**Thouless Responds:** The observation that a gauge transformation can be used to show the equivalence of the constant-$J$, random-($\pm h$)-field Ising model to the random-($\pm J$), constant-field spin-glass[1,2] has been made to me by a number of other people.[3] Although the argument given is clearly correct for a finite Cayley tree, I do not think that it is correct for the Bethe lattice, which can be regarded either as an infinite system, or as the limit of a finite Cayley tree with boundary conditions designed to give homogeneous bulk properties. For the ferromagnetic Ising model in a weak $\pm h$ random field it is easy to show that my treatment of the Bethe lattice[4] leads to a ferromagnetic critical transition shifted down from the pure critical point at $K \tanh(J/T) = 1$ by an amount proportional to $h^2$, as was found in an earlier paper by Bruinsma.[5] The spin-glass model with $\pm J$ coupling and constant field $h$ has a lower critical temperature with a quite different field dependence. It is therefore clear that the method I use is not invariant under the type of gauge transformation given in Eq. (2) of Bruinsma's Comment.[2]

One important difference between a finite Cayley tree and a Bethe lattice is that for a Cayley tree the number of sites and bonds is essentially equal. For a Bethe lattice the ratio of sites to bonds is $2/(K+1)$ since each bond connects two sites, while each site has $K+1$ bonds. This ratio has to be used, for example, in the calculation of the relative contributions of sites and bonds to the internal energy. A recent careful treatment using this approach can be found in a paper by Peruggi, di Liberto, and Monroy.[6] The equality of sites and bonds for the finite Cayley tree allows the random-bond problem to be transformed into a random-field problem by this gauge transformation applied to the sites, but the excess of bonds over sites for the infinite Bethe lattice means that a gauge transformation applied to the sites does not have enough degrees of freedom to do this.

If the Bethe lattice is to be regarded as the limit of a finite Cayley tree, the boundary conditions used are not general boundary conditions, but only those which could be proved by an embedding of the tree in a similar but larger tree. The boundary conditions must therefore satisfy a generalization of the criterion originally used by Bethe.[7] From this approach also the random-field and the random-coupling models are different. Stability of the solution means that small perturbations of the boundary conditions lead to the same solution, but it does not imply that almost all boundary conditions lead to that solution. I am sorry if a remark I made in my paper gave that impression. We have prepared a much longer discussion of the effect of boundary conditions on such problems.[8]

Since I do not accept that the gauge transformation can be applied to the Bethe lattice I do not agree with the other conclusions drawn by these authors.

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