Nonrenormalization Theorems in Superstring Theory

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(Received 15 October 1986)

A simple proof of the nonrenormalization theorems in string theory is given. We prove (with the assumption that space-time supersymmetry is nonanomalous) that the space-time superpotential is not renormalized by string loops. As corollaries of this result we argue that in string perturbation theory, Calabi-Yau vacua (or any other classical solution with space-time supersymmetry) are solutions of the quantum equations of motion with zero cosmological constant, massless particles remain massless, and Yukawa couplings are not renormalized, to all orders.

PACS numbers: 11.17.+y, 11.30.Pb

One of the features of superstring theories which makes them so attractive compared to point-particle theories is their good ultraviolet behavior. They are, apparently, finite theories to all orders of perturbation theory. Related to this is the fact that they have no free parameters.[1] Supersymmetric field theories also have interesting properties. Four-dimensional, $N=1$ supersymmetric theories, in particular, are subject to a nonrenormalization theorem.[2] This theorem asserts that the superpotential, which describes masses, Yukawa couplings, scalar potentials, and the like, does not receive perturbative corrections. Analogous statements are true in other than four dimensions. The proof of this theorem involves a detailed examination of properties of superspace perturbation theory.[2] It is this nonrenormalization theorem which is the basis of the hope that $N=1$ supersymmetry is the key to the gauge hierarchy problem.[3] It would be surprising if superstring theories, with their good high-energy properties, were not subject to similar theorems. Some progress has been made in proving this. In particular, Martinec[4] has shown, with the assumption that space-time supersymmetry is not anomalous, that the space-time superpotential is not renormalized to all orders in the loop expansion. In this note, we will give an extremely simple proof of this fact. Like Martinec, we will have to assume that space-time supersymmetry is not anomalous.

The reasoning we will give is very similar to that of a proof given by Witten for a nonrenormalization theorem at tree level in string theory,[5] so it is instructive to review his result here. Wen and Witten[6] noted that in compactifications of string theory there are always certain nonlinearly realized U(1) symmetries. These are associated with modes of the antisymmetric tensor $BB_{i\bar{i}}$, where i and $\bar{i}$ are indices in the internal space (we will assume, for definiteness, that the compactification is to $M_4 \times K$, where M_4 is flat four-dimensional Minkowski space, while K is some compact, Kahler, six-dimensional space). To each closed, harmonic two-form on this space, $b^n_{i\bar{j}}$, there corresponds a massless particle in four dimensions, a_n. There is always at least one such form, the Kahler form, with components given by

$$b_{i\bar{i}} = i g_{i\bar{i}} = -b_{\bar{i}i}, \tag{1}$$

where $g_{i\bar{i}}$ denotes the metric of K. We will refer to the corresponding particle as a_0. In the heterotic superstring theory,[7] the vertex operator for the particle a_n is

$$V_{a_n} = \int d^2\xi\, b^n_{i\bar{i}}(x)\,\partial x^i(\bar{\partial} x^{\bar{i}} + i k_\mu \psi^\mu \psi^{\bar{i}}) e^{i k_\mu x^\mu} - i \leftrightarrow \bar{i}, \tag{2}$$

where ψ denotes right-moving Ramond-Neveu-Schwarz fermions. At zero momentum ($k_\mu = 0$), only the bosonic term survives. This term is topological, and vanishes in σ-model perturbation theory. Thus to any finite order in α', the zero-momentum mode of a_n decouples and the theory is invariant under the Peccei-Quinn–type symmetries,

$$a_n \to a_n + c_n. \tag{3}$$

Therefore, the particles a_n behave like axions. This symmetry is explicitly broken nonperturbatively in the σ model,[6,8] but this will not concern us here.

Having observed the existence of this symmetry, one can immediately prove a nonrenormalization theorem, at string tree level.[5] Consider a four-dimensional, low-energy effective action, from which all massive (string and Kaluza-Klein) modes have been integrated out. This effective action must be supersymmetric. To any finite order in σ-model perturbation theory, it must also obey the Peccei-Quinn symmetries described above. It is easy to show that the field a_0 lies in a chiral supermultiplet R whose scalar component is $r+ia_0$, where r is the "breathing mode" or dilitational mode of the metric. r describes the size of the internal space (r^3 is the volume of the compact space in string units) and $r^{-1/2}$ is the coupling constant of the σ model describing the propagation of the string on K. As we have stated above, we will focus on the space-time superpotential. This function is an analytic function of the various chiral superfields. In particular, it must be an analytic function of the field R. However, the Peccei-Quinn symmetry forbids any dependence on a_0, and thus the superpotential must be independent of R. But this means that the superpotential is independent of r, the σ-model coupling. Thus the superpotential receives no corrections in any finite order of σ-model perturbation theory![5]

This is a simple proof of a very powerful result. Among several striking corollaries of this theorem is the statement that the compactifications of Witten and coworkers[9] are good starting points for construction of solutions of the classical string equations of motion to all finite orders in α'. This follows simply because, if they were not, there would necessarily be a tadpole for some massless field in the effective Lagrangean. But such a tadpole cannot be generated if there are no corrections to the superpotential. From a microscopic point of view, this is a highly nontrivial statement since at four loops in the σ model the β function for the Ricci flat background does not vanish.[10] As a result, the vacuum expectation values of the massive fields must be shifted if one is to solve the equations of motion.[11] The previous argument, which involves only the fields which are massless in four dimensions guarantees that such a shift is always possible. The proof of this statement by direct examination of the σ-model β function, or equivalently by study of the full ten-dimensional equations of motion, is considerably more difficult.[11] (Similar arguments involving properties of the four-dimensional effective action were used in Ref. 8 to argue that one can construct out of the original Calabi-Yau vacuum an exact solution to the string classical equations of motion, even nonperturbatively in the σ-model coupling.)

In string theory, the coupling constant is given by the expectation value of a dynamical field,[1] the "dilation" ϕ. This field is massless, and its expectation value is undetermined at string tree level. As a result, it will appear in any low-energy effective action. Let us consider compactifications of any of the superstring theories to four dimensions, and suppose that the compactification preserves at least $N=1$ supersymmetry. Then the field ϕ will lie in a chiral supermultiplet. Its pseudoscalar superpartner b is the massless mode of the antisymmetric tensor $B_{\mu\nu}$, with indices in M_4. Because of the gauge symmetry

$$B \rightarrow B+d\Lambda \tag{4}$$

(Λ is a space-time-dependent one-form), only the one degree of freedom out of the six in $B_{\mu\nu}$ is a physical propagating field. This can be shown, for instance, by use of a duality transformation ($db={}^*dB$). The vertex operator for the pseudoscalar field b is different in the different string theories. In the heterotic theory, it is

$$V_b=\int d^2\xi\, e_{\mu\nu}\partial x^\mu(\bar{\partial} x^\nu+ik_\rho\psi^\rho\psi^\nu)e^{ik_\mu x^\mu}, \tag{5}$$

where $e_{\mu\nu}$ is an antisymmetric transverse tensor. (Notice that since it originates from B, b has a nontrivial "polarization" tensor, even though it has spin zero.) At zero momentum, none of the terms survives, since the first is a total derivative. Note that there are no particular subtleties here since all of the two-dimensional fields appearing in this vertex operator are free and the bosonic fields are noncompact. We wish to stress that the fact that a zero momentum insertion of the b field vanishes is true regardless of the topology of the two-dimensional world sheet. It is therefore true to all orders in the loop expansion. While we have exhibited here only the vertex for the heterotic string theory, it is a simple matter to show that the b vertex vanishes at zero momentum in all of the string theories. As for the axions a_n, the vanishing of such zero momentum insertions means that the zero momentum mode of b decouples and therefore the effective action is invariant under the Peccei-Quinn symmetry

$$b \rightarrow b+c. \tag{6}$$

This symmetry of the b field was first pointed out as a remnant of the gauge invariance (4) in Ref. 1. Here we have shown that this symmetry persists to all orders in string perturbation theory. Note that this symmetry is anomalous—its currents $j_\mu=\partial_\mu b$ suffers from an $F\tilde{F}$ anomaly[1]—but in perturbation theory this does not mean that it is explicitly broken.

We would like to examine the implications of this result for the low-energy effective theory. We will consider compactifications which preserve $N=1$ space-time supersymmetry in the leading approximation (tree level). Hence, there is a light (if not massless) gravitino in the spectrum. Because of this, and with our assumption that supersymmetry is a true symmetry of string theory, the low-energy effective Lagrangean must be supersymmetric to all orders of perturbation theory. This is so even if supersymmetry should turn out to be spontaneously broken at some order. In this case, it would still be a symmetry of the Lagrangean (but not a symmetry of the vacuum) and we would be able to describe the spontaneous breaking in the effective Lagrangean. This fact was also crucial in the discussion of Ref. 5. Before discussion of the implications of the Peccei-Quinn symmetry for this effective theory, it will be helpful to describe in somewhat more detail the multiplets in which ϕ, r, a_0, and b lie. By use of the vertex operators of these fields, it is easy to show that they belong to two chiral superfields,

$$R = r + i a_0, \tag{7}$$

$$Y = r^3 \phi^{-2} + i b. \tag{8}$$

The expectation value of the field Y is related to the gauge coupling in four dimensions $-g^{-2} = \mathrm{Re}\langle Y \rangle$, thus reflecting the fact that the ten-dimensional coupling constant is related to ϕ. The nonrenormalization theorem of Ref. 5 concerned the perturbative expansion in the σ model and the superfield R. We will now use the superfield Y to prove the nonrenormalization theorem of the loop expansion.

We now come to the main point of this note. String loop corrections to the superpotential would necessarily involve the field Y. However, as we have shown, the Peccei-Quinn symmetry of Eq. (6) is an exact symmetry of string perturbation theory. Since the superpotential, being analytic in chiral fields, cannot depend on Y, it cannot receive corrections in string perturbation theory!

This result also has dramatic corollaries:

(1) Any configuration which is a solution of the classical string equations of motion and which preserves at least an $N=1$ supersymmetry in four dimensions leads to a good ground state with unbroken supersymmetry and with vanishing cosmological constant to all orders in the topological expansion.[12] Note that our proof does not imply that every single diagram vanishes—only their sum. Our argument, being macroscopic, is not sensitive to shifts of heavy fields. It is perfectly possible that the solution of the quantum equations of motion at a given order differs from the original classical configuration. As in the case of the nonrenormalization theorem of Ref. 5, our proof only shows that the heavy fields can shift so as to achieve a solution.

(2) Since infinities in string theory signal vacuum instability, and we have just shown that our supersymmetric vacua are perturbatively stable, we conclude that the perturbation expansion around these vacua is finite.

(3) Particles which are massless at the tree level remain massless to all orders, even if they are not protected by ordinary symmetries. In a more general sense, we confirm that string theories preserve all of the features of ordinary supersymmetric field theories which make them candidates for the solution of the hierarchy problem.

(4) Yukawa couplings and coefficients of higher dimension operators in the superpotential are not renormalized by string loops. (They were also not renormalized by σ-model loops but were modified at string tree level by world-sheet instantons.[8])

This proof of the nonrenormalization theorem also leads naturally to considerations of how the theorem might break down nonperturbatively. There will certainly be nonperturbative effects which explicitly (not spontaneosuly) break the Peccei-Quinn symmetry. An example of such effects is provided by space-time instantons (as opposed to world-sheet instantons). Although we do not presently know how to construct string instantons, it is presumably true that in the presence of such instantons, the integration by parts used in Eq. (5) is illegal. There are presumably other effects which explicitly break the symmetry. The gluino condensation mechansim of supersymmetry breaking[13,14] is an example. Here the symmetry is broken by strong interactions in the "hidden" E_8. As stressed earlier,[15] we can advance no argument, at present, that there are not effects in string theory which break the symmetry even more strongly. Certainly it is clear from this analysis that any effort to understand supersymmetry breaking should focus on the manner in which the Peccei-Quinn symmetry is explicitly broken. Once this symmetry is broken, a superpotential for Y of some nonperturbative form (such as e^{-fY} where f is a constant) will be generated. Such a superpotential will lead to a potential for Y which tends to zero for large Y—weak coupling.[16]

This analysis has implications for ground states of string theory in higher dimensions as well. By taking the radius of the internal space to infinity, for any supersymmetry-preserving compactification, we learn that the flat ten-dimensional ground state has unbroken supersymmetry with vanishing cosmological constant to all orders in perturbation theory. Therefore, the pertubative expansion around this vacuum is finite.

We can also use our arguments to learn about coupling-constant renormalization in perturbation theory around some vacua. Consider a toroidal compactification to four dimensions. The resulting four-dimensional effective action has $N=4$ supersymmetry. If we write this action in $N=1$ superspace, the $N=4$ invariance is not manifest but it leads to relations between couplings. In particular, as in global $N=4$ supersymmetric theories the gauge coupling is related to the coefficient of the superpotential (to show that explicitly, we need to rescale the fields). Since the superpotential is not renormalized, the gauge coupling is not renormalized as well. By taking the radius of the compact space to infinity, we can also learn that there is no coupling-constant renormalization in perturbation theory around flat ten-dimensional space. This result was also obtained in Ref. 4 where it was shown that the three-point coupling of massless fields is not renormalized by loop corrections once we expand around flat space.

As we have stressed, our arguments rely on the strong assumption that there is no anomaly in space-time supersymmetry. In order to prove this, considerably more work is needed. A more detailed understanding of the fermion vertex operator[17] and the "splitting and joining" operation of Ref. 4 as well as a complete analysis of the structure of the perturbative expansion both in light-cone gauge and in the Polyakov approach will be necessary.

In many ways, this result is not particularly surprising.

It may, as we have noted, be useful for a focusing of thinking about supersymmetry breaking. It does not, unfortunately, provide any clues to the biggest question of all: How can the cosmological constant vanish in the presence of supersymmetry breaking?

The work of one of us (M.D.) was supported in part by the U. S. Department of Energy Grant No. DE-AC02-83ER40107, and by an Alfred P. Sloan Foundation fellowship; that of the other (N.S.) by U. S. department of Energy Contract No. DE-AC02-76ER02220 and by The Fund for Basic Research Administered by The Israel Academy of Sciences and Humanities. We thank E. Witten for discussions.

[1]E. Witten, Phys. Lett. **149B**, 351 (1984).

[2]M. J. Grisaru, W. Siegel, and M. Rocek, Nucl. Phys. **B159**, 429 (1979).

[3]E. Witten, Nucl. Phys. **B188**, 513 (1981).

[4]E. Martinec, Phys. Lett. **171B**, 189 (1986).

[5]E. Witten, Nucl. Phys. **B268**, 79 (1986).

[6]X. G. Wen and E. Witten, Phys. Lett. **166B**, 397 (1986).

[7]D. J. Gross, J. A. Harvey, E. Martinec, and R. Rohm, Phys. Rev. Lett. **54**, 502 (1985); Nucl. Phys. **B256**, 253 (1985), and **B267**, 75 (1986).

[8]M. Dine, N. Seiberg, X.-G. Wen, and E. Witten, to be published.

[9]P. Candelas, G. Horowitz, A. Strominger, and E. Witten, Nucl. Phys. **B258**, 46 (1985); E. Witten, Nucl. Phys. **B258**, 75 (1985).

[10]D. Zanon, M. Grisaru, and A. Van de Ven, Phys. Lett. **173B**, 423 (1986); Harvard University Reports No. HUTP-86/A026, 86/A027, 86/A046 (to be published); C. N. Pope, M. F. Sohnius, and K. S. Stelle, Imperial College Report No. Imperial/TP/85-86/16, 1986 (to be published); M. D. Freeman and C. N. Pope, Phys. Lett. **174B**, 48 (1986).

[11]E. Witten, unpublished; D. Nemeschansky and A. Sen, Stanford Linear Accelerator Center Report No. SLAC-PUB-3925 (to be published); A. Sen, Stanford Linear Accelerator Center Report No. SLAC-PUB-3956 (to be published).

[12]There is one possible exception to this statement. If a compactification yields a semisimple group containing anomalous unbroken U(1)'s, it is possible that one might generate Fayet-Illiopoulos D terms for these [P. Fayet and J. Illiopoulos, Phys. Lett. **51B**, 461 (1974)], perhaps yielding tadpoles for the field Y in the process. This possibility was pointed out to us by E. Witten.

[13]J. P. Derendinger, L. E. Ibanez, and H. P. Nilles, Phys. Lett. **155B**, 65 (1985).

[14]M. Dine, R. Rohm, N. Seiberg, and E. Witten, Phys. Lett. **156B**, 55 (1985).

[15]M. Dine and N. Seiberg, Nucl. Phys. **B273**, 109 (1986).

[16]M. Dine and N. Seiberg, Phys. Rev. Lett. **55**, 366 (1985), and Phys. lett. **162B**, 299 (1985); R. Rohm and E. Witten, Ann. Phys. (N.Y.) **170**, 454 (1986).

[17]D. Friedan, E. Martinec, and S. Shenker, Phys. Lett. **160B**, 55 (1985), and Nucl. Phys. **B271**, 93 (1986).