Heavy Quarks and Electroweak Symmetry Breaking

Bob Holdom
Department of Physics, University of Toronto, Toronto, Ontario M5S 1A7, Canada
(Received 26 August 1986)

It is proposed that a dynamical mass for a fourth family of quarks is responsible for breaking of the weak interactions, and for breaking of the extended technicolor interactions which provide mass to the third family. This greatly simplifies the extended technicolor sector and avoids fermions with exotic quantum numbers. A technicolor sector is still required but it is lighter than usual. The most attractive-channel hypothesis for symmetry breakdown is relied on.

PACS numbers: 12.50.Lr, 12.15.−y

The basic technicolor idea[1] is a simple and appealing way to understand the W and Z masses, without the introduction of elementary scalar fields and their associated naturalness problems. It was tempting to try to extend this idea by introduction of additional gauge interactions with mass scales above the weak scale, in such a way as to also provide masses for quarks and leptons.[2] The dream was to deal with the origin of fermion masses completely within the context of gauge theory dynamics, without reliance on adjustable Yukawa couplings. The dynamics would be at scales low compared with the Planck mass and thus the problem of fermion masses would be separated from the problem of gravity.

But in practice, the idea of extended technicolor (ETC) proves cumbersome. Of concern is that the hierarchies among quark and lepton masses suggest several different mass scales in the ETC sector. It is difficult to see how the necessarily complicated pattern of symmetry breaking in the ETC sector can arise.

In this paper, my aim is to reduce the required complexity of the ETC sector. Besides wanting fewer ETC mass scales, we also wish to avoid the introduction of new fermions with exotic quantum numbers under $SU(3)\otimes SU(2)\otimes U(1)$. We will be led to a situation in which all technifermions are less massive than a fourth family of quarks. New physics appears at lower energies than in the usual ETC picture making "upside-down technicolor" easier to test experimentally.

Let us focus on the problem of heavy quarks within the ETC picture. Heavy quarks imply an ETC mass scale not too far above the weak scale. At this scale an ETC gauge symmetry, which is itself a subgroup of a larger ETC symmetry at a higher scale, breaks down further to the technicolor group, and in the process splits off the heavy quarks as technicolor singlets. But what is the order parameter which describes this symmetry breaking? Any bilinear form involving fermions with standard $SU(3)\otimes SU(2)\otimes U(1)$ quantum numbers breaks $SU(2)\otimes U(1)$ and therefore must not develop a vacuum expectation value above the weak scale. Thus, not only do we need new strong interactions somewhat above the weak scale, but these interactions must involve new exotic fermions in order that a bilinear condensate can break the ETC group without breaking $SU(2)\otimes U(1)$.

This observation is disturbing for model building and we look for an alternative. We wish to remain in the simple picture in which all fermions and technifermions are in families with standard $SU(3)\otimes SU(2)\otimes U(1)$ quantum numbers. The full ETC group is then a simple *sideways* symmetry which acts horizontally between the various families of quarks/leptons and technifermions. At mass scales somewhat above the weak scale some subgroup of this sideways symmetry must remain, and we refer to this subgroup as the *subsideways* symmetry. It acts on what will become at lower energies the technifermion families of heavy fermions. [We have argued in the past that a suitable order parameter for the breaking of the full sideways symmetry is a dynamical mass for the right-handed (RH) neutrino. Then RH neutrinos (and RH technineutrinos) do not appear at lower energies and can play no role in the symmetry breaking close to the weak scale.]

The possibility I wish to consider is that the condensate which breaks the subsideways symmetry is the same condensate which breaks $SU(2)\otimes U(1)$. We thereby avoid the introduction of exotic fermions. In particular, I will eventually present a model in which the third and fourth families belong to subsideways multiplets. The subsideways-breaking condensate will be a dynamical mass for the fourth family quarks.

The first observation is that although the subsideways breaking condensate can break $SU(2)\otimes U(1)$, it may not be sufficient to produce light-fermion masses. Light-fermion masses rely on terms in the effective Lagrangean of the form $\bar{\Psi}\Psi\bar{\psi}\psi$, where Ψ transforms under the subsideways symmetry and ψ is a light fermion, a singlet under subsideways symmetry. The effective Lagrangean must be subsideways invariant and therefore the $\bar{\Psi}\Psi$ factor in any such operator must be a subsideways singlet. Thus, we may need a further condensate besides the subsideways-symmetry violating one to provide masses for the light fermions.

I would like to arrange that this other condensate is due to the unbroken subgroup of the subsideways group, which I still refer to as the technicolor group. This technicolor condensate can form below the subsideways- and $SU(2)\otimes U(1)$-breaking scale, thus giving us smaller technifermion masses than usual. In my model, I will show how the mass difference between the technifermions and the fourth family can induce a suitable mass for the third family. This mass is fed down from the technifermions via broken subsideways symmetry. The mass for the first two families, on the other hand, if fed down via broken sideways, symmetry (i.e., via the terms $\bar{\Psi}\Psi\bar{\Psi}\Psi$).

I first present a toy model illustrating mass generation for technifermions and heavy families. I assume that condensates form only in the most attractive channel of strong gauge forces. But the model will also highlight a problem due to a new source of flavor-changing neutral currents. It will be necessary to introduce another ingredient to build a successful model of this kind.

The toy model ignores the light families, the leptonic sector, and the weak interactions. I take the strong interactions near the weak scale to be chiral in the sense that any bilinear condensate breaks some part of the strongly interacting gauge group. The gauge group and fermion content is

$$U(1)\otimes\tilde{U}(1)\otimes SU(3)_{SS}\otimes SU(3)_c;$$

$$\Psi_{1L}:(1,1,3,3)_L,$$

$$\Psi_{1R}:(0,1,3,3)_R,$$

$$\Psi_{2L}:(0,-1,3^*,3)_L,$$

$$\Psi_{2R}:(1,-1,3^*3)_R.$$

The first three factors are taken to be strong at the weak scale. $SU(3)_{SS}$ is the subsideways group and $SU(3)_c$ is ordinary color. [This choice of $\hat{U}(1)$ charges produces a $\hat{U}(1)^2\otimes\tilde{U}(1)$ gauge anomaly, but this will be dealt with below.]

We see that the $\hat{U}(1)$ forces compete with the $\tilde{U}(1)\otimes SU(3)_{SS}$ forces in the production of a bilinear condensate. The former would prefer a nonzero $\langle\bar{\Psi}_{2R}\Psi_{1L}\rangle$ while the latter would prefer a nonzero $\langle\bar{\Psi}_{1R}\Psi_{1L}\rangle$ and $\langle\bar{\Psi}_{2R}\Psi_{2L}\rangle$ (and/or $\langle\bar{\Psi}_{1L}\Psi_{2R}^c\rangle$, but these break color). We may assume that $\langle\bar{\Psi}_{2R}\Psi_{1L}\rangle$ forms for some range of gauge couplings. This condensate preserves $\hat{U}(1)$ but it transforms as $3\otimes 3$ under $SU(3)_{SS}$.

Let us consider the breakdown of $\tilde{U}(1)\otimes SU(3)_{SS}$ to $U(1)_{tp}\otimes SU(2)_{tc}$ where $U(1)_{tp}$ is a combination of $\tilde{U}(1)$ and the λ_8 generator of $SU(3)_{SS}$. In other words, $\Psi_1\rightarrow(Q_1,q_1)$ and $\Psi_2\rightarrow(Q_2,q_2)$ where Q_1 and Q_2 (q_1 and q_2) are doublets (singlets) under $SU(2)_{tc}$, and the vector $U(1)_{tp}$ charges are $Q_1[\frac{3}{2}]$, $Q_2[-\frac{3}{2}]$, $q_1[0]$, and $q_2[0]$. We have $\langle\bar{q}_{2R}q_{1L}\rangle\neq 0$ which corresponds to $\langle\bar{\Psi}_{2R}\Psi_{1L}\rangle\in(3\times 3)_{sym}$.

Thus far we have the dynamical mass $\langle\bar{q}_{2R}q_{1L}\rangle\equiv m_{SS}^3$ and the gauge symmetries are preventing any other bilinear condensates. At a somewhat lower mass scale we have a similar situation to before. The $\hat{U}(1)$ forces are now competing with the $U(1)_{tp}\otimes SU(1)_{tc}$ forces. The former would prefer a nonzero $\langle\bar{Q}_{2R}Q_{1L}\rangle$ while the latter would prefer a nonzero $\langle\bar{Q}_{1R}Q_{1L}\rangle$ and $\langle\bar{Q}_{2R}Q_{2L}\rangle$ (again $\langle\bar{Q}_{1L}Q_{2R}^c\rangle$ and $\langle\bar{Q}_{2R}Q_{1L}^c\rangle$ would break color). Now let us assume that the $U(1)_{tp}\otimes SU(1)_{tc}$ forces win and that the color-preserving techniquark condensates, $\langle\bar{Q}_{1R}Q_{1L}\rangle\approx\langle\bar{Q}_{2R}Q_{2L}\rangle\equiv m_{tc}^3$, form. The $\hat{U}(1)$ symmetry breaks while $U(1)_{tp}\otimes SU(1)_{tc}$ remains unbroken.

In the effective Lagrangean below m_{SS}, we have terms of the form $\bar{Q}_{1R}Q_{1L}\bar{q}_{1L}q_{1R}$ and $\bar{Q}_{2R}Q_{2L}\bar{q}_{2L}q_{2R}$ due to the broken $SU(3)_{SS}$ interactions, with coefficients of order m_{SS}^{-2}. Thus the quark-mass matrix receives further contributions of order $m_q\approx m_{tc}^3/m_{SS}^2$. I now give the mass matrices for the techniquarks and quarks. I use a Majorana mass notation and choose the bases $(Q_1,Q_2,Q_1^c,Q_2^c)_L$ and $q_1,q_2,q_1^c,q_2^c)_L$ to simplify the respective mass matrices. They are

$$\begin{bmatrix}0 & A\\ A^T & 0\end{bmatrix}_Q \text{ with } A=\begin{bmatrix}m_{tc} & 0\\ 0 & m_{tc}\end{bmatrix}$$

and

$$\begin{bmatrix}0 & B\\ B^T & 0\end{bmatrix}_q \text{ with } B=\begin{bmatrix}m_q & M_{SS}\\ 0 & m_q\end{bmatrix}.$$

There is a unitary transformation on the quark mass matrix which keeps it in the same form but with B replaced by

$$\tilde{B}\approx\begin{bmatrix}m_{SS} & 0\\ 0 & m_q^2/m_{SS}\end{bmatrix}.$$

We have a quark mass matrix for two Dirac masses, one heavy and one as much as $(m_{tc}/m_{SS})^6$ times lighter, corresponding in a more realistic model to the masses of the fourth and third families. The fact that we have Dirac masses is ensured by color conservation. It is interesting that a small mass splitting between m_{SS} and m_{tc} will give a reasonable mass to a third family.

We may now identify the problem with a model like this. We have seen that the technicolor condensate breaks a gauge symmetry, in our case the $\hat{U}(1)$. This is not surprising since we have required that the strong interactions at the weak scale be chiral to ensure the breakdown of the subsideways symmetry. In a more realistic theory we have the $\bar{T}T\bar{\psi}\psi$ terms (T=technifermion) to supply masses to the light fermions ψ. Such terms must be invariant under the same gauge symmetry [the $\hat{U}(1)$] which we have shown is broken by the technicolor condensate $\langle\bar{T}T\rangle$. We are forced to conclude that the light fermions ψ must carry $\hat{U}(1)$ charges. We are faced with a new relatively light neutral gauge boson coupling to light fermions. This is trouble enough, but we also find it difficult to avoid family-dependent couplings and horrible flavor-changing problems.

In this model, a $\hat{\mathrm{U}}(1)$ provided the attractive force in the production of the $\langle \bar{\Psi}_{2R}\Psi_{1L}\rangle$ condensate which broke the subsideways symmetry. I suggest that we may do without such a $\hat{\mathrm{U}}(1)$ (and its flavor-changing and gauge-anomaly problems) if instead we had a term of the form $\bar{\Psi}_{2R}\Psi_{1L}\bar{\Psi}_{1L}\Psi_{2R}$ appearing in the effective Lagrangean. Normally, if such a term were generated at some scale above the weak scale, then it would be suppressed by inverse powers of the higher mass scale. In this case, it would be irrelevant for the breakdown of the subsideways symmetry. But there is a situation in which such terms are substantially enhanced.

I invoke a mechanism proposed previously to help alleviate the general problem of flavor-changing neutral currents (FCNC's) in extended technicolor theories (and to raise the mass of technipions).[3-5] Let us suppose that the running coupling of the subsideways interactions may be treated as effectively constant up to some scale $\tilde{\Lambda}$ above the weak scale before becoming significantly smaller. Now we must be more careful in defining mass scales. We know that whatever happens, there must be a spontaneous breaking of the approximate chiral symmetries of the weak interactions. This defines an F_π value which must be $(250/N^{1/2})$ GeV, where N is the number of weak doublets to be summed over in the weak symmetry-breaking condensate $\langle\bar{\Psi}\Psi\rangle$. An analysis of the Schwinger-Dyson equation in the ladder approximation for the case of a constant or nearly constant coupling indicates that $\langle\bar{\Psi}\Psi\rangle$ is enhanced[4] by a factor[5] or order $\tilde{\Lambda}/F_\pi$ over naive expectations.

We now consider the effect of the $\bar{\Psi}_{2R}\Psi_{1L}\bar{\Psi}_{1L}\Psi_{2R}$ term in the standard effective action formalism[6] used to study chiral symmetry breaking. It will contribute a term in the effective action which has an enhancement factor $(\tilde{\Lambda}/F_\pi)^\gamma$ where $\gamma\approx 2$. We thus conclude that if the $\bar{\Psi}_{2R}\Psi_{1L}\bar{\Psi}_{1L}\Psi_{2R}$ term originates at a scale of order $\tilde{\Lambda}$ (or less) then it may play an important role in determination of whether $\langle\bar{\Psi}_{2R}\Psi_{1L}\rangle$ forms.

Thus, we return to our toy model, throw away the $\hat{\mathrm{U}}(1)$ gauge factor, and postulate that $\tilde{\mathrm{U}}(1)$ and/or $\mathrm{SU}(3)_{\mathrm{SS}}$ is roughly scale invariant up to some scale $\tilde{\Lambda}$. $\tilde{\Lambda}$ is bounded from above by the sideways scale. Without knowledge of the sideways physics there are many four-fermion operators which could appear in the effective theory, all preserving the gauge symmetries we have been considering. If the operator $\bar{\Psi}_{2R}\Psi_{1L}\bar{\Psi}_{1L}\Psi_{2R}$ is generated with the appropriate sign and is sufficiently enhanced compared to other operators, then it may play the same role as the $\hat{\mathrm{U}}(1)$ gauge factor. We expect that the gauge group at the sideways scale will contain a number of diagonal generators and that the symmetry breaking will produce a nontrivial mass matrix for the associated gauge bosons. When these gauge bosons are integrated out, we would not be surprised to obtain the desired operator, among others.

To be more explicit, we would like to incorporate our toy model [without the $\hat{\mathrm{U}}(1)$] factor into a complete model describing the sideways physics. We put back in the light families, the leptonic sector, and the weak interactions. The resulting model has in fact been outlined elsewhere.[7] It was presented as a way to suppress the worst FCNC problem of generic extended technicolor theories, the $\Delta S=2$ effects. The suppression followed from symmetry arguments due to the structure of the theory, and it was found that all new $\Delta S=2$ amplitudes were suppressed by at least a factor $\theta^2_{\mathrm{Cabibbo}}\alpha_{\mathrm{weak}}$. Thus, this model gives us the bonus of not having to depend entirely on anomalous scaling arguments to suppress $\Delta S=2$ FCNC's.

The gauge symmetry above the sideways scale is

$$\mathrm{U}(1)_A\otimes\mathrm{U}(1)_V\otimes\mathrm{SU}(4)_S\otimes\mathrm{SU}(4)_{\mathrm{P}}\otimes\mathrm{SU}(2)_L\otimes\mathrm{SU}(2)_R$$

and the fermions transform as

$$\Psi_{1L}:\ (1,1,\mathbf{4},\mathbf{4},\mathbf{2},\mathbf{1})_L,$$

$$\Psi_{1R}:\ (-1,1,\mathbf{4},\mathbf{4},\mathbf{1},\mathbf{2})_R,$$

$$\Psi_{2L}:\ (-1,-1,\mathbf{4}^*,\mathbf{4},\mathbf{2},\mathbf{1})_L,$$

$$\Psi_{2R}:\ (1,-1,\mathbf{4}^*,\mathbf{4},\mathbf{1},\mathbf{2})_R.$$

The model is gauge-anomaly free. $\mathrm{SU}(4)_S$ is the sideways symmetry and $\mathrm{SU}(4)_{\mathrm{P}}$ is the Pati-Salam symmetry connecting quarks to leptons. These group factors and the two U(1)'s may all be strongly interacting at the sideways scale. At some scale below the sideways scale, we must assume that the remaining gauge symmetry is

$$\tilde{\mathrm{U}}(1)\otimes\mathrm{SU}(3)_{\mathrm{SS}}\otimes\mathrm{SU}(3)_c\otimes\mathrm{SU}(2)\otimes\mathrm{U}(1). \qquad (1)$$

The $\mathrm{SU}(1)\otimes\mathrm{U}(1)$ is the standard weak gauge group and this ensures that only right-handed neutrinos gain a mass at the sideways scale. The masses are of the form $\nu_{1R}\nu_{1R}$, $\nu_{2R}\nu_{2R}$, $\nu_{1R}\nu_{2R}$, and $N_{1R}N_{2R}$ where ν (N) is a singlet (triplet) under $\mathrm{SU}(3)_{\mathrm{SS}}$ subsideways. $\tilde{\mathrm{U}}(1)$ is the linear combination of $\mathrm{U}(1)_v$ and the λ_{15} generator of $\mathrm{SU}(4)_S$ which is unbroken by the masses $\nu_{1R}\nu_{1R}$ and $\nu_{2R}\nu_{2R}$.

We may compare this to our toy model by considering how colored and technicolored particles transform under (1). We have the representations $(\frac{4}{3},3,3,2,\frac{1}{6})_L$, $(\frac{4}{3},3,3,1,\frac{2}{3})_R$, $(\frac{4}{3},3,3,2,-\frac{1}{3})_R$, $(-\frac{4}{3},3^*,3,2,\frac{1}{6})_L$, $(-\frac{4}{3},3^*,3,1,\frac{2}{3})_R$, and $(-\frac{4}{3},3^*,3,1,-\frac{1}{3})_R$. Thus, except for the addition of the weak interactions and the absence of $\hat{\mathrm{U}}(1)$, this looks like our toy model with the $\tilde{\mathrm{U}}(1)$'s identified. And there is at least enough structure in the sideways physics to produce the effective operators able to play the role of $\hat{\mathrm{U}}(1)$. We see this by noting that a linear combination of $\mathrm{U}(1)_A$ and the λ_{15} generator of $\mathrm{SU}(4)_{\mathrm{P}}$ has charges (1,0,0,0,1,1) for the above representations, respectively. This would correspond to an attractive force in the desired channel.

Thus, we assume the same pattern of symmetry breaking as in our toy model, yielding $U(1)_{tp} \otimes SU(2)_{tc} \otimes SU(3)_c \otimes U(1)_{em}$ as the final unbroken gauge symmetry. The fourth-family quarks t' and b' receive the largest mass and $m_{t'} \approx m_{b'}$, in order that $M_Z \cos\theta / M_W \approx 1$. The lightest two families receive their mass from the condensates $\langle \bar{T}_{1R} T_{1L} \rangle$ and $\langle \bar{T}_{2R} T_{2L} \rangle$, respectively. Since the mass is fed down via the same broken $SU(4)_S$ interaction in each case, the mass difference between the lightest two families must arise from the difference between $\langle \bar{T}_{1R} T_{1L} \rangle$ and $\langle \bar{T}_{2R} T_{2L} \rangle$.

The $U(1)_{tp}$ techniphoton only couples to technifermions with charges $T_1[2]$ and $T_2[-2]$. There is also the broken combination of the $\tilde{U}(1)$ generator and the λ_8 generator of $SU(3)_{SS}$; this a neutral gauge boson receiving mass from the same source as the W and Z. This gauge boson couples to technifermions and to the two heaviest families but not to the two lightest families (in the limit of no mass mixing between heavy and light families).

Of course, for all this to work we need $\tilde{U}(1)$ and/or $SU(3)_{SS}$ to be strongly interacting and roughly scale invariant above the weak scale. It is amusing to note that at least the perturbative β function of $SU(3)_{SS}$ is small; $\beta(g) = -bg^3$ where $b = (33-30)/48\pi^2$.

Our model incorporating "upside-down technicolor" illustrates how simple the extended technicolor sector can become. There is now only one new mass scale above the weak scale for interactions [in our case broken $SU(4)_S$] which connect fermions to technifermions. We feel that it is of experimental interest that some of the physics relevant to the origin of fermion masses can thereby be transferred to lower energies.

This research was supported in part by the Natural Sciences and Engineering Research Council of Canada.

[1]L. Susskind, Phys. Rev. D **20**, 2619 (1979); S. Weinberg, Phys. Rev. D **13**, 1025 (1976); Phys. Rev. D **19**, 1277 (1979).

[2]S. Dimopoulos and L. Susskind, Nucl. Phys. **B155**, 237 (1979); E. Eichten and K. Lane, Phys. Lett. **90B**, 125 (1980).

[3]B. Holdom, Phys. Rev. D **24**, 1441 (1981); H. Georgi and S. Glashow, Phys. Rev. Lett. **47**, 1511 (1981).

[4]B. Holdom, Phys. Lett. **150B**, 301 (1985).

[5]T. Akiba and T. Yanagika, Phys. Lett. **169B**, 432 (1986); K. Yamawaki, M. Bando, and K. Matumoto, Phys. Rev. Lett. **56**, 1335 (1986); T. Appelquist, D. Karabali, and L. Wijewardhana, Phys. Rev. Lett. **57**, 957 (1986).

[6]J. Cornwall, R. Jackiw, and E. Tomboulis, Phys. Rev. D **10**, 2428 (1974).

[7]B. Holdom, Phys. Lett. **143B**, 227 (1984).