar{P} = \int_0^r P(r') r'\, dr'/rN$. The interesting feature that emerges from this equation is that the cube root of the source term strongly suppresses its r dependence. Furthermore for low q_L, since $q^{1/3}$ is now a very weak function of r, we can treat it as a constant so that the solution to Eq. (15) is the same as that of Eq. (9).

Thus the electron temperature profile is similar to the Ohmic case for almost any heating profile of an auxiliary source for low-q_L discharges. But to predict the scalings for the total energy confinement time, the ion equation also has to be considered since there is a significant fraction of the power that goes into the ions and the ion temperature $T_i \gtrsim T_e$, the electron temperature. Details involving study of the combined ion and electron transport equations are in progress and will be presented elsewhere.

Thus, in conclusion, we have shown that for the thermal-conduction model based on the electron temperature-gradient mode, the elements of profile consistency are a consequence of the transport coefficient. Also for the Ohmic case the scalings and magnitude of the central electron temperature with plasma parameters are in reasonable agreement with experimental observation.

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