Contribution to the Muon Anomaly from Superstring-Inspired Models

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We calculate the contribution to the muon anomaly induced by new Yukawa couplings involving heavy-matter E_6 fields predicted in the framework of superstring theories. We analyze a few models found in the current literature and show that for some of them the effect is on the order of the standard weak contribution. We produce a specific example where the effect could be even larger. Thus, we conclude that an improved $(g-2)_\mu$ experiment should be sensitive to such effects, if they happen to exist.

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The agreement to an almost incredible degree of accuracy between theory[1] and experiment[2] attained in the measurement of the anomalous magnetic moment of the electron and the muon has not been matched in any other field of physics. For this very same reason the electron (muon) anomaly serves as a very precise probe for new phenomena. Indeed, it has been used, for instance, to constrain supersymmetric particle masses[3] and to survey different aspects of various composite scenarios.[4] At the present level of precision, the electroweak effects cannot be checked yet. However only one order of magnitude improvement would suffice for those effects to be tested. In fact, the contribution to the muon anomaly $a_\mu \equiv (g-2)_\mu/2$ coming from weak-boson exchange amounts to[5] $\sim 2\times 10^{-8}$, whereas the present experimental accuracy is at the level of[2] $\sim 10^{-8}$. Thus, it is clear that the new generation of $g-2$ experiments, such as the newly proposed Brookhaven National Laboratory experiment, (with an estimated factor-of-20 improvement in precision) will start revealing structure associated to physics at the Fermi scale and beyond. This is extremely interesting since modern particle theory predicts a handful of qualitative changes at the ~ 1 TeV scale.

In this Letter we show that the nowadays fashionable superstring theories[6] might also produce effects which could be detected in the next generation of improved $g-2$ experiments. The effects which we are referring to are generic to superstring-motivated models and should be added to the strictly supersymmetrical ones.

In a class of favorite superstring models,[7] after dimensional reduction (from ten to four space-time dimensions) via a compactification on a Calabi-Yau manifold, one is left with a four-dimensional E_6 gauge group that contains, as a subgroup, the low-energy sector of the theory. Each generation of matter particles is contained in a **27** representation of E_6. Its decomposition into the familiar SO(10) and SU(5) subgroups reads explicitly

$$(\mathbf{27})_{E_6}=\begin{cases}(\mathbf{16}\oplus\mathbf{10}\oplus\mathbf{1})_{\mathrm{SO(10)}},\\ [(\mathbf{10}\oplus\mathbf{5}^*\oplus\mathbf{1})\oplus(\mathbf{5}\oplus\mathbf{5}^*)\oplus\mathbf{1}]_{\mathrm{SU(5)}}.\end{cases} \tag{1}$$

We see from this that, apart from the well known 15 standard-model particles in each generation, we are led to twelve new extra particles of the fundamental E_6 representation, which are the following:

$$\nu^c(1,1),\ H(1,2),\ \bar{H}(1,2),\ D(3,1),\\ D^c(3^*,1),\ N(1,1), \tag{2}$$

where in parentheses we display their $\mathrm{SU}(3)_c\otimes\mathrm{SU}(2)_L$ content. So, in particular, we see that there is an extra color-triplet quark D and D^c.

The D quark participates in the following trilinear Yukawa couplings to conventional standard model and supersymmetrical standard model particles as required by the most general E_6-invariant superpotential:

$$\alpha DQQ+\beta D^c u^c d^c \tag{3a}$$

and

$$aD^cLQ+bDe^cu^c+cD\nu^cd^c, \tag{3b}$$

where

$$L\equiv\begin{pmatrix}\nu\\ e\end{pmatrix}_L,\quad Q\equiv\begin{pmatrix}u\\ d\end{pmatrix}_L$$

with corresponding expressions for the other two generations.

Now, the coexistence of Eqs. (3a) and (3b) would lead to an almost instantaneous proton decay.[8] It has been argued therefore that either $\alpha,\beta\neq 0$ or $a,b,c\neq 0$ because of topological reasons or because of some discrete symmetry.[8] Since we are interested in effects involving leptons, we shall assume that $\alpha=\beta=0$. A further requirement that one usually imposes is that $c=0$ or is small. This choice is made in order not to get dangerous D-d mixing and to avoid trouble with flavor-changing neutral-current constraints.[9] Actually, we are not concerned with this term here. Just for simplicity we have set $c=0$.

The Yukawa couplings a and b in Eq. (3b) give contribution to the muon magnetic moment through the quantum loop effects shown in Fig. 1. It is obvious from the structure of the diagrams that the resulting contribution to $(g-2)_\mu$ is proportional to the D-quark mass. Also, it is worth mentioning that one needs left-right mixing in the c-scalar-quark sector in order to get a nonzero answer.

A calculation of the diagrams in Fig. 1 leads to the result

$$\left(\frac{g-2}{2}\right)_\mu = \frac{ab}{8\pi^2}\sin 2\chi_c \sum_{i=1,2}(-1)^i \frac{m_\mu m_D}{m_i^2 - m_D^2}\left[\frac{3}{2} + \frac{m_i^2 + 2m_D^2}{m_i^2 - m_D^2}\left(1 - \frac{m_i^2}{m_i^2 - m_D^2}\ln\frac{m_i^2}{m_D^2}\right)\right], \tag{4}$$

where $m_{1,2}$ stand for the c-scalar-quark masses (recall that scalar quarks come in two chiralities) and χ_c is the angle that diagonalizes the c-scalar-quark mass matrix in left-right space

$$\begin{pmatrix} m_{LL}^2 & m_{LR}^2 \\ m_{LR}^2 & m_{RR}^2 \end{pmatrix}. \tag{5}$$

The above mass matrix is nondiagonal because supersymmetry breaking introduces off-diagonal entries which in the context of supergravity[10] models have the structure

$$m_{LR}^2 = A m_c m_0, \tag{6}$$

where A is a model-dependent unknown constant, m_c is the c-quark mass, and m_0 is the supersymmetry breaking scale.

Before evaluating formula (4) for some particular cases let us make a few comments. We expect, *a priori,* larger effects than in the purely supersymmetric case because the supersymmetric contributions involve scalar leptons propagating in the loops. Scalar leptons, however, have tiny L-R mixings since off-diagonal terms in the mass matrix (5) happen to be proportional to the lepton mass m_l [see Eq. (6), with m_c replaced by m_l]. Furthermore, the Yukawa couplings a and b in Eq. (3) are unconstrained and can be rather large whereas, in supersymmetry, the gauge couplings are bound to be small ($\sim e$).

Also, the muon anomaly will be more sensitive to superstring effects than the electron anomaly because $(g-2)_\mu$ is proportional to the muon mass and because it involves L-R mixings for the second generation of scalar quarks.

Therefore we expect

$$(g-2)_e \sim (m_e m_u / m_\mu m_c)(g-2)_\mu.$$

From the purely phenomenological point of view, we could choose a great variety of masses, couplings, and mixings. However, just to provide a scenario, we shall evaluate formula (4) in a few presently popular superstring-inspired models. Two of our choices correspond to the so-called "hybrid dimensional transmutation" models[11] and three are taken from the work of Ibanez.[12]

In Table I we display the values obtained. We see from the numbers that for $|ab|$ not too small, say $ab \sim O(g_w^2)$, the effect could well be present at the 10^{-9} level for some of the models exhibited. If this is so, a future $(g-2)_\mu$ experiment should see such an effect.

Furthermore, it is possible to get even more dramatic effects by just feeding formula (4) with other sets of parameters which, although not embodied in any specific model, do not contradict known phenomenological constraints[13] nor bluntly violate sacred theoretical prejudices.[8,10]

As an illustration take

$$m_1 = 65\text{ GeV}, \quad m_2 = 60\text{ GeV}, \quad m_D = 50\text{ GeV}, \tag{7}$$

and $A = 3$. Then the resulting value for the anomaly reads

$$[(g-2)/2)]_\mu \simeq 2.2\times 10^{-7} ab \tag{8}$$

Even allowing for $|ab|$ as small as $\alpha = \frac{1}{137}$, the resulting $[(g-2)/2]_\mu$ value is $\sim 1.6\times 10^{-9}$, i.e., comparable to the standard electroweak contribution.

To conclude, we have demonstrated that a rather substantial contribution to the muon anomaly is to be ex-

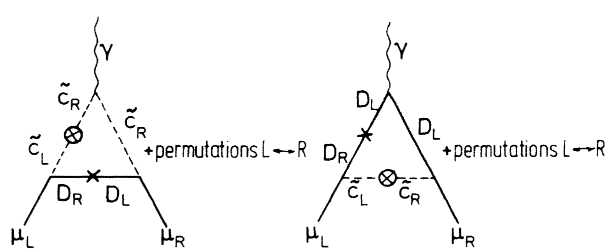

FIG. 1. Feynman diagrams contributing to the muon anomaly. The cross on a fermionic line stands for a mass insertion and a cross on a scalar line stands for left-right mixing.

TABLE I: Values for $a_\mu \equiv [(g-2)/2]_\mu$ for the various models quoted. Since Table 2 of Ref. 12 does not separately specify left-right scalar quark masses, we have assumed a $\sim$5% splitting (consistent with the splittings given in Table 4 of Ref. 11). All masses are given in gigaelectronvolts.

	m_1	m_2	m_D	A	$(a_\mu/ab)\times 10^9$
Model (a), Ref. 11	1600	1500	530	3	0.13
Model (b), Ref. 11	610	560	180	3	1.5
Model (a), Ref. 12	263	250	222	2.6	11.5
Model (e), Ref. 12	316	300	438	−0.86	2.7
Model (h), Ref. 12	131	125	206	−2.1	35

pected from those superstring-inspired models which contain, as low-energy remnants, extra isosinglet color-triplet quarks.

Indeed, some of the models in the market deliver values for $[(g-2)/2]_\mu$ which, if not suppressed by small Yukawa couplings, could be detected in future experiments. We have also produced a particular example where even small Yukawa couplings $[O(\sqrt{\alpha})]$ give rise to a contribution tantamount to the weak contribution. This result suggests that it is certainly worth improving the precision of the $g-2$ experiment, for it could reveal hints of new physics, the physics of the superstring. This is welcome in a theory rather meager in experimental consequences.

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