Massive, Unitary, Renormalizable Yang-Mills Theory without Higgs Bosons

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We present a variant of the Stueckelberg formalism for Yang-Mills theory where the Stueckelberg scalar field is eliminated in favor of the gauge potential. Normally this leads to nonpolynomial interactions. However, we point out that it is possible to pick a gauge where the nonpolynomiality disappears and the resulting theory is power-counting renormalizable and unitary. This procedure allows one to construct an electroweak model without the Higgs boson.

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The electromagnetic inverse-square law of force calls for a massless photon. Yet there is no theoretical reason why an Abelian gauge theory should not allow for a massive photon, since the resulting construct is both unitary and renormalizable, by virtue of current conservation; nature has simply not availed itself of this possibility. On the other hand, the weak force is short range and the weak-boson carriers are massive. Not surprisingly, therefore, the few years before the advent of the standard electroweak model saw considerable efforts devoted towards construction of massive, renormalizable, non-Abelian gauge models. All these efforts failed, basically for two reasons: Either the model was nonpolynomial[1] (and nonrenormalizable if expanded in powers of the coupling) or it was nonunitary[2] (physical ghosts). It is no wonder then that, apart from a few sporadic efforts,[3] incidental to other research, the enormous success of the spontaneously broken electroweak theory has weakened all subsequent investigations into massive Yang-Mills theory. Yet the nondiscovery of the Higgs boson, so essential to the standard theory, argues for continued attempts to construct a massive non-Abelian gauge model. In this Letter, we shall describe such an attempt which seems to evade all the previous difficulties and may deserve serious investigation as an alternative to the Higgs mechanism.

It is well known that a "mass term" m^2A^2 added to the Yang-Mills Lagrangean is not gauge invariant; for electromagnetism this hardly matters because of U(1) current conservation, but in Yang-Mills theory the problem is fatal and is one of the reasons advanced for insistence upon *bare, massless* chromodynamics or flavor dynamics—it is only thanks to the Higgs field or perhaps because of spontaneous symmetry breaking that the final vector states acquire mass. There is, however, another way of constructing a massive gauge-invariant term and this is by introduction of a Stueckelberg scalar field ϕ, in the adjoint representation of the gauge group, transforming according to

$$U \rightarrow US;\quad U = \exp(ig\phi);\quad S = \exp(i\Lambda). \tag{1}$$

It is then easy to show that the combination

$$A - U^{-1} i\,\partial U/g$$

transforms *homogeneously* under gauge transformations S. Consequently, the squared term

$$\mathrm{Tr}[(A - U^{-1} i\,\partial U/g)^2] \tag{2}$$

is gauge invariant. Unfortunately, one has now augmented the original Yang-Mills theory and introduced nonpolynomial interactions of the Stueckelberg field into the bargain, causing havoc with renormalizability, given our current perturbative technology.

However, suppose that we eliminate[4] the Stueckelberg field in favor of the vector field A, by solving the equation of motion for ϕ,

$$\partial^2\phi + \partial\cdot A - \tfrac{1}{2} ig([A^\mu,\partial_\mu\phi] + \partial_\mu[A^\mu,\phi]) + O(g^2) = 0. \tag{3}$$

In principle, this can be done order by order in perturbation theory. Equivalently, it is more efficient to solve the simpler equation

$$iD_\mu(U^{-1}\partial^\mu U) = g\,\partial\cdot A \tag{3'}$$

in powers of the coupling. The combination $U(A + i\partial/g)U^{-1}$, *with* U *determined by* A according to the above condition, then transforms gauge invariantly. That the field U satisfying Eq. (3′) has the correct transformation property (1) is straightforwardly checked by note of two facts: first, that for infinitesimal transformations,

$$\delta(U^{-1}\partial U) = [\partial + U^{-1}\partial U, U^{-1}\delta U];$$

and, second, that by a variation of Eq. (3′),

$$\{D^\mu(A), \delta(U^{-1}\partial_\mu U) - [\partial_\mu + U^{-1}\partial_\mu U, \Lambda]\} = 0.$$

This means we are back to the original number of fields, albeit with a *nonlocal but gauge-invariant* mass term

$$m^2\mathrm{Tr}[U(A)(A + i\partial/g)U^{-1}(A)]^2.$$

Another way to arrive at this term is to require that

$\frac{1}{2}A^2+G(A)$ be gauge invariant, or

$$\partial\cdot A = D_\mu \delta G/\delta A_\mu, \tag{4}$$

and look for solutions where $G(A)\neq -\frac{1}{2}A^2$. A solution is

$$G(A)=\tfrac{1}{2}\partial\cdot A\partial^{-2}\partial\cdot A+\tfrac{1}{2}igA^\mu[\partial_\mu\partial^{-2}\partial\cdot A,\partial^{-2}\partial\cdot A]+O(g^2) \tag{4'}$$

and it coincides with the Stueckelberg method. In any event, the fact that the resulting expression is a complicated nonpolynomial would indicate that we are no better off. Nevertheless, once we realize that the Stueckelberg ϕ, or equivalently G, vanishes in the Landau gauge (it is proportional to $\partial\cdot A$) then we can claim that a real advance has been made because we have at one stroke *a massive, gauge-invariant, polynomial theory which is clearly power-counting renormalizable.* It is important to emphasize that at this point we are no longer dealing with a conventional Stueckelberg theory. Cornwall[4] has thoroughly appreciated the significance of this approach and exploited this idea in connection with nonperturbation behavior in QCD, whereas here we are advocating its use in traditional Yang-Mills theory.

Our quantum version of massive non-Abelian gauge theory includes gauge fixing and ghost terms. Its Lagrangean reads

$$L=-\tfrac{1}{4}F^2+\tfrac{1}{2}m^2\mathrm{Tr}[(A-e^{-ig\phi}i\,\partial e^{ig\phi}/g)^2]+B\,\partial\cdot A+\tfrac{1}{2}aB^2+\omega\,\partial\cdot D(A)\omega, \tag{5}$$

with ϕ expressed in terms of A through its equation of motion. Only when we go to the covariant gauge $a=0$ (Landau) can we treat the model in a realistic way, because all the nonpolynomiality then evaporates. In that limit, the B field acts like a multiplier field, enforcing the condition $\partial\cdot A=0$ and removing all problems with the mass term. The vector propagator reduces to the naive one,

$$\Delta_{\mu\nu}(k)=(-\eta_{\mu\nu}+k_\mu k_\nu/k^2)/(k^2-m^2),$$

and it has *acceptable high-energy behavior.* Furthermore, the entire theory is invariant under the *conventional* Becchi-Rouet-Stora-Tyutin (BRST) transformations and satisfies the same gauge identities as in the massless case. The nilpotency of the BRST variations then ensures the unitarity of the theory in the asymptotic limit for the physical subspace of state vectors (determined by zero BRST charge) according to the Kugo-Ojima[5] proof. The only reservation one might have about the proof is that the mass term and the ghost propagators are nonlocal in time because they carry $1/\partial^2$; however, it is possible to choose other gauges, such as Coulomb or axial (see below) which are time local, and the objection does not have the same force. In any event, for a number of processes in one-loop order we have verified in detail that the cancellation of zero-mass singularities takes place, thereby confirming physical unitarity.

The only change to the renormalization program for the massless case is that we must include a mass^2 counterterm in addition to the usual wave-function and coupling-constant renormalizations [there is no need in version (5) to modify the massless A^4 or A^3 renormalizations, which are in fact related]. A simple calculation gives the self-mass renormalization to be

$$\delta m^2=m^2(13g^2C/64\pi^2)\ln(\Lambda^2/m^2),$$

which may be compared with the wave-function renormalization to this order,

$$Z=1+(3g^2C/96\pi^2)\ln(\Lambda^2/m^2).$$

The significance of this construction will not have escaped the reader and it is appropriate to mention an important application now. Primarily one would like to avoid the Higgs mechanism (at least as long as the Higgs boson remains undiscovered) for the electroweak model.[6] Let $A=\frac{1}{2}[IW_0+\mathbf{W}\cdot\boldsymbol{\tau}]$ and interpret S in (1) as a U(2) gauge transformation. Because the combination $U(A+i\partial/g)U^{-1}$ is gauge invariant, we are able to construct terms

$$\tfrac{1}{2}m_M^2\mathrm{Tr}\{[\tau_1U(A+i\,\partial/g)U^{-1}]^2+[\tau_2U(A+i\,\partial/g)U^{-1}]^2\}+\tfrac{1}{2}m_Z^2\mathrm{Tr}\{[(\tau_3\cos\theta-\sin\theta)U(A+i\,\partial/g)U^{-1}]^2\},$$

which cause the Z^0 and the $W^\pm$ (but not the photon, by construction) to become massive. We have no reason to doubt that the renormalization program can be pushed through and we strongly suspect that, apart from mass renormalizations, the other renormalizations are exactly as in the standard Higgs version, at least to order e^2. We shall report on details of these quantum corrections in subsequent work and provide the explicit proof of unitarity. One big bonus of our mechanism is the absence of quadratic infinities (connected with the Higgs boson) which makes the need for supersymmetry less compelling; it cannot be denied that the present evidence for supersymmetry in particle physics is tenuous, at best.

The impression we have given so far is that the whole scheme will only work in Landau limit of the covariant gauge class (5). Actually, the method can be generalized to other gauges, such as the axial gauge. Provided that U is still given in terms of A, we can also impose constraints like

$$n\cdot(A-U^{-1}\partial U)=0, \tag{6}$$

where n is a nonderivative operator. As before, the U satisfying (6) can be shown to have the required transformation property (1). In this example, $U=\exp(ig\phi)$, where

$$\phi=(n\cdot\partial)^{-1}\{n\cdot A-g[(n\cdot\partial)^{-1}n\cdot A,n\cdot A]+O(g^2)\}. \tag{6'}$$

This kind of gauge can therefore be implemented via the Lagrangean

$$L=-\tfrac{1}{4}F^2+\tfrac{1}{2}m^2\mathrm{Tr}[(A-U^{-1}i\partial U/g)^2]+Bn\cdot A. \tag{7}$$

A nice application of this approach is to massive pure QCD in two dimensions, which is known to be either nonunitary or not conventionally renormalizable if treated in the normal way.[7] Suppose instead that we take the Lagrangean to be (7). Let $n=(1,0)$ to make $A_0=0$ and, of course, insist upon U being fixed in terms of A, as in Eq. (6); this choice then removes the A interaction terms and *all that survives is a free* A_1 *with mass* m. It is as trivial as that.

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