Onset of the Critical Velocity Regime in Superfluid ^{4}He at Low Temperature

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We have measured the critical velocity of superfluid ^{4}He flow through a submicron orifice from 1.2 K down to 5 mK. The velocity for the onset of phase-slip events is found to be independent of pressure, to be strongly altered by minute traces of ^{3}He impurities, and to vary with temperature as $1-T/T_0$ with $T_0=2.46$ K. These results imply the existence at $T=0$ of a hydrodynamic instability which is thermally activated at finite temperature and takes place in a very small volume.

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The phenomenon of a critical velocity above which a different, more dissipative hydrodynamic regime arises is very commonly met in nature. Well-known examples, such as the surfing speed of sailboats or the vibration onset of the reed of wind musical instruments, are usually understood as resulting from the combined effects of dissipation and nonlinear terms in the equations of motion of these systems. The critical velocity v_{c0} for local destruction of the superfluid state at absolute zero, while also involving eddies, or vortices, differs from the classical case in two important ways. First, no intrinsic dissipation processes exist in the quantum-mechanical ground state. Second, the hydrodynamic circulation has to be quantized along any contour encircling the created defects. The conceptual difficulties associated with this long-standing problem of the creation (and destruction) of quantized vortices in superfluid ^{4}He have been concisely but lucidly analyzed in a review by Fetter.[1]

Previous experimental observations of critical flows through small orifices[2,3] support, at least in a qualitative manner, two types of nucleation models. One mechanism is extrinsic in the sense that it depends on the geometry of the flow path. It involves the existence of some form of pinned vorticity which can either twist on itself and grow unstable against loop formation, as in the "vortex mill" model suggested by Glaberson and Donnelly,[4] or generate by reconnections a "self-sustaining tangle" as proposed by Schwarz.[5] These temperature-independent mechanisms lead to Feynman's formula[1] for the critical velocity:

$$v_c=(f\kappa_0/2\pi d)\ln(d/\xi_0), \tag{1}$$

in which κ_0 is the quantum of circulation, ξ_0 is the coherence length (0.15 nm), d is a characteristic dimension of the orifice, and f is a coefficient dependent on the nucleation model[4,5] and on experimental conditions. Equation (1) is verified experimentally in a number of cases[6] for channels over a micrometer in size and for which no guard is used to filter out remnant vorticity.[2]

The other mechanism, put forward by Iordanskii[7] and by Langer and Fisher,[8,9] pertains to temperature-dependent critical velocities. This Iordanskii-Langer-Fisher (ILF) model assumes an Arrhenius law

$$\Gamma=\Gamma_0\exp(-E_a/k_BT) \tag{2}$$

for the rate of vortex formation, driven over an energy barrier E_a by thermal fluctuations characterized by an attempt frequency Γ_0. The general dependence of the energy barrier on temperature T and superfluid velocity v_s as deduced from heuristic arguments and as justified in a classical vortex ring model is

$$E_a=\beta_F\rho_s(T)/\rho v_s, \tag{3}$$

where the value of the fluctuation parameter β_F is calculated to be about 50×10^{-12} erg cm/s and $\rho_s(T)/\rho$ is the superfluid fraction at temperature T. The ILF model satisfactorily explains the logarithmic time decay of persistent currents[9] and does reproduce, as will be discussed critically below, the observed temperature dependence of v_c, at least for submicronic pores and between T_λ and ~1.1 K.[10,11] However, it yields quantitative predictions for β_F which are too large by an order of magnitude and, more fundamentally, clearly fails to account for a proper zero-temperature behavior of v_c.[1] In this Letter, we present evidence, from the effect of temperature, pressure, and ^{3}He impurities on the onset threshold of phase-slip events in superfluid ^{4}He down to the millikelvin range, that neither mechanism, extrinsic[4,5] or thermally driven,[7-9] provides a satisfactory description of the full experimental situation.

Our experiments were conducted in the same Helmholtz resonator[12] with the same 0.3-μm-wide slit as used previously for the observation of singly quantized dissipation events.[13] These phase-slip events give an unambigu-

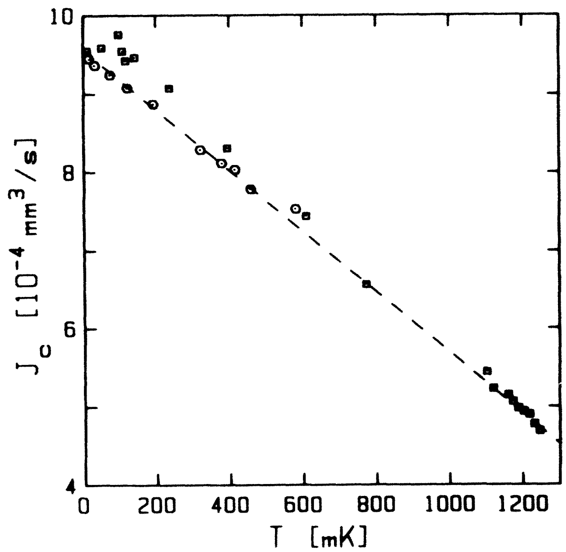

FIG. 1. Critical volume flow rate vs temperature for the two runs with the purest ^{4}He sample ($x_3 < 10^{-8}$) at zero pressure. The dashed line corresponds to Eq. (4) in the text with $T_0 = 2.46$ K. A small effect of ^{3}He contamination can be seen in the second run, identified by the squares.

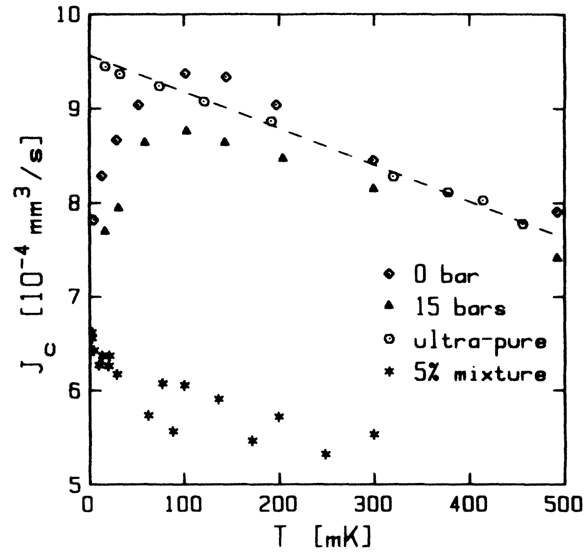

FIG. 2. Critical volume flow rate vs temperature for nominal purity ($x_3 \sim 4\times10^{-7}$) ^{4}He at 0 and 15 bars, for the first run with the ultrapure sample and for a 5.0% mixture. The dashed line is the same as in Fig. 1. The experimental scatter from one run to another is of the order of 5%. The scatter during the same run is smaller, except for the 5.0% mixture because of the strong effect of viscosity and osmotic compressibility in the ^{3}He quasiparticle gas.

ous signature of the onset of the critical flow regime. A cold, superfluid-tight valve has been added on the fill line to provide improved isolation of the experimental cell against external influences. We have thus been able to follow the phase slips in temperature from a few millikelvins up to 1.2 K at the pressures of 0 and 15 bars. Since, as is the case for ion motion[14] and film transfer-rate experiments,[15,16] minute traces of ^{3}He impurities can have a pronounced effect on the critical velocity, we purified our nominally pure ^{4}He to obtain "ultrapure" ^{4}He[17] which was used for two runs at zero pressure. To obtain further information on the effect of ^{3}He and to monitor the cell operation and its ability to track phase slips in the presence of strong damping, we have also made a run on a 5.0% ^{3}He-^{4}He mixture at zero pressure.

The data points collected in the various runs are shown in Fig. 1 over the full temperature range for the two runs with the purest sample and in Fig. 2, in which the effect of ^{3}He impurities and applied pressure can be seen up to 500 mK. The important experimental findings are the following: (1) Even minute traces of ^{3}He strongly affect the critical velocity close to absolute zero. (2) The effect of hydrostatic pressure is quite small. (3) The temperature dependence of the critical velocity is linear, going as

$$v_c = v_{c0}(1 - T/T_0), \tag{4}$$

with $T_0 = 2.46$ K, all the way down to the millikelvin range ($\sim$5 mK) for ultrapure ^{4}He and down to 100 mK for nominal-purity ^{4}He. The critical velocity v_c is reproducible from run to run provided the cell is kept at liquid-helium temperatures. We point out that the readily measured quantity is the total phase difference across the orifice, which is obtained from the size of the phase change (i.e., 2π) divided by the experimentally observed relative velocity jump $\Delta v/v_c$. This total phase difference was observed to vary from 53 to 38 times 2π upon warming to room temperatures. This change was accompanied by a variation of the Helmholtz frequency, hence of the slit geometry, because of contamination. The absolute value of the velocity is uncertain within a factor 3 because of these contamination effects but is of the order of 1.5 m/s. In spite of extreme precautions taken against the ambient mechanical noise in the apparatus, we have not been able to reduce the noise on the determination of the critical threshold below the level obtained in the early runs.[13] We cannot decide definitively whether this noise level is intrinsic or setup dependent but are sure that it does not depress significantly the values of v_c. Finally, the overall dissipation in the resonator was measured. The power necessary to maintain the oscillation amplitude close to the critical level in pure ^{4}He was found to be $P_c = 3.2\times10^{-21}\exp(26.5T/T_\lambda)$ W.

These results have important implications on the mechanisms for phase-slip nucleation. The influence of less than one ^{3}He atom in a volume of (70 nm)^3 is felt below 100 mK when the isotopic impurity condenses on the vortex core[18,19] and not above where they move freely through the superfluid. This effect seems to be a rather strong indication that vortex nucleation is linked either to some form of preexisting vorticity so that ^{3}He atoms have time to collect on the vortex core, or to the near-instantaneous formation of a bound state between one ^{3}He atom and a nascent, very small-scale vortex. The insensitivity of v_c to pressure changes suggests that the nucleation mechanism does not involve density-dependent parameters such as the roton energy or the speed of sound and is essentially hydrodynamic origins. Finally, the most striking and unexpected result is the linear temperature dependence of v_c expressed by Eq. (4), which extends unwaveringly to millikelvin temperatures where no property of superfluid ^{4}He still varies significantly (apart from phonon density). We need to seek a nucleation mechanism which remains effective at absolute zero and whose rate is exponentially sensitive to temperature. Quantum tunneling through a finite barrier would provide a natural extension at $T=0$ of the high-temperature ILF mechanism.[20] However, neither when dissipation is absent[21] nor when any reasonable damping process is included[22] can such a tunnel effect lead to a linear dependence of v_c with temperature starting from $T\sim 0$. The only possibility which will produce such a behavior seems at present to be provided by thermal activation over an energy barrier of the form

$$E_a = E_0(1 - v_s/v_{c0}), \tag{5}$$

where E_0 and v_{c0} are phenomenological parameters. Although we shall not attempt to justify Eq. (5) formally here, we note that a number of different mechanisms lead to similar expressions, e.g., Anderson's flux creep in superconductors[23] or the effect of rough boundaries on superfluid vortex formation.[24] Equation (5) indeed yields the temperature dependence of v_c required by Eq. (4) with the following identification:

$$E_0 = k_B T_0 \ln(\Gamma_0/\Gamma_{obs}). \tag{6}$$

The observed rate Γ_{obs} of phase-slip events is known experimentally[13] to be not less than 10^3 s^{-1}. From Eq. (6) we can evaluate an upper bound for E_0. The attempt frequency Γ_0 is at most the sum of the zero-point fluctuation rate ($\sim 10^{12}$ s^{-1}) of all atoms present in the volume relevant for vortex nucleation. Taking the full volume of the orifice ($\sim 1\ \mu$m^3), we find an upper bound for Γ_0 of 2.2×10^{22} s^{-1} which, using the measured value of T_0, yields a maximum possible value for E_0 of 110 K. This value, while quite high for an atomic energy in superfluid helium, is rather small for a collection of atoms and indicates that the nucleation volume cannot be large. As a basis for comparison, the energy of a classical vortex ring with the smallest critical radius given by the ILF theory ($r_c \sim 2$ nm) is 307 K. If, building on this remark, we reduce the nucleation volume to 1 nm^3, which seems to be the smallest conceivable value, we find $E_0 = 58$ K. These low values of E_0 fix a very small length scale for the nucleated vortex.

Before commenting on these results, let us mention that the linear temperature dependence of v_c expressed by Eq. (4) has also been observed by other workers[10,11,25] with nearly identical values of T_0, although different situations may also occur (see, e.g., Refs. 2 and 3). The recent observations of Zimmermann and Beecken,[25] also carried out in a high-sensitivity resonator with submicron holes between 0.4 and 1.9 K, very nicely confirm and complement the present work. In particular, these authors report a value for T_0 of 2.45 ± 0.1 K, in excellent agreement with the present work. The studies of Refs. 10 and 11 of pressure-driven superflows have led to the conclusion that v_c was proportional to $\rho_s(T)/\rho T$ as predicted by the ILF theory on the basis of Eqs. (2) and (3), at least for small ($d < 100$ nm) pores and above ~ 1.1 K, and to a value for β_F of $\sim 10^{-12}$ erg cm/s. But, since $\rho_s(T)T_\lambda/\rho T$ is represented to a good approximation by $3.64[1 - T/(2.35\text{ K})]$ between 1.1 and 2 K, these earlier results can also be reinterpreted in terms of a plain linear law with $T_0 = 2.35$ K. Thus, they can be compared in a meaningful way to the present work, which would lead to a value of $\beta_F = v_{c0}E_0T_\lambda/3.64,\ T_0 = (0.3\text{–}0.6)\times 10^{-12}$. These various sets of data are in quite reasonable agreement with one another and yield converging pieces of information.

Would it be possible to reconcile the physical models underlying the energy barriers given by Eqs. (3) and (5), as seems to be the case for the high- and low-temperature experimental data? A hint along this line could be provided by the empirical energy barrier used by Kukich, Henkel, and Reppy[26] to account for low-temperature deviations with respect to the ILF theory. However, the functional form of the energy barrier in this theory, which stems from the assumption of bulk, homogeneous nucleation, cannot be made to depart much from Eq. (3). Also, the barrier heights remain too high. The bulk, homogeneous formation of vortices has to be assisted in some way, e.g., by the presence of rough boundaries[24] or other types of inhomogeneities. But the very presence of inhomogeneities, necessary to introduce a length scale in the problem (and hence v_{c0}), breaks the translational-invariance argument invoked by ILF and alters drastically the essence of their approach. Large-scale vortices, pinned across the orifice, as in the vortex mill model,[4] would involve fluctuation energies E_0 which are much larger than those determined from Eq. (6) and lead to critical velocities of the type described by Eq. (1). Vortex creep[23] or depinning[27] do provide alternative energy-dissipation mechanisms worth further investigation. But to us, in their present form, they seem to lack the ability to explain the remarkable reproducibility of v_c and of the phase-slip events that we observe over such a wide range of temperatures and ^{3}He concentrations. Thus, conventional models are not vindicated by the experiments reported here. Having ruled out nucleation in the bulk of the fluid, we are led to invoke the presence of small-scale nucleating centers at the boundaries. This assertion is not in conflict with the experimental evidence reported here and is further supported by the following remarks. First, velocities in potential flow are larger close to the orifice walls. Second, the presence of fine-grained surface defects further increases the local velocity. Finally, the depression of ρ_s over a few healing lengths from the walls generates the current-density gradients necessary for a boundary-layer instability. Should this be so, the quantum problem of vortex nucleation would not be much different from the classical one.

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