Evidence for Lorenz-Type Chaos in a Laser

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(Received 18 April 1986)

Observations of the dynamics of a single-mode, traveling-wave laser under bad-cavity conditions are reported. The sequence of instabilities occurring on resonator tuning corresponds in detail to the transition to chaos of the logistic equation. Period-doubling cascade, reverse ("noisy") cascade, and the regular period-3 and -5 windows in the chaotic range are observed. At presumably homogeneous broadening conditions the transition from cw to chaotic emission is abrupt on pump variation. All of the observed features including instability pump thresholds and characteristics of the chaotic laser pulses agree with predictions of the Lorenz equations.

PACS numbers: 42.50.Tj, 42.55.−f

We have recently found instabilities and chaotic emission of an optically pumped NH_3 laser.[1] For this laser transition we estimate[2] that the conditions for the chaos of the Lorenz model[3] can be met. An experiment was therefore done to observe the characteristic features of the Lorenz-model chaos on this optical system. These are bifurcation behavior, magnitude of second threshold, and in particular chaotic pulse shape, determined by the form of the chaotic Lorenz attractor.

The laser transition is the $aR(7,7)$ rotational transition in the $V_2=1$ vibrational state of $^{14}NH_3$, which can be pumped optically with the $P(13)$ line of the N_2O laser via the vibrational $aQ(8,7)$ transition. The ratio of population to polarizational relaxation rate is 0.25.[4]

The laser resonator is a unidirectional ring designed to approach most closely the conditions of spatially uniform material and field parameters of the laser Lorenz equations. Details have been given in Ref. 1. To ensure reproducible pumping conditions, i.e., pumping of always the same velocity group of molecules, the frequency of the N_2O pump laser was controlled with respect to a Lamb dip of the vibrational $aQ(8,7)$ NH_3 transition. The homogeneous linewidth of the laser medium is in all cases smaller than the resonator linewidth so that the "bad cavity" condition is always fulfilled.

The laser emission at 81 μm wavelength was detected by a micrometer-sized Schottky-barrier diode, preamplified, and displayed on a storage oscilloscope or a radiofrequency spectrum analyzer. Figure 1 shows spectra of the laser output at different laser resonator settings. The period-doubling cascade of the logistic equation is clearly observed in Fig. 1(a).

The reverse cascade of the logistic equation, in which the stable periodic orbits successively become chaotic until full chaos sets in, is also clearly observed [Fig. 1(b)]. After onset of full chaos we can observe the well known period-3 and period-5 "windows" of regular motion in the chaotic range shown in Figs. 2(a) and 2(b) in the time picture.

Figure 3 gives the measured threshold pump powers for cw laser emission and instability as functions of laser gas pressure, and their ratio (pump parameter). Whereas for lower working pressures the instability threshold pump parameter lies around 4—too low to be compatible with the Lorenz model, and thus presumably indicating some inhomogeneous broadening due to the ac Stark effect of the pump transition[5]—the values at the higher pressures, where pressure broadenings should exceed ac Stark broadening, are in the range expected a homogeneously broadened band-cavity laser.[3] From the relaxation data of the laser transition[4] and estimated resonator losses of 5% we calculate an instability threshold pump parameter of 11 to 12 for the Lorenz case. The fact that we observe somewhat higher values is not surprising since the Gaussian field distribution in the laser is known to raise the thresholds with respect to the plane-wave case. The resonator loss may also be higher than the estimate which again raises the thresholds.

In the pressure range where we find the low instability pump thresholds, the instability sequence leading to chaos as the laser is centrally tuned and the pump strength is increased is also the period-doubling sequence, incompatible with the Lorenz case. On the other hand, in the pressure range above 9 Pa we find an *abrupt* transition from continuous emission to chaotic emission as predicted by the Lorenz model[3,6] without any regular pulsing between them. Hysteresis between the chaotic state and the cw state with respect to the pump power, which is predicted by the Lorenz model, was observed. The chaos "onset" pump power appeared to be 20% higher than the pump power at which the chaos disappears. It was, however, not possible to measure this amount of hystersis reliably because of the combined effects of mechanical hysteresis of the pump attenuator and thermally dependent sensitivity of the pump power detector. For detuned cases this transition proceeds via period doublings, also as predicted for the Lorenz case.[6] The number of period doublings to the pump power at which line-center chaotic emission is reached decreases with detuning until at large detunings any periodic puls-

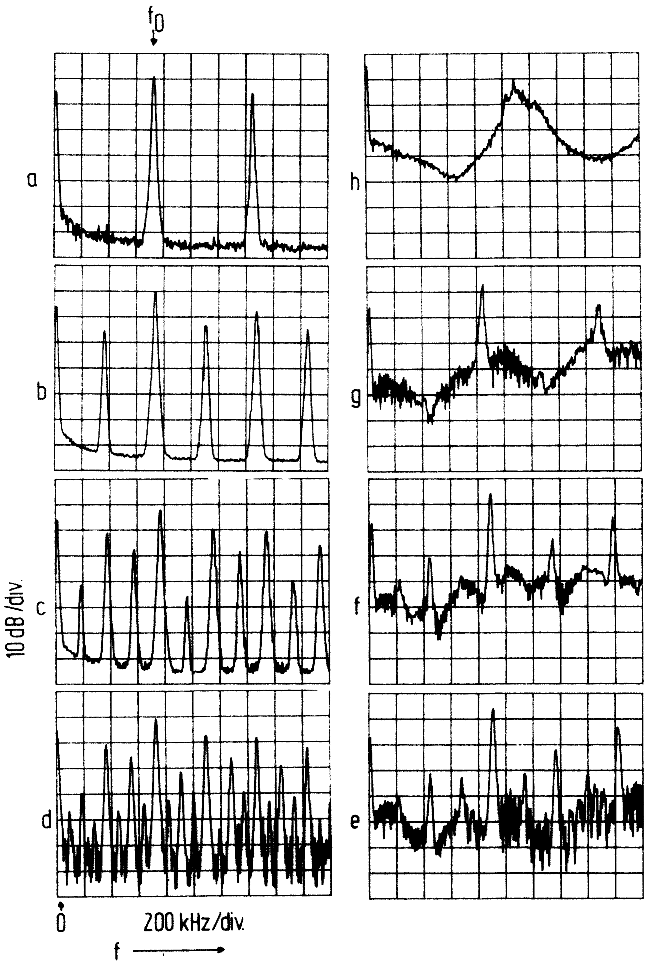

FIG. 1. rf spectra of laser intensity at fixed pump intensity for different laser resonator settings. (Left) A period-doubling sequence followed by (right) a reverse doubling sequence as laser resonator is tuned towards gain line center, in the reverse sequence the subharmonics become progressively noisy [(e) noisy period 8; (f) noisy period 4; (g) noisy period 2; (h) noisy period 1). Pump intensity: 2 W/cm, ~14 times above first laser threshold (see Fig. 3); NH_3 pressure: 9 Pa. Ratio of laser field decay rate to polarization decay rate estimated to be 2 (resonator loss can only be estimated). Laser observed in backward emission direction. Pumping is ~30 MHz off NH_3 pump absorption line center. Quantitative detuning data cannot be given, as precise enough resonator length measurements or (heterodyne) laser frequency measurements are not possible with the present apparatus.

ing disappears and only continunous emission remains.

In short we observe a bifurcation diagram (variables: pump parameter and resonator tuning) in agreement with the one calculated for the Lorenz equations.[6] The instability pump parameter is in addition in the high range characteristic for the Lorenz case.

It is worth noting that even in the case of chaotic pulsing the basic pulsing period is still well defined. The temporal evolution of the chaotic pulse train in Fig. 4(a)

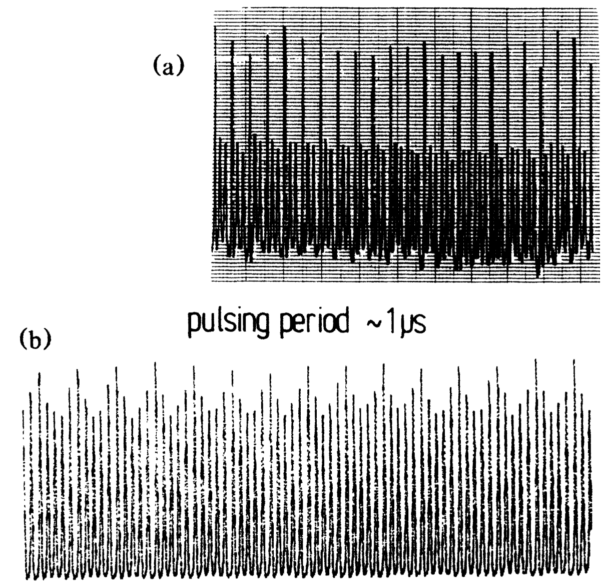

FIG. 2. (a) Regular period-3 pulsing and (b) period-5 pulsing in the chaotic range (time picture). Conditions as in Fig. 1. Pump power 15 times above first laser threshold, central tuning.

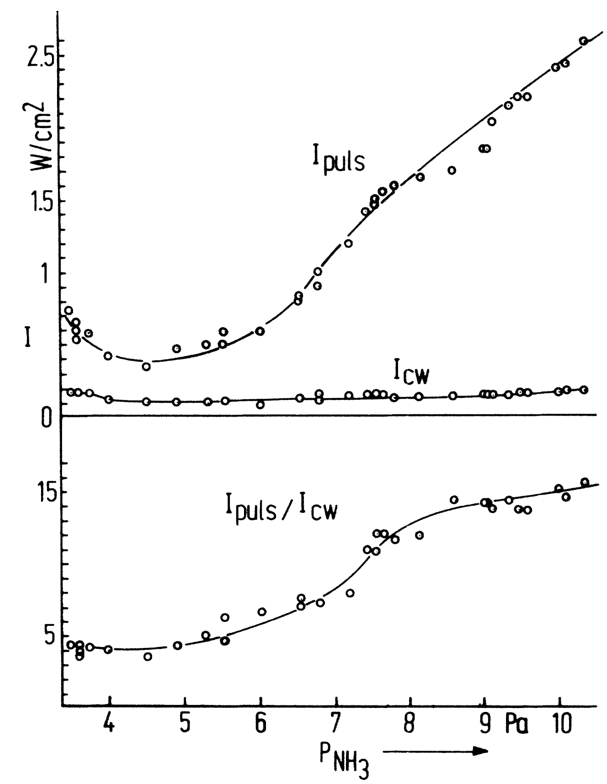

FIG. 3. cw laser and instability threshold pump intensity and their ratio as functions of NH_3 pressure for the centrally tuned laser. At low pressure the instability thresholds are too low for compatibility with the Lorenz model, while they are high enough to be compatible with the Lorenz model for $p > 9$ Pa. Laser emission observed in backward direction, pumping ~30 MHz off NH_3 pump absorption line center.

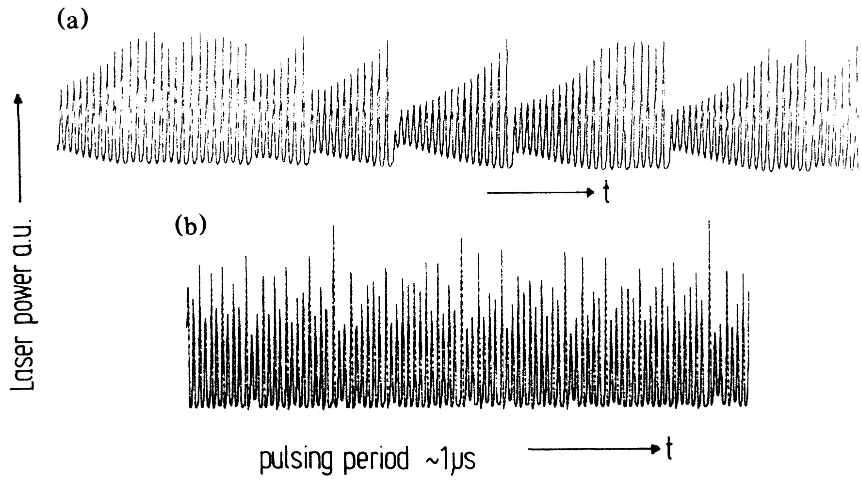

FIG. 4. Chaotic pulsing (time picture). (a) Conditions as in Fig. 1, but for a pressure of 8 Pa, central tuning. (b) Conditions as in Fig. 1, but for a pressure of 3 Pa, central tuning. Ratio of laser field decay rate to polarization decay rate: estimated to be 2 for (a), 7 for (b). (See corresponding remark Fig. 1.) Ratio of pump power at first laser threshold 14 for (a), 10 for (b).

shows the typical motion of the Lorenz system, the spiraling around two centers (e.g., in the field-polarization plane) with random jumps from one center to the other. The field values of the centers differ essentially in their sign, which, of course, cannot be seen in Fig. 4(a) (recording of laser pulse intensities). The outward spiraling and the random jumps are, however, clearly visible. Note the completely different chaotic pulsing in the low-pressure range which does not correspond to the Lorenz case [Fig. 4(b)].

We conclude that real homogeneously broadened lasers show the dynamics of the Maxwell-Bloch equations (Lorenz model for central tuning). The spatial inhomogeneity of the laser field and material parameters does not cause qualitative changes from the theoretical predictions.

This work was supported by Deutsche Forschungsgemeinschaft

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