

Au and Drachman Respond: Although it was not evident from the original Letter of Manson and Ritchie,¹ we agree that their “conservative imaginary contribution” is similar to our first nonadiabatic correction² but not identical. It is exactly the “small terms” in Eq. (3) of the Comment³ which represent what we consider to be a significant difference; these are particularly important at zero energy where Manson and Ritchie’s expression vanishes identically. To see how large these terms may be, let us write the asymptotic form of the general scattering function as

$$\Psi \sim e^{i\mathbf{k}\cdot\mathbf{R}} + f(\theta)e^{ikR}/R. \quad (1)$$

Then the gradient term in our formalism gives the result

$$\partial\Psi/\partial R = ik\Psi - f(\theta)e^{ikR}/R^2. \quad (2)$$

We see that the extra term involving the scattering amplitude is, in fact, not necessarily small. In particular, the gradient term at zero energy yields the following:

$$\partial\Psi/\partial R \sim a/R^2, \quad (3)$$

where a is the scattering length, which may be large. In the same way, one might ask how the effective potential could be used in a bound-state calculation. Our method could be applied almost without modification.⁴ (This is of more than academic interest, since the Ps₂ and PsH molecules have recently been reexamined.⁵) It is the virtue of our method that the complete and correct expression for this term appears naturally.

Perhaps a more interesting problem concerns the transformation we used to convert the non-Hermitian gradient terms into a local potential.⁶ (The k -dependent term is evidently also not Hermitian.) The remarkable fact is that the transformation results in a Hermitian Hamiltonian for the “quasiparticle,” giving rise to a more correct form of the Schrödinger equation. The transformation takes into account the changing normalization as the two systems approach each other, thereby avoiding spurious current nonconservation.

The most important difference between the two methods is in the basic philosophy employed in the formulation. We have taken the point of view, implemented through the Feshbach-projection-operator formalism, that the goal is to reduce a many-body Schrödinger equation to an effective one-body equation. This does not require any perturbation expansion, and at no time is the one-body Schrödinger equation itself ever replaced by a Born series. On the other hand, if one wishes to take a perturbative approach, one definition of an effective potential is the Fourier transform of the scattering amplitude in the momentum transfer Q . It is well known that the scattering

amplitude depends on only two independent variables, commonly chosen to be the energy and Q^2 . Hence the effective potential can always be written as a function of the energy and separation R , and need not contain any dependence on \mathbf{k} . The “conservative imaginary contribution” of Ref. 1 derives from the $\mathbf{k}\cdot\mathbf{Q}$ terms in the scattering amplitude. But overall energy conservation enables us to eliminate $\mathbf{k}\cdot\mathbf{Q}$. This provides another way to convert the momentum-dependent term into a true local potential, identical to the one derived in Ref. 2, while retaining the perturbative approach. One simply transforms the $i\mathbf{k}\cdot\mathbf{R}$ term in (\mathbf{k}, \mathbf{R}) space back into (\mathbf{k}, \mathbf{Q}) space and makes the substitution $\mathbf{k}\cdot\mathbf{Q} = -Q^2/2$. On performing one more Fourier transform from \mathbf{Q} to \mathbf{R} , one recovers the local form. Here we see from another angle why the momentum-dependent term, which appears to vanish at threshold, actually does not.

Finally, we would prefer not to argue about terminology, and for that reason did not use the word “recoil” in the title of our Letter. We would like to remark, however, that earlier workers⁷ have interpreted the short-range effects, attributed in Ref. 1 to recoil, as due to zero-point oscillations.

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C. K. Au

Department of Physics and Astronomy
University of South Carolina
Columbia, South Carolina 29208

Richard J. Drachman

Laboratory for Astronomy and Solar Physics
Goddard Space Flight Center
Greenbelt, Maryland 20771

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⁶Such a transformation has been used often in the electron-atom problem: J. Callaway, R. W. LaBahn, R. T. Pu, and W. M. Duxler, Phys. Rev. **168**, 12 (1968); M. J. Seaton and L. Steenman-Clark, J. Phys. B **10**, 2639 (1977); D. W. Norcross, Phys. Rev. A **28**, 3095 (1983).

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