Experimental Evidence of MHD Surface Waves

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MHD surface waves of $m = -1$ (poloidal mode number of left-hand rotation) compressional Alfvén waves in cylindrical finite-β plasma have been observed for the first time to propagate together with shear Alfvén waves. These modes also show a distinctive feature of the dispersion merging with that of shear Alfvén waves at the center of a plasma column when a limiting frequency below the ion cyclotron frequency is reached. The experimental results confirm a recent prediction concerning surface-wave properties of the first radial eigenmode of a nonaxisymmetric compressional wave in a plasma surrounded by an insulating boundary.

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The MHD surface waves have been one of the most important subjects of recent theoretical investigations associated with Alfvén-wave heating.[1-5] According to recent papers, the first radial eigenmodes of nonaxisymmetric compressional waves in a cylindrical plasma surrounded by an insulating boundary have surface-wave properties at frequencies below the ion cyclotron frequency even in a plasma with a spatial density profile. These surface waves are classified into two kinds. One is the eigenmode having a frequency much less than the ion cyclotron frequency and a real radial wave number which is smaller than $|m/r|$ in a uniform cylindrical system. We are, however, interested here in another type of the MHD surface waves in which only the eigenmode with a negative poloidal mode number m is involved. The reason is that for the latter mode there is a reversion from bulk- to surface-wave properties with wave fields in the plasma eventually becoming completely concentrated at the plasma surface when a limiting axial wave number k_l and a limiting frequency f_l are reached. This interesting feature of the surface wave is attributed to nothing but a pure-imaginary radial wave number. At f_l, which is always less than f_{ci}, the dispersion relation curve of this eigenmode merges with that of the shear Alfvén wave at the maximum density, and disappears in the range of frequencies above f_l. Therefore, this MHD surface wave is of particular interest for Alfvén-wave heating not only because it is easily excited by an antenna, but also because it experiences a spatial Alfvén resonance, and the resulting damping beyond the Alfvén resonance point, which moves towards the position of the maximum density in an inhomogeneous density profile as the limiting frequency of the wave is approached; this should be then quite beneficial for efficient central heating of a plasma column.

The importance of a magnetic field in heating the solar atmosphere is being increasingly recognized,[6] and a kind of surface wave, which is excited at the "foot points" of a magnetic loop, is considered one of the candidates to couple with the kinetic Alfvén waves[7] at the Alfvén resonance point in a coronal loop. On the other hand, some geomagnetic pulsations generated in the magnetosphere have recently been tried to be interpreted by assumption of a simple surface mode of compressional Alfvén waves penetrating from the magnetopause[8]; this mode has to have a real wave number in the east-west direction, so that it should be asymmetrical and then essentially correspond to MHD surface waves discussed here. Thus the MHD surface wave has possible applications to magnetospheric or photospheric plasmas as well as to nuclear fusion devices. To our knowledge, however, this MHD surface wave has not been clearly identified so far from the experimental viewpoint.

The purpose of this Letter is to report the first observation of the $m = -1$ MHD surface wave which has a distinctive feature of the dispersion merging with that of the shear Alfvén wave at a limiting frequency.

The experiments were carried out in the TPH device of Shizuoka University, which has been described in detail in an earlier publication[9] (chamber diameter, 15 cm; chamber length, 2 m). A quasisteady (~ 1 ms), current-free, high-density ($\lesssim 5\times 10^{15}$ cm^{-3}), streaming plasma has been produced by development of a pulsed magneto-plasma-dynamic (MPD) arcjet with an anode of 4-cm diameter. Plasma parameters were measured with an HCN laser inteferometer ($\lambda = 337$ μm), a spectrometer, double probes, magnetic probes, and diamagnetic loops. Typical plasma parameters used here are as follows: electron and helium-ion temperatures $T_e \simeq T_i \sim 7$ eV, plasma density $n_0 \simeq 5.5\times 10^{14}$ cm^{-3} at the center of the column; ionization degree $\gtrsim 80\%$; the constant axial magnetic field of 3.0 kG; β value $\sim 1\%$. The density profile in the radial direction is approximately given by $n = n_0[1-(r/6)^2]/[1+(r/6)^2]$, where the radius r is in centimeters. In order to excite the $m = -1$ (left-hand rotation) of a frequency below f_{ci}, we used a convenient helical antenna with 12.0-cm diameter, which can successfully launch only a preferred m mode as described in the Ref. 9. In the present experiment, a pair of small movable θ-magnetic probes (100-

turn wire with 5-mm diameter) were used to detect magnetic fields for every 5 mm along the radial direction. One probe used as a reference was radially inserted at 30 cm down from the antenna, and another probe was inserted at 20 cm further down from the reference probe. The data were processed digitally by use of a data acquisition system described previously in detail.[10] The Nyquist frequency of the analog-to-digital converter is 15 MHz for two channels, so that we do not have aliasing problems for signals of frequencies below the ion cyclotron frequency 1.14 MHz. The time series data obtained were averaged over at least five shots per each position, even though the reproducibility of plasma is excellent. In order to obtain phase velocities and attenuation lengths of the waves propagating along the magnetic field, auto- and cross-correlation functions, their power spectrum densities and coherency of correlations were computed from converted data by use of the fast-Fourier-transform technique. In this paper the data of cross correlation keep the coherency more than 90%.

Figure 1 shows two examples of the parallel phase velocity profile in the radial direction, of launched waves below the ion cyclotron frequency. We can see from this figure that near the plasma center the waves propagate with the velocity depending on the density inhomogeneity, whereas outside of this region to the plasma edge $r \sim 5$ cm, they obviously have an eigenfrequency almost independent of the nonuniform density, and at the edge near $r = 6$ cm, their phase velocity varies again versus radius, where the measured wave numbers must be altered very much by the eigenmode, as will be mentioned later in the paper. From these facts the helical antenna used with $m = -1$ rotation is likely to provide shear Alfvén waves (SAW's) near the center, and compressional waves on the edge of the plasma column. Thus we can expect the eigenmode observed on the surface to have the characteristic of the MHD surface waves.

First of all, the peculiar dispersion relation of $m = -1$ compressional waves is demonstrated by measurement of axial wave numbers of excited waves with a pair of θ probes at some fixed radii. Figure 2 shows the dispersion relations at the radial positions $r = 0.0$, 1.5, and 4.0 cm. Obviously the dispersion near the center of the plasma has the form of shear Alfvén waves, while the dispersion curve at $r = 4.0$ cm is *merging* with that of the SAW at $r = 0.0$ cm when the angular frequency goes up to 3.93×10^6 rad/s (625 kHz). The waves of frequencies above this value, however, follow the dispersion of the SAW, indicating that as predicted by the theory there does not exist any eigenmodes of $m = -1$ above the limiting frequency and limiting axial wave number. From Fig. 2 one can deduce the limiting wave number $k_l \simeq 0.34$ cm^{-1}. Using the average plasma density 2.1×10^{14} cm^{-3} or average Alfvén velocity 2.3×10^7 cm/s, one can evaluate a normalized limiting wave number of the surface wave ε_{l0} defined in the literature[2] to be 0.72, and so one approximately obtains the wave number of slow waves at the merging point to be 0.37 cm^{-1}; considering the fact of a nonuniform plasma in the radial direction, this is in good agreement with the former observed limiting value. Here we should note that compared to the theoretical curve, measured wave numbers of the SAW in Fig. 2 are somewhat modified in the frequency range where the $m = -1$ eigenmode exists. This is

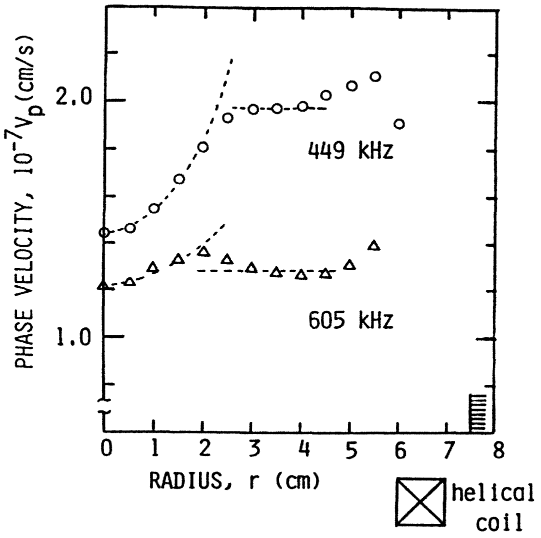

FIG. 1. Plot of parallel phase-velocity distribution for two frequencies vs radius, averaged over 20 cm. The position of the helical antenna is 30 cm up from the reference probe.

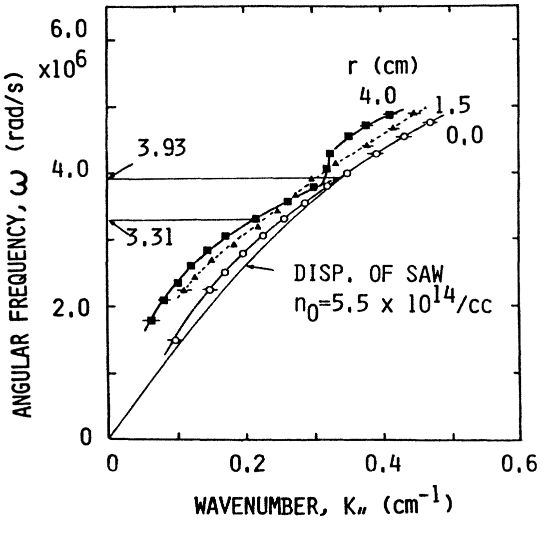

FIG. 2. Dispersion relation $(\omega, k_{\parallel})$ at $r = 0.0$ (circles), 1.5 (triangles), and 4.0 cm (squares), demonstrating the merging of the MHD surface waves. $\omega_{ci} = 7.19 \times 10^6$ rad/s. Theoretical curve of the shear Alfvén wave for the density of 5.5×10^{14} cm^{-3} is also drawn.

caused by computation of the complex cross-power spectrum as a function of frequency from the time-series data which include a small amount of component of a wave number of eigenmode even near the center.

An important check on another point of the theoretical predictions, i.e., the feature of the surface wave, or $k_r^2 < 0$, is provided by the observation of the wave attenuation length (*e*-folding length) as a function of radius. Figure 3 shows the attenuation length of the propagating waves together with the axial wave number as a function of radius for two typical frequencies, 605 kHz which is just below the limiting frequency, and 449 kHz at which the compressional waves are not expected to behave like the surface waves. We find clearly that the wave of $f = 605$ kHz and $k_\parallel = 0.30$ cm^{-1} propagates on the surface of the column with the attenuation rate much less than that of the wave of $f = 449$ kHz and $k_\parallel = 0.14$ cm^{-1}. This surface-wave behavior was observed, certainly as expected from the theory, in the range of angular frequencies between 3.93×10^6 and 3.31×10^6 rad/s as indicated in Fig. 1. Furthermore, inside the plasma column, the attenuation length of the wave of 449 kHz has a peak around $r = 3.5$ cm, where the axial wave number of the compressional wave matches with that of the SAW, indicating that this peak corresponds to the spatial Alfvén resonance. On the other hand, the attenuation length curve for 605 kHz appears to have two peaks inside the plasma column at which the axial wave number of the eigenmode is equal to the local axial wave numbers; the bump around $r = 1.0$ cm corresponds to the spatial Alfvén resonance, so that it is unexpected that there is another sharp peak of the attenuation length around $r = 2.7$ cm.

In conclusion, I have observed the $m = -1$ MHD surface waves propagating with the SAW's for the first time, their properties being consistent with the recent prediction. However, it should be noted that I have made experiments by excitation of all possible modes at the antenna and the obtained correlation functions of propagating waves at the distance of the order of one wavelength from the antenna; in other words, we have observed the waves in a transitional period. Hence, unless we excite only one discrete eigenmode (e.g., the surface wave) or we observe the waves at a much larger distance from the antenna where the SAW disappears already by suffering a stronger damping, we cannot compare the field profiles, polarizations, and so on exactly with the existing theories in which an eigenvalue problem is dealt with. The anomalous peak of the attenuation length in Fig. 3 may be due to the transitional effect, but further experiments and considerations treating an initial value problem must be required to clarify its mechanism.

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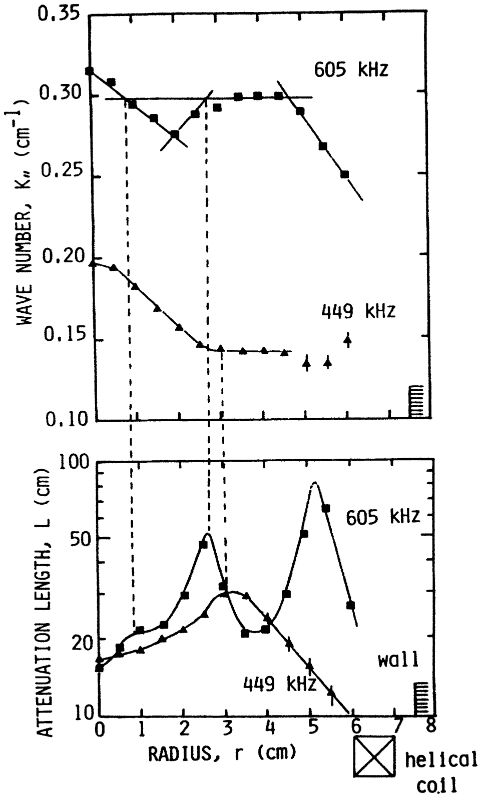

FIG. 3. Measured attenuation length (*e*-folding length) and axial wave number vs radius for 605 and 449 kHz. Dashed lines indicate that each resonance peak of attenuation lengths corresponds to a bend of $k_\parallel$ curves.

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