Universal Scattering Theorems for Strongly Interacting W's and Z's

Michael Chanowitz and Mitchell Golden
Lawrence Berkeley Laboratory, University of California, Berkeley, California 94720

and

Howard Georgi
Department of Physics, Harvard University, Cambridge, Massachusetts 02138
(Received 25 September 1986)

The experimentally verified relationship $M_W = M_Z \cos\theta_W$ implies universal low-energy theorems for scattering of longitudinally polarized W and Z bosons provided that the symmetry-breaking sector contains no particles much lighter than 1 TeV.

PACS numbers: 12.15.−y

Although the $SU(2)_L \otimes U(1)$ theory of electroweak interactions is extremely successful, we know nothing about the mechanism that breaks the gauge symmetry and gives the W and Z their masses. In particular, the physics of the symmetry-breaking sector may be weakly or strongly interacting. Examples of the former are models with elementary Higgs bosons much ligher than 1 TeV, including supersymmetric models in which the scale of supersymmetry breaking is of order $M_W \sim 100$ GeV. Examples of the latter are the minimal Higgs model[1] with heavy Higgs-boson mass, $m_H \gtrsim 1$ TeV, or dynamical theories such as technicolor models[2] with technihadron spectrum of order 1 TeV and above. It may also be possible, in ultracolor models,[3] to obtain a composite Higgs boson with $m_H \lesssim 1$ TeV and with the ultrahadron spectrum at masses much greater than 1 TeV. The purpose of this Letter is to derive theorems for "low energy" (compared to 1 TeV) scattering of longitudinally polarized W and Z bosons that hold in *all* experimentally viable strongly interacting symmetry-breaking models which have no light (compared to 1 TeV) scalars in their spectrum.

As discussed elsewhere,[4,5] the low-energy theorems are the basis of a general probe of the symmetry-breaking sector that could be implemented at a proton-proton collider with the energy and luminosity proposed for the Superconducting Super Collider (SSC). The central qualitative point is that WW fusion provides a significantly enhanced yield of longitudinally polarized gauge-boson pairs if and only if the symmetry-breaking sector is strongly interacting. The low-energy theorems can be used as the basis of an estimate of this gauge-boson-pair signal. Given the likely ability to detect gauge-boson pairs, it seems that the signal could be detected over backgrounds (e.g., $\bar{q}q \rightarrow WW$) at a pp collider with $\sqrt{s} = 40$ TeV and $\mathcal{L} = 10^{33}$ cm^{-2} sec^{-1}, as proposed for the SSC, but not at a collider with half the energy or a tenth the luminosity. This implies a "no-lose corollary" for a collider with the parameters proposed for the SSC: Either we would see the gauge-boson pairs signaling a strongly interacting symmetry-breaking sector at or above 1 TeV and/or there must be light particles from the symmetry-breaking sector that could be produced and studied directly. A more careful look at the experimental implications is planned for the future.

The intuitive basis for the scattering theorems is the observation that W_L and Z_L, the longitudinally polarized W and Z bosons, are essentially Goldstone bosons generated by the spontaneous breaking of a global symmetry of the symmetry-breaking sector. This observation is made precise by the "equivalence" theorem, proved to all orders in the weak and symmetry-breaking interactions,[4] which asserts for gauge-boson energies much greater than M_W that

$$\mathcal{M}(W_L(p_1), W_L(p_2), \ldots)_U = \mathcal{M}(w(p_1), w(p_2), \ldots)_R + O(M_W/E_i). \tag{1}$$

Here the left side is evaluated in the U gauge, the right side in an R gauge, and the $w(p_i)$ are the R-gauge Goldstone bosons associated with the longitudinal modes $W_L(p_i)$.

In Ref. 4 the W_L, Z_L scattering theorems were obtained for the class of models in which the symmetry-breaking sector has a chiral $SU(2)_L \otimes SU(2)_R$ symmetry that breaks spontaneously to the diagonal $SU(2)_{L+R}$ subgroup. The latter "custodial" SU(2) is sufficient[6] (but has not been proved necessary) to protect the tree relation

$$\rho \equiv (M_W / M_Z \cos\theta_W)^2 = 1 \tag{2}$$

against possibly strong quantum corrections from the symmetry-breaking sector. In this Letter we shall obtain the scattering theorems without making any assumptions about the symmetry structure of the symmetry-breaking sector [except of course that the $SU(2)_L \otimes U(1)$ gauge symmetry not be explicitly violated]. Instead we show directly that the experimentally verified constraint $\rho\cong 1$ implies that universal form for the scattering theorems that was found in Ref. 4.

We have verified this result by three different methods. The most intuitively accessible, presented below, bypasses the equivalence theorem, Eq. (1), and makes use of a power-counting analysis in the unitary gauge. Two other derivations. to be presented in a longer paper,[7] rely on symmetry considerations that follow from gauge invariance. One uses $SU(2)_L$ current algebra and PCLC (partial conservation of left-handed current), as opposed to $SU(2)_L\otimes SU(2)_R$ current algebra and PCAC (partial conservation of axial-vector current) used to derive the pion low-energy theorems[8] on which Ref. 4 is based. The other derivation uses effective chiral-Lagrangean methods. Our longer paper will also include examples, analyzed by the effective-Lagrangean method, that show how low-energy W_L,Z_L scattering can be affected by light spin-zero scalars and pseudo-Goldstone bosons.

Although the current-algebra and chiral-Lagrangean derivations may seem more generally appropriate to dynamical models, we present here the U-gauge, power-counting analysis because it reveals a simple interpretation of the underlying physics. From the U-gauge analysis we see explicitly that the universal form of low-energy W_L,Z_L scattering is determined by $SU(2)_L\otimes U(1)$ gauge invariance alone, provided that the quanta of the symmetry-breaking sector are all heavy.

To see how this works, consider first the minimal Higgs model.[1] Though we are ultimately interested in working to order g^2 in the electroweak gauge coupling and to all orders in the Higgs-boson coupling λ_{SB}, we begin by examining $W_L^+W_L^-\to Z_LZ_L$ scattering to tree approximation in both couplings. The tree-approximation amplitude in unitary gauge can be decomposed into the sum of a gauge sector term and a symmetry-breaking sector term,

$$\mathcal{M}(W_L^+W_L^-\to Z_LZ_L)=\mathcal{M}_{\rm gauge}+\mathcal{M}_{\rm SB}. \tag{3}$$

We will evaluate the amplitude for $s\gg M_W^2$. The first term, $\mathcal{M}_{\rm gauge}$, is given by the sum of t and u channel W exchanges and by the four-point contact interaction. Independent of the nature of the symmetry-breaking sector it is a universal function of M_W and ρ,

$$\mathcal{M}_{\rm gauge}=g^2s/4\rho M_W^2. \tag{4}$$

The second term, $\mathcal{M}_{\rm SB}$, is in the tree approximation just given by s-channel Higgs-boson exchange,

$$\mathcal{M}_{\rm SB}=-(g^2s/4M_W^2)[s/(s-m_H^2)]. \tag{5}$$

$\mathcal{M}_{\rm gauge}$ has the famous "bad" high-energy behavior that is canceled at infinite s by $\mathcal{M}_{\rm SB}$ (since $\rho=1$ in the minimal Higgs model). However, for $s\ll m_H^2$, $\mathcal{M}_{\rm SB}$ is negligible and so for the low-energy domain $M_W^2\ll s\ll m_H^2$ we have

$$\mathcal{M}(W_L^+W_L^-\to Z_LZ_L)\cong\mathcal{M}_{\rm gauge}\cong g^2s/4\rho M_W^2. \tag{6}$$

Similarly we also find

$$\mathcal{M}(W_L^+W_L^-\to W_L^+W_L^-)\cong-\frac{g^2u}{4\rho M_W^2}(4\rho-3), \tag{7}$$

$$\mathcal{M}(Z_LZ_L\to Z_LZ_L)\simeq 0. \tag{8}$$

The other W_L,Z_L scattering amplitudes (elastic scattering of $W_L^\pm Z_L$, $W_L^+W_L^+$, and $W_L^-W_L^-$) follow from Eqs. (6) and (7) by crossing symmetry. With the experimental constraint that $\rho\cong 1$ these amplitudes agree precisely with those deduced in Ref. 4 from $SU(2)_L\otimes SU(2)_R$ current algebra.

Though to this point we have only derived Eqs. (6)–(8) in the tree approximation for the minimal Higgs model, they are actually valid to all orders in λ_{SB}, the potentially strong coupling of the symmetry-breaking sector, for any symmetry-breaking sector with no light particles. By power counting we will see that the only strong corrections to Eqs. (6)–(8) are absorbed as renormalizations of M_W and ρ. All other quantum corrections due to the symmetry-breaking sector are screened by an extra power of the electroweak coupling constant, $\alpha_W/\pi=g^2/4\pi^2$, or they are suppressed by powers of s/M_{SB}^2, where M_{SB} is the characteristic scale of the spectrum of the symmetry-breaking sector.

The effective-Lagrangean approach[7] suggests that the large-s corrections are actually $O(s/\Lambda_{SB}^2)$, where

$$\Lambda_{SB}=\min\{M_{SB},4\pi v\} \tag{9}$$

and $v\equiv 2M_W/g=\frac{1}{4}$ TeV is the standard-model vacuum expectation value $4\pi v$ is the scale set by the one-loop corrections computed with a phenomenological Lagrangean.[9] The successful applications of chiral symmetry in hadron physics support the analogous contention that $4\pi F_\pi$ sets the scale for the corrections to soft-pion theorems.[10] A similar scale, $4\sqrt{\pi}v$, is also imposed[4] by partial-wave unitarity in the $I=J=0$ channel, $W_L^+W_L^-\to Z_LZ_L$. (However, the $I=J=1$ and $I=2$, $J=0$ channels are unitary up to larger scales—see Ref. 4.)

As a concrete example of the power-counting analysis, consider a general Higgs model of heavy scalar bosons with a large mass scale $M_{SB}\gtrsim 1$ TeV. The scalar one-loop contributions to the gauge-current vacuum-polarization tensor are shown in Figs. 1(a)–1(c). In unitary gauge all are quadratically divergent and it is easy to see that the leading behavior in large M_{SB} is proportional to $g^2M_{SB}^2$. For instance, schematically we see on dimensional grounds that Fig. 1(a) contributes finite terms

$$g^2\int d^4p\frac{(2p-q)^\mu(2p-g)^\nu}{(p^2-M_{SB}^2)[(q-p)^2-M_{SB}^2]}\sim g^{\mu\nu}(g^2AM_{SB}^2+g^2Bq^2+\dots)+q^\mu q^\nu\left[g^2C+g^2D\frac{g^2}{M_{SB}^2}+\dots\right],$$

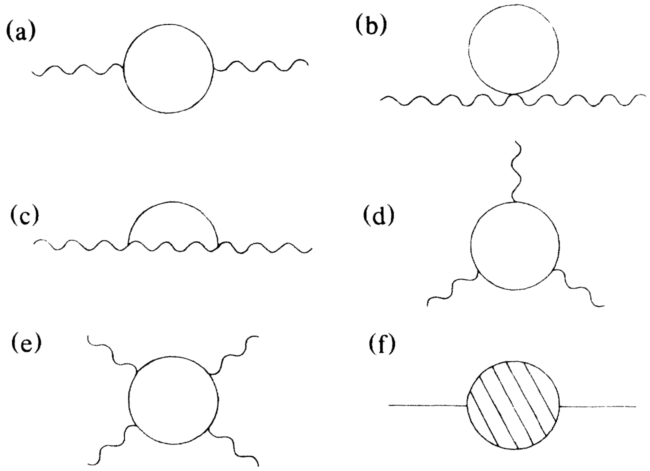

FIG. 1. Quantum corrections from the symmetry-breaking sector. Curly lines represent gauge bosons and straight lines represent physical scalar or pseudoscalar bosons from the symmetry-breaking sector.

where A, B, C, and D are dimensionless numbers. (Any possible logarithmic dependence on $M_{\rm SB}$ is neglected.) The $g^2 A M_{\rm SB}^2$ term contributes to the gauge-boson self-energy; as a fraction of the tree-level mass it is just an $O(\lambda_{\rm SB})$ correction: $\delta M_W^2/M_W^2 \sim g^2 M_{\rm SB}^2/g^2 v^2 \sim \lambda_{\rm SB}$ (cf. the standard-model tree-level relation $\lambda = m_H^2/2v^2$). The $g^2 B q^2$ term gives an $O(\alpha_W/\pi)$ [or at most $O((\alpha_W/\pi) \times \ln M_{\rm SB})$] contribution to the wave-function renormalization. Adding internal scalar lines to the loops of Figs. 1(a)-1(c) modifies these one-loop results by additional powers of $\lambda_{\rm SB}$. Thus the gauge-boson masses are strongly renormalized while the corrections to the wave-function renormalization are screened (modulo logarithms) by one power of $\alpha_W/\pi = g^2/4\pi^2$.

Like the wave-function renormalization, the one-loop scalar corrections to the three-gauge-boson vertex [Fig. 1(d) and other diagrams with one or two internal scalars replaced by gauge bosons] are only logarithmically divergent, and on dimensional grounds the leading finite contribution to the gauge-coupling-constant renormalization is at most $O((\alpha_W/\pi)\ln M_{\rm SB})$. Again, to higher orders in the strong scalar interactions this correction is multiplied by additional powers of $\lambda_{\rm SB}$. Finite-momentum, form-factor effects are suppressed by powers of $Q^2/M_{\rm SB}^2$, where Q is the external momentum scale.

The one-particle irreducible scalar loop contributions to the $W_L W_L$ scattering amplitude [Fig. 1(e) and other diagrams with one, two, or three scalar propagators replaced by gauge bosons] make an at most logarithmically divergent contribution to the amplitudes at threshold, which are linear in s, t, or u [cf. Eqs. (6)-(8)]. Dimensionally the dependence on $M_{\rm SB}$ of this contribution to the threshold amplitudes is at most logarithmic, so that Fig. 1(e) also makes no $O(\lambda_{\rm SB})$ correction to the tree approximation. Relative to the tree amplitudes, Eqs. (6)-(8), it contributes corrections of order $O(\alpha_W/\pi)$ or $O((\alpha_W/\pi)s/M_W^2)$, multiplied by factors $(s/M_{\rm SB}^2)^n$ for integer $n \geq 0$. The $O((\alpha/\pi)s/M_W^2)$ terms correspond to the $O(s/(4\pi v)^2)$ terms expected in the phenomenological-Lagrangean approach.[9,10]

Finally there are strong renormalizations of the scalar-boson self-energies, indicated generically in Fig. 1(f). These could affect the leading threshold behavior of the $W_L W_L$ scattering amplitudes if they were to induce a pole in the complex energy plane with real and imaginary parts small compared to $M_{\rm SB}$. This possibility is excluded by the assumption that there be no physical particles in the spectrum of the symmetry-breaking sector that are light compared to the typical scale $M_{\rm SB}$. If light scalars do exist, they can in different cases increase, decrease, or leave unaffected the low-energy amplitudes, Eqs. (6)-(8) as we will illustrate with examples elsewhere.[7]

The conclusion is that M_W and M_Z are renormalized by order-$\lambda_{\rm SB}$ effects while all other corrections to the low-energy $W_L W_L$ amplitudes from a strongly interacting Higgs sector are screened by a power of α_W/π or suppressed by powers of $s/\Lambda_{\rm SB}^2$. This establishes the validity of the low-energy amplitudes, Eqs. (6)-(8), where ρ and M_W are the physical values, incorporating possible strong quantum corrections from the Higgs sector.

The same power-counting analysis applies if the symmetry-breaking sector also contains massive fermions, perhaps carrying the non-Abelian charge of a new strongly coupled gauge interaction as in technicolor models. The one-fermion-loop contributions to the vacuum polarization induce order-$\lambda_{\rm SB}$ renormalizations of M_W and M_Z [where now $\lambda_{\rm SB} \sim (gM_F/2M_W)^2 \sim (M_{\rm SB}/v)^2$ with $M_F \sim M_{\rm SB}$ the heavy fermion mass], while the one-loop contributions to the three- and four-point gauge boson amplitudes are higher order in α_W/π or suppressed by $s/M_{\rm SB}^2$.[11] Higher-order corrections due to the strong gauge interactions modify these one-loop corrections by powers of the strong gauge-coupling constant. Therefore to any finite order in perturbation theory we reach the same conclusion as for the Higgs models.

It should, however, be said that for models of dynamical symmetry breaking, in which the physical spectrum is not explicit in the Lagrangean but must emerge from the solution, the current-algebra and effective-Lagrangean methods may be a more suitable language than the perturbation expansion. In particular, using effective Lagrangeans we can analyze the effect of pseudo-Goldstone bosons on low-energy scattering as a function of the assumed patterns of symmetry breaking.[7]

We have shown for $M_W^2 \ll s \ll_{\rm SB}^2$ that the low-energy theorems, Eqs. (6)-(8), describe the scattering of longitudinally polarized gauge bosons if there are no light spin-zero particles (compared to 1 TeV) in the spectrum of the symmetry-breaking sector. The experimental consequence of these low-energy theorems is a "no-lose corollary"[5] that holds if we can probe the symmetry-breaking physics with a hadron collider of the energy and luminosity for the SSC, $\sqrt{s} = 40$ TeV and $\mathcal{L} = 10^{33}$ cm^{-2} sec^{-1}. By extrapolating the low-energy theorems we can conservatively predict[4] the order of magnitude of the enhanced *WW*, *WZ*, and *ZZ* pair yields they imply, via the *WW* fusion mechanism. The signal would be observable over the backgrounds at the SSC but probably not at a *pp* collider with half the energy or one with a tenth the luminosity.[5] With a facility like the SSC we would therefore be in a no-lose situation, since either we would see the enhanced gauge-boson pairs that signal a strongly interacting symmetry-breaking sector at or above 1 TeV or we could conclude that there must be light particles from the symmetry-breaking sector that could be produced and studied directly.

We wish to thank Michael Dugan for several helpful discussions. This work was supported by the Director, Office of Energy Research, Office of High Energy and Nuclear Physics, Division of High Energy Physics of the U.S. Department of Energy under Contract No. DE-AC-03-76SF00098 and in part by the National Science Foundation under Grant No. PHY-82-15249.

[1]S. Weinberg, Phys. Rev. Lett. **19**, 264 (1967); A. Salam, in *Elementary Particle Physics,* edited by N. Svartholm (Almquist and Wiksell, Stockholm, 1968), p. 367.

[2]S. Weinberg, Phys. Rev. D **13**, 974 (1976), and **19**, 1277 (1977); L. Susskind, Phys. Rev. D **20**, 2619 (1979).

[3]D. B. Kaplan and H. Georgi, Phys. Lett. **136B**, 183 (1984); D. B. Kaplan, H. Georgi, and S. Dimopoulos, Phys. Lett. **136B**, 187 (1984); M. Dugan, H. Georgi, and D. B. Kaplan, Nucl. Phys. **B254**, 299 (1985).

[4]M. Chanowitz and M. K. Gaillard, Nucl. Phys. **B261**, 379 (1985).

[5]M. Chanowitz, Lawrence Berkeley Laboratory Report No. LBL-21973 in Proceedings of the Twenty-third International Conference on High Energy Physics, Berkeley, 1986 (to be published); and in *Observable Standard Model Physics at the SSC: Monte Carlo Simulation and Detector Capabilities,* edited by H.-U. Bengtsson *et al.* (World Scientific, Singapore, 1986), p. 183.

[6]P. Sikivie, L. Susskind, M. Voloshin, and V. Zakharov, Nucl. Phys. **B173**, 189 (1980).

[7]M. Chanowitz, H. Georgi, and M. Golden, to be published.

[8]S. Weinberg, Phys. Rev. Lett. **17**, 616 (1966).

[9]S. Weinberg, Physica **9A**, 327 (1979).

[10]A. Manohar and H. Georgi, Nucl. Phys. **B234**, 189 (1984).

[11]M. Veltman, Nucl. Phys. **B123**, 89 (1977); M. Chanowitz, M. Furman, and I. Hinchliffe, Phys. Lett. **78B**, 25 (1978), and Nucl. Phys. **B153**, 402 (1979).