

Flow of Polarized Fermi Gases through Narrow Channels

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The flow of a polarized Fermi gas through channels from one cell to another can be used to study its transport properties. If the mean free path λ in the gas is smaller than the radius r of a channel, the time constant for the filling of an empty chamber can yield the viscosity of the gas in both the Boltzmann and degenerate cases. If $\lambda > r$ and the gas is degenerate, a novel effect is predicted: Because the Fermi velocities of the two spin species are different, the polarization in the second cell will initially have a value greater than the equilibrium value in the first cell.

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Polarized gases, such as ^3He gas or dilute solutions of ^3He in superfluid ^4He , are expected to have substantial increases in viscosity and thermal conductivity as the polarization is increased.¹⁻³ It seems natural to measure the viscosity by studying the flow rate of the gas through a narrow channel. We consider theoretically the situation in which a gas is polarized in one chamber and then allowed to flow through a channel into a second chamber which is empty or partially full of polarized gas. We will show how the experimenter can infer the viscosity from the time dependence of the density of gas in the second chamber. A nice feature of this setup is that the density in the second cell can be monitored either by measuring the polarization in the second cell by magnetic techniques or by using the usual capacitance gauge.

An assumption in the above discussion is that the conditions for Poiseuille flow hold, namely that the mean free path λ is smaller than the radius r of the channel. However, in the Knudsen limit, $\lambda > r$, the rate of diffusion of particles down the channel is governed by collisions with the wall and depends on the average particle velocity. For degenerate gases the relevant velocities are different for up and down spins because the Fermi momenta are different. This implies that the dominant spin species will diffuse faster. The result is that the polarization in the initially empty chamber will be enhanced over that of the full chamber for the early portions of the fill time. This situation is very analogous to the separation of isotopes of uranium by gaseous diffusion. Of course, in that case the velocity difference is created by a mass difference rather than by degeneracy effects. In principle, the cascade technique used in isotope separation could be used here to produce very large nonequilibrium polarizations—if the spin-relaxation time can be kept sufficiently long.

Some preliminary viscosity measurements of polarized dilute solutions of ^3He in liquid ^4He were carried out by Schmiedeshoff⁴ who used a vibrating-wire technique. Experimental difficulties made these measurements uncertain. Bouchaud and Lhuillier⁵ have proposed measuring the product of the thermal conductivity and the viscosity by examining the Rayleigh-Bénard instability.

Our approach to the viscosity provides a simple alternative method.

We first consider a polarized gas in the limit of Poiseuille flow. Two volumes V_1 and V_2 are separated by a filter (or perhaps a capillary tube) as shown schematically in Fig. 1. V_1 is filled with gas of density n_1 and initial polarization M_{10} ($-1 < M < 1$). The filter separating the two volumes has a total of C channels each of radius r and length L . The current density of atoms flowing through one of these channels is⁶

$$J = -(nr^2/8\eta)dP/dz, \quad (1)$$

where η is the viscosity and n the gas density. dP/dz is the pressure gradient in the tube. From the current expression we find that the density in cell 1 decreases according to

$$\frac{dn_1}{dt} = \frac{-C\pi r^2}{V_1} J \Big|_{z=0} = \frac{\pi r^4 C}{8V_1 \eta_1} n_1 \frac{dP}{dz} \Big|_{z=0} \quad (2)$$

The filter channels start at $z=0$ and end at $z=L$. A subscript has been put on η because, in general, the viscosity can depend on density which can change considerably from one end of the channel to the other. A similar equation holds for the density n_2 of gas in the other chamber V_2 ; we simply insert a minus sign, change

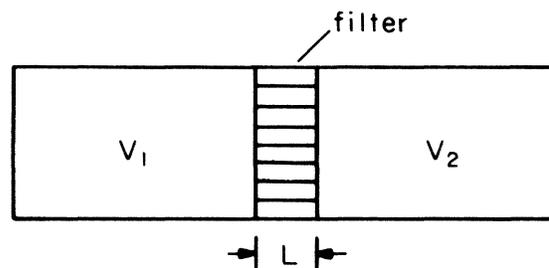


FIG. 1. Two cells of volumes V_1 and V_2 are connected by a filter of thickness L . Polarized gas flows from V_1 into the less completely filled V_2 . The filter has C channels each of radius r .

1 to 2 everywhere, and evaluate the pressure gradient at $z=L$.

The pressure gradient in the channel can be determined by use of the equation of continuity for the density: $\partial n/\partial t = -\partial J/\partial z$. It is easy to show that as long as the total volume of the channels is much smaller than that of either chamber the density quickly reaches the quasistatic situation $\partial n/\partial t = 0$ within the channels. Thus we need only solve the equation

$$(d/dz)[(n/\eta)dP/dz] = 0. \quad (3)$$

In the case of a polarized Boltzmann gas, we have $P = nkT$ and we use the expression for the viscosity in

the low-temperature approximation¹ which is

$$\eta = (5\sqrt{\pi}mkT/32\pi a^2)(1 + \frac{3}{5}M^2)/(1 - M^2). \quad (4)$$

Here m is the atomic mass and a is the s -wave scattering length. Note that this expression is independent of the density. Thus the density in a channel satisfies $(d/dz)(n dn/dz) = 0$. It is easy to solve this equation subject to the boundary conditions $n(0) = n_1$, $n(L) = n_2$. The pressure gradient follows immediately so that Eq. (2), and the corresponding one for n_2 , become

$$dn_i/dt = (-1)^i \alpha_i (n_1^2 - n_2^2), \quad (5)$$

where $\alpha_i = \pi r^4 CkT/(16\eta LV_i)$. The solution of these equations for the density in the filling chamber 2 is

$$n_2(t) = n(\infty) - \frac{\alpha_2}{\alpha_1 + \alpha_2} \frac{\gamma [n_1(0) - n_2(0)] e^{-\gamma t}}{\gamma + (\alpha_1 - \alpha_2) [n_1(0) - n_2(0)] (1 - e^{-\gamma t})}, \quad (6)$$

where $n_i(0)$ is the initial density in the chamber i , $n(\infty)$ is the final equilibrium density equal to $N(V_1 + V_2)$, and $\gamma = 2n(\infty)(\alpha_1 + \alpha_2)$ is the inverse of the time constant. The density in the emptying chamber 1 obeys a very similar formula which follows easily from the fact that $n_1 V_1 + n_2 V_2$ is a constant. Note that Eq. (6) reduces to a simple exponential relation if $V_1 = V_2$.

For ³He gas at 1 K or dilute solutions at a concentration of, say, 10^{-4} , the mean free path is on the order of 25 μm . Therefore, a channel radius somewhat larger than this is in order. However, we note that Eq. (1) contains no correction for slip.^{6,7} Such a correction is density dependent and would complicate the analysis considerably. Since the slip correction is proportional to λ/r ,⁷ we need to make this as small as possible while maintaining a reasonable time constant. This is somewhat difficult because $\gamma \sim r^4$. Nevertheless, with $V_1 = V_2 = 10 \text{ cm}^3$, a single channel having $r \approx 30\lambda$ and $L = 2 \text{ cm}$, we find a time constant of about a tenth of a second. Slip should then cause only a few percent error if the scattering at the wall is diffuse. Another major consideration

here is to have $\gamma^{-1} \ll T_1$, the spin relaxation time, which can be several minutes in cells having ⁴He-coated walls.⁸

In the degenerate case the viscosity has a quite complicated dependence³ on polarization M which we do not repeat here. However, the density dependence can be written as $\eta = K_\eta n^{5/3}$ with K_η containing the polarization and temperature dependences. The pressure, in the ideal-Fermi-gas limit, can also be put in the form $P = K_P n^{5/3}$, where K_P is a polarization-dependent parameter. When these expressions are put into Eqs. (2) and (3) we find that the density distribution in a channel is linear and that the densities in volumes 1 and 2 obey the simple relations

$$dn_i/dt = (-1)^i \xi_i (n_1 - n_2), \quad (7)$$

in which $\xi_i = (5\pi r^4 C/24LV_i)(P/\eta)$. Note that the ratio P/η is independent of density but, of course, depends on polarization and temperature. The result is that in the degenerate case the filling chamber density is given by

$$n_2(t) = n(\infty) - [\xi_2/(\xi_1 + \xi_2)] [n_1(0) - n_2(0)] \exp[-(\xi_1 + \xi_2)t]. \quad (8)$$

The time constant gives the ratio P/η ; P can be gotten independently by means of a capacitance gauge. Again slip (which is treated for Fermi gases in Ref. 7) needs to be minimized. For a ³He concentration of 10^{-4} in a dilute solution and the experimental parameters quoted above for the Boltzmann case, a time constant of about a tenth of a second results here too.

In the Knudsen limit, $\lambda > r$, which might be achieved by use of a Nuclepore filter, channel flow does not provide a measure of the viscosity. Nevertheless, a rather novel effect can be observed in this case when the gas is degenerate. The idea is based on the fact that for a degenerate gas the flow down a channel is impeded by collisions of those particles at the Fermi surface with the walls. However, in a polarized gas up and down spins

have different Fermi velocities v_σ so that the dominant spin species diffuses down a channel at a faster rate, and the polarization in the second cell will be initially larger than the original polarization. We can write equations somewhat analogous to those above except now we consider them for individual spin-component densities n_σ . In a channel the current density for particles of spin σ can be shown to be $J_\sigma = -D_\sigma \partial n_\sigma / \partial z$ where the diffusion constant is (up to constants of order unity) $D_\sigma = r v_\sigma$.⁹ The spin densities in cell 1 are therefore given by

$$\frac{\partial n_{1\sigma}}{\partial t} = \frac{-C\pi r^2}{V_1} \left[D_\sigma \frac{\partial n_\sigma}{\partial z} \right]_{z=0}, \quad (9)$$

with a similar equation for $n_{2\sigma}$. To find the spin-density

profiles in the channel we use the continuity equation $\partial n_\sigma / \partial t = -\partial J_\sigma / \partial z$. (T_1 is assumed to be very long.) Again we can consider this in the quasistatic case, which gives us

$$(\partial / \partial z)(D_\sigma \partial n_\sigma / \partial z) = 0. \quad (10)$$

We cannot take D_σ out of the differentiation because $D_\sigma \sim v_\sigma = \hbar(6\pi^2 n_\sigma)^{1/3} / m$ depends on the density. The density gradient is found from Eq. (10) to be given by $n_\sigma^{1/3} dn_\sigma / dz = 3(n_{2\sigma}^{4/3} - n_{1\sigma}^{4/3}) / 4L$. Putting this result into Eq. (9) and into the corresponding one for $n_{2\sigma}$ gives

$$dn_{i\sigma} / dt = (-1)^i \mu_i [n_{1\sigma}^{4/3} - n_{2\sigma}^{4/3}], \quad (11)$$

in which the coefficients are $\mu_i = 3C\pi r^3 \hbar (6\pi^2)^{1/3} / 4mLV_i$. These equations can be reduced to an integral for $n_{2\sigma}$ that can be evaluated numerically. However, before we present the results of such an analysis, we look at a simpler linear case that can be handled analytically and yet gives good qualitative results. What we do is to replace the quantity in square brackets in Eq. (11) by $n_{1\sigma}^{1/3}(0)[n_{1\sigma} - n_{2\sigma}]$. In this case the equations for $n_{1\sigma}$ and $n_{2\sigma}$ become

$$dn_{i\sigma} / dt = (-1)^i \zeta_{i\sigma} (n_{1\sigma} - n_{2\sigma}), \quad (12)$$

where $\zeta_{i\sigma} = \pi r^3 C v_\sigma(z=0, t=0) / (LV_i)$. These equations are similar to Eqs. (7) except that the coefficients are now spin dependent. The solutions for the second-cell spin densities are

$$n_{2\sigma}(t) = n_\sigma(\infty) - [\zeta_{2\sigma} / (\zeta_{1\sigma} + \zeta_{2\sigma})][n_{1\sigma}(0) - n_{2\sigma}(0)] \exp[-(\zeta_{1\sigma} + \zeta_{2\sigma})t]. \quad (13)$$

The time constants of n_{2+} and n_{2-} differ as expected. If $v_+ > v_-$ the gas initially flowing into cell 2 will be mostly up spins and the polarization will become larger than in cell 1. This is most clearly seen by considering the case in which the cell 2 is empty.¹⁰ It is straightforward to show that, for times small enough that the exponentials can be expanded and terms beyond those linear in t dropped, the polarization in chamber 2 is then given by

$$M_2 = \frac{(1 + M_{10})^{4/3} - (1 - M_{10})^{4/3}}{(1 + M_{10})^{4/3} + (1 - M_{10})^{4/3}}, \text{ for small } t \quad (14)$$

where M_{10} is the initial polarization in cell 1. If the initial polarization in cell 1 is small Eq. (14) reduces to

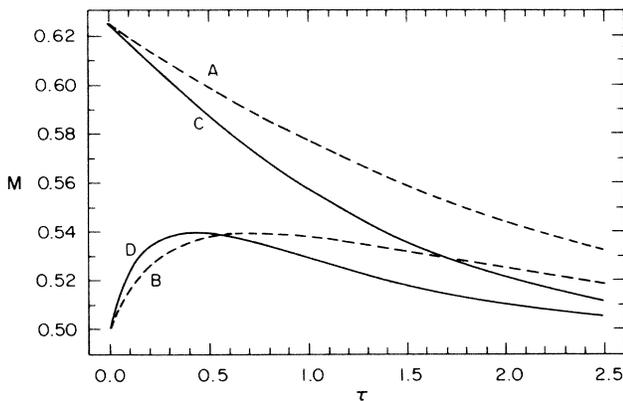


FIG. 2. Polarization vs time of a degenerate Fermi gas in cell 2 which is being filled by flow through a filter from cell 1. Curves *A* and *B* are computed on the basis of the simple approximation, Eq. (12). Curves *C* and *D* are the results of a numerical solution of the exact equations. The conditions in cases *A* and *C* are that cell 1 has an initial polarization of 0.5 and cell 2 is initially empty. In cases *B* and *D* cell 2 initially contains $\frac{1}{4}$ of the density of particles as in cell 1 and the initial polarization in both cells is 0.5. In all cases the volumes of both cells are equal to V . The time parameter is $\tau = tC\pi r^3 \hbar \times [3\pi^2 n_{1\sigma}(0)]^{1/3} / mVL$.

$M_2 = \frac{4}{3} M_{10}$. This is the largest polarization enhancement that can occur in a single pass through a filter, even in the more complicated model given by Eq. (11). The polarization as a function of time is shown as curve *A* in Fig. 2.

If the density of the filling cell is not initially empty, then the polarization reaches a maximum before settling back down to the initial equilibrium value. Curve *B* in Fig. 2 shows this situation. It should be easy to adjust the physical parameters in these experiments to establish a reasonable time scale.

We now consider the exact case of Eq. (11). Numerical results are shown in curves *C* and *D* in Fig. 2 which have the same initial conditions as curve *A* and *B*, respectively. The approximate treatment gives qualitatively correct results but persists at high polarization too long.

There is an alternative experimental arrangement that can also lead to a polarization enhancement. Instead of Knudsen flow through a tube, we can consider effusion through one or more small holes. The current density of particles having spin σ through a hole from a chamber containing density n_σ can be shown to be $\frac{3}{16} n_\sigma v_\sigma$; the predominant spin species flow out of the chamber more quickly. The result for flow from one chamber to another turns out to be described by Eq. (11) but with μ_i now simply missing a factor of $2r/L$.

We have shown how the viscosity of a polarized gas can be measured by letting the gas flow from one chamber into another through channels that are larger in radius than the mean-free-path length. Such a measurement is important in order to test theoretical predictions that tell us that this quantity will increase dramatically with increasing polarization for either a degenerate or a Boltzmann gas.

When a degenerate gas flows through a channel or an opening smaller than the mean-free-path length, then the flow rate is different for the two spin species. This can

lead to a polarization enhancement.¹¹ Although this enhancement can be as large as $\frac{4}{3}$ it may be difficult to use in creating highly polarized samples for further experiments because the density is small when the gas polarization is large. One might think of trying to set up a multistage process in which the polarization is enhanced many times in a continuous process analogous to that used for purifying ^{235}U . Whether this is feasible depends on whether spin relaxation, including that taking place in the filters, provides sufficient time.

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¹⁰We should point out that if the cell-2 density initially is too small, the gas there may not be degenerate, nor will all of the gas in the channels. This situation could probably be taken into account only with difficulty. One way to avoid this is to make sure to start the filling process with an amount of gas in the second cell sufficient that there is degeneracy everywhere.

¹¹We note that the polarization enhancement process is a refrigerator. The spin temperature of the enhanced gas is lower than that of the original gas. The small density of the gas should also limit any useful applications of this effect.