# Theoretical Model of Fishbone Oscillations in Magnetically Confined Plasmas

B. Coppi and F. Porcelli[a]
*Massachusetts Institute of Technology, Cambridge, Massachusetts 02139*
(Received 14 November 1985)

The onset of electromagnetic oscillations that are observed in magnetically confined plasmas where beams of fast neutrals are injected is associated with the excitation of a mode with poloidal wave number $m^0=1$ and phase velocity equal to the core-ion diamagnetic velocity. The resonant interaction of the mode with the beam ions is viewed as a form of dissipation that allows the release of the mode excitation energy, related to the gradient of the plasma pressure.

PACS numbers: 52.35.−g, 52.55.−s

A new type of instability has been observed in magnetically confined toroidal plasmas where beams of fast neutrals are injected nearly perpendicular to the equilibrium magnetic field. The poloidal magnetic field fluctuations produced by this instability have a characteristic signature and are called "fishbone oscillations."[1,2] Particle bursts corresponding to loss of energetic beam particles are correlated with fishbone events, reducing the beam heating efficiency and thus limiting the maximum achievable $\beta$ [=(kinetic pressure)/(magnetic pressure)] by this technique.

We present the results of an analysis that supports one of the interpretations[3] advanced when these experimental observations were first reported. This consisted of proposing the following: (a) The excited mode, whose spatial structure is dominated by the component with poloidal wave number $m^0=1$, has a frequency related to the ion diamagnetic frequency and is one of the two $m^0=1$ modes that are found under the conditions for ideal MHD instability, but are rendered marginally stable by finite ion Larmor radius effects[4]; (b) the mode "excergy" (excitation energy) is related to the plasma pressure gradient; and (c) the presence of a "viscous" dissipative process (e.g., produced by a mode-particle resonance that scatters the beam ions) is required for the instability to develop.

We refer, for simplicity, to a large-aspect-ratio axisymmetric toroidal confinement configuration with circular magnetic surfaces, and consider perturbations of the equilibrium field that are dominated by the $m^0=1$, $n^0=1$ poloidal and toroidal components. The relevant model dispersion relation is[4]

$$[\omega(\omega-\omega_{\rm di})]^{1/2}=i\,\gamma_{\rm MHD} \tag{1}$$

when we omit mode-particle resonances and other dissipative processes. Here, $\omega_{\rm di}=-(c/eBrn)dp_{i\perp}/dr$ is the ion diamagnetic frequency evaluated at the surface $r=r_0$ where the pitch angle of the unperturbed equilibrium magnetic field equals that of the perturbation, and $p_{i\perp}$ is the transverse ion pressure. The ideal MHD growth rate $\gamma_{\rm MHD}$ is given by[4] $\gamma_{\rm MHD}=\omega_A\lambda_H$, where $\omega_A\equiv v_{A\theta}\hat{s}/r_0$ with $v_{A\theta}=B_\theta/(4\pi m_i n_i)^{1/2}$, $\hat{s}=d\ln q/d\ln r$, $q=rB_\zeta/RB_\theta$, $R$ is the major radius of the torus, $\lambda_H=\lambda_0(r_0/R)^2(\beta_p^2-\beta_{p,\rm crit}^2)$, $\lambda_0$ is a finite numerical factor, $\beta_p\equiv-(R/r_0^2)^2\int_0^{r_0}dr\,r^2(d\beta/dr)$, and, for a parabolic $q$ profile,[5] $\beta_{p,\rm crit}=\sqrt{13}/12$. The ideal stability parameter $\lambda_H$ is the negative of the (normalized) minimum value of the perturbed potential energy[4] $\delta W_{\rm min}$ and the dispersion relation (1) is valid for $\lambda_H>0$. In the realistic limit where $\gamma_{\rm MHD}<\omega_{\rm di}$, the dispersion relation (1) yields two (marginally) stable roots: one with $\omega\approx(\gamma_{\rm MHD}/\omega_{\rm di})^2\omega_{\rm di}$, and the other with $\omega\approx\omega_{\rm di}$ that we argue to be the mode excited by the injected beam. Following an observation given by Coppi, Rosenbluth, and Sudan[6] we may argue that the two roots of Eq. (1) are waves with energies of opposite sign. Thus, one of them will be damped and the other destabilized when a dissipative process is introduced.

In the case of plasmas where fast neutrals are injected, the relevant mode-particle resonance[7,8] $\omega_{Dh}^{(0)}(\varepsilon,\mu)=\omega$ involves beam ions with energies $\varepsilon=m_h v^2/2$ and magnetic moments $\mu=m_h v_\perp^2/2B$, where $\omega_{Dh}^{(0)}(\varepsilon,\mu)$ is the average (along the orbit) magnetic drift frequency of the energetic ($h$ denotes hot) ions that have magnetically trapped orbits. This dissipative process will be more important for the root $\omega\simeq\omega_{\rm di}$, as $\omega_{\rm di}/\omega_{Dh}^{(0)}\sim(T_i/T_h)(R/r_n)(1+\eta_i)\sim1$ for typical beam parameters, where $T_{i\,(h)}$ is the core-ion (hot-ion) temperature, $r_n=|d\ln n_i/dr|^{-1}$, and $\eta_i=d\ln T_i/d\ln n_i$. In this case, the dispersion relation (1) including the effect of viscosity can be put in the form[6]

$$(\omega+i\,\nu_v)(\omega-\omega_{\rm di})=-\gamma_{\rm MHD}^2, \tag{2}$$

where $\nu_v$ is introduced to represent the rate of resonant momentum exchange between the beam and the mode. The solution of (2) with $\omega=\omega_{\rm di}+\delta\omega$, $|\delta\omega|<\omega_{\rm di}$, corresponds to an unstable mode with growth rate $\gamma\equiv{\rm Im}\,\delta\omega=(\gamma_{\rm MHD}/\omega_{\rm di})^2\nu_v$. A small damping term related to the plasma resistivity has been neglected.

The low-frequency solution of Eq. (1) with $\omega\simeq\gamma_{\rm MHD}^2/\omega_{\rm di}$ does not interact efficiently with the beam, as $\omega<\omega_{Dh}^{(0)}$, but is destabilized by the longitudinal resistivity $\eta_\parallel$. In this case, the dispersion relation (1) is modified into[4]

$$\omega(\omega-\omega_{\rm di})=-\gamma_{\rm MHD}^2-i\,\nu_\eta\omega_A^2/(\omega-\omega_{*e}'), \tag{3}$$

where $\nu_\eta^{-1}\sim4\pi r^2/\eta_\parallel c^2$, $\omega_{*e}'=\omega_{*e}+(1.71c/eBr_0)dT_e/dr\sim\omega_{\rm di}$, and $\omega_{*e}=(cT_e/eBr_0)d\ln n/dr$ is the electron drift wave frequency. The relevant growth rate is $\gamma\simeq\nu_\eta\omega_A^2/$

 

$|\omega_{di}\omega_{*e}'|$. Thus, two roots of the dispersion relation for $m^0=1$ modes can be simultaneously unstable: One root, with $\omega \simeq \omega_{di}$, corresponds to the fishbone oscillations, while the second is the usual (resistive) internal kink mode (in the regime $0 \le \gamma_{MHD} < \omega_{di}$) that is responsible for the "crash" phase of sawtooth oscillations.[4] The fishbone mode, which has a larger growth rate, is not as hard to excite as the resistive internal kink. Accordingly, fishbone oscillations are observed to grow, saturate, and damp on a time scale considerably shorter than the sawtooth period.

We describe now the main analytical steps to support our theoretical model. The structure of $m^0=1$ modes is characterized by the presence of a transition layer of thickness $\delta$, centered around the surface $r=r_0$, where inertial as well as nonideal (such as resistive) effects become important.[4] Away from this layer, field and fluid are coupled as in the infinite-conductivity case, so that in this "outer" region a simplified ideal-MHD description of the fluid motion is feasible. We derive a normal-mode equation by matching the solution of the normal-mode equation in the boundary layer to the outer MHD solution.

In the outer region, the following MHD equations apply to the core plasma:

$$m_i n_i \, d\mathbf{u}/dt = -\nabla p_c + \mathbf{J}_c \times \mathbf{B}/c, \tag{4}$$

$$\mathbf{E} + \mathbf{u} \times \mathbf{B}/c = 0, \tag{5}$$

where $p_c = p_e + p_i$, $\mathbf{J}_c = \mathbf{J}_e + \mathbf{J}_i = e(n_i \mathbf{u}_i - n_e \mathbf{u}_e)$, and $\mathbf{u} \simeq \mathbf{u}_i$ is the core-plasma fluid velocity. These are coupled with Maxwell's equations $\nabla \times \mathbf{E} = -(1/c)\partial \mathbf{B}/\partial t$ and $\nabla \times \mathbf{B} = (4\pi/c)\mathbf{J}$, where $\mathbf{J} = \mathbf{J}_c + \mathbf{J}_h$ is the total current, $\mathbf{J}_h = e\int d^3v\, \mathbf{v} f_h$ is the beam current, and the beam distribution function $f_h$ satisfies the Vlasov equation. Applying the operator $\mathbf{e}_\parallel \cdot \nabla \times$ to the linearized Eq. (4), neglecting the core-plasma inertial term, we obtain

$$\mathbf{e}_\parallel \cdot \nabla \times (\mathbf{B} \cdot \nabla)\mathbf{B} + (\mathbf{B} \cdot \nabla)\hat{\mathbf{B}} = (4\pi/c)\mathbf{e}_\parallel \cdot \nabla \times (\hat{\mathbf{J}}_h \times \mathbf{B}). \tag{6}$$

We consider a normal mode structure of the type

$$\hat{\mathbf{B}} = [\tilde{\mathbf{B}}(r)e^{-i\theta} + \bar{\mathbf{B}}(r,\theta)]\exp(i\phi - i\omega t),$$

where we can order $\bar{B}(r,\theta)/\tilde{B}(r) \sim \varepsilon_0 \equiv a/R$, given the poloidal modulation of the low-$\beta$ toroidal equilibrium ($a$ is the torus minor radius). This ordering allows a truncation of the set of coupled equations generated by the poloidal expansion of Eq. (6). In particular, the $m^0=0$ and $m^0=2$ satellite harmonics can be obtained explicitly in terms of the $m^0=1$ component.[9] We also consider relatively low values of the beam poloidal beta, that is $\beta_{p,h} < \varepsilon_0 \beta_p$. With this ordering, only the hot-particle resonant (dissipative) term need be retained. This procedure leads to the following equation for the radial displacement $\xi_r(r) \equiv i\tilde{u}_r(r)/\omega$:

$$(d/dr)[r^3 F^2 (d\xi_r/dr)] = [g(r) + b(r)]\xi_r, \tag{7}$$

where $F = -B_\theta(1-q)/r$, $g(r)$ is such that[4,9] $\lambda_H = -[\pi/(B_\theta \hat{s})^2_{r=r_0}]\int_0^{r_0} g(r)dr$, and $b(r)$ is defined as

$$b(r)\xi_r(r) = -(4\pi i r^2/c)\langle \mathbf{e}_\parallel \cdot \nabla \times (\hat{\mathbf{J}}_h \times \mathbf{B})\exp(i\theta - i\phi + i\omega t)\rangle_{\text{res}}, \tag{8}$$

with $\langle A \rangle = \oint A(d\theta/2\pi)$ and the subscript "res" indicates that we consider only the resonant hot-particle contribution. By analogy with the definition of $\lambda_H$, we introduce $i\lambda_h = [\pi/(B_\theta \hat{s})^2_{r=r_0}]\int_0^{r_0} b(r)dr$, and $\lambda_H' = \lambda_H - i\lambda_h$. We observe that the beam contributes to the real part of $\lambda_H'$ both indirectly, by raising the temperature of the core plasma, and directly through the nonresonant part of $\hat{\mathbf{J}}_h$ that, however, we neglect, given the ordering of $\beta_{p,h}$ that we consider.

To evaluate $\lambda_h$, we consider for simplicity an equilibrium magnetic field of the type $\mathbf{B} = [B_\theta(r)\mathbf{e}_\theta + B_{\phi 0}\mathbf{e}_\phi]/h(r,\theta)$, where $h(r,\theta) = 1 + (r/R)\cos\theta$. Expanding the right-hand side of Eq. (8), we find

$$\mathbf{e}_\parallel \cdot \nabla \times (\hat{\mathbf{J}}_h \times \mathbf{B}) \simeq -ieB\omega[\hat{n}_h - \varepsilon_{\text{res}}^{-1}(1 - \Lambda_0/2h)^{-1}(\bar{\omega}_{Dh}/\omega)\hat{p}_{\perp h}],$$

where $\omega_{Dh} = -i\mathbf{v}_{Dh} \cdot \nabla$, $\mathbf{v}_{Dh}$ is the magnetic drift velocity, $\bar{\omega}_{Dh} = \omega_{Dh}(\varepsilon = \varepsilon_{\text{res}}, \Lambda = \Lambda_0)$, $\varepsilon_{\text{res}}$ is the energy of the resonant beam particles [defined in such a way that $\bar{\omega}_{Dh}^{(0)} \equiv \omega$ and the superscript (0) indicates average along the unperturbed particle orbit], $\Lambda = \mu B h/\varepsilon$, $\Lambda_0 = [1 + (r/R)]\cos^2\alpha_{\text{inj}}$, and $\alpha_{\text{inj}}$ is the beam injection angle. We have used the hot-particle continuity equation to relate $\nabla \cdot \hat{\mathbf{J}}_h$ to $\hat{n}_h$. The transverse beam pressure $\hat{p}_{\perp h}$ is evaluated directly from the relevant perturbed distribution function and we have verified that, to lowest order in $\rho_h/a$, $\hat{\mathbf{J}}_h \simeq c\mathbf{e}_\parallel \times \nabla \hat{p}_{\perp h}$. For mode frequencies below the bounce frequency of the trapped beam particles, and for $\text{Im}(\omega) < \text{Re}(\omega)$, the resonant part of the perturbed beam distribution function is approximated by

$$\hat{f}_{h,\text{res}} \simeq -i\pi\varepsilon(\partial f_{h0}/\partial\varepsilon)(\omega - \omega_{*h})\hat{\psi}^{(0)}\delta(\varepsilon - \varepsilon_{\text{res}}),$$

where $f_{h0}(r;\varepsilon,\Lambda)$ is the equilibrium distribution function for the beam

$$\omega_{*h} \simeq \frac{\partial \ln f_{0h}/\partial r}{\partial \ln f_{0h}/\partial \varepsilon}(m_h \Omega_h r)^{-1}$$

and

$$\hat{\psi} = [(\omega_{Dh}/\omega)\hat{\phi} + (v_\perp^2/2\Omega_h c)\hat{B}_\parallel]e/\varepsilon.$$

When the ideal-MHD relation (5) and Faraday's law are used to relate the perturbed electrostatic potential $\hat{\phi}$ and $\hat{B}_\parallel$ to $\boldsymbol{\xi}$, we obtain $\hat{\psi} \simeq [(3\Lambda/h) - 2]\hat{\boldsymbol{\xi}} \cdot \boldsymbol{\tau}$, where $\boldsymbol{\tau} = (\mathbf{e}_\parallel \cdot \nabla)\mathbf{e}_\parallel$. Then, we use a beam distribution function $f_{0h} = S(r)\varepsilon^{-3/2}\theta(\varepsilon_{\text{inj}} - \varepsilon)\delta(\Lambda - \Lambda_0)$, where

$$S(r) = (r/R)^{1/2} p_{\perp h}(r)/4K m_h \varepsilon_{\text{inj}},$$

$\varepsilon_{inj}$ is the beam injection energy, $\theta(x)$ is the step function, and $K$ is the complete elliptic integral of the first kind of argument $\kappa^2=(R/2r)\sin^2\alpha_{inj}$. We also notice that $\omega\sim\omega_{di}<\omega_{*h}$. The final result for $i\lambda_h$ is

$$i\lambda_h=(i\pi^2/2\hat{s}_0^2)(r_0/R)\omega/\varpi_{Dh})\beta_{p,h}, \tag{9}$$

where $\varpi_{Dh}=\omega_{Dh}^{(0)}(r=r_0,\varepsilon=\varepsilon_{inj},\Lambda=1+r_0/R)$ and

$$\beta_{p,h}=-(R/r_0^2)^2\int_0^{r_0}dr\,r^2(d\beta_h/dr)\theta(\varepsilon_{inj}-\varepsilon_{res}).$$

(Notice that $\varepsilon_{res}$ goes to infinity near the radius where $\kappa^2\simeq 1$ and the beam particles become untrapped.)

In the transition layer, we consider the effects of inertia, resistivity, core-ion diamagnetic frequency $\omega_{di}$, and electron drift frequency $\omega_{*e}$. The hot-particle contribution is not important here. This is principally due to the fact that, for the type of mode-particle resonance that we consider, the dominant part of the associated "dissipative" term does not involve radial derivatives of the plasma displacement which would become very large near the surface $r=r_0$. Thus, we refer to Ref. 4 for the form of the normal-mode equation in the transition layer. The asymptotic matching procedure, which is well known in the literature,[4] involves the quantity $\lambda_H'$ that we have previously defined. This leads to the dispersion relation[10]

$$[-\hat{\omega}(\hat{\omega}-\hat{\omega}_i)]^{1/2}=(\lambda_H-i\lambda_h)(Q^{3/2}/8)\{\Gamma((Q-1)/4)/\Gamma((Q+5)/4)\}, \tag{10}$$

where $Q^2\equiv i\hat{\omega}(\hat{\omega}-\hat{\omega}_i)(\hat{\omega}-\hat{\omega}_e)/\varepsilon_\eta$, $\hat{\omega}\equiv\omega/\omega_A$, $\hat{\omega}_i=\omega_{di}/\omega_A$, $\hat{\omega}_e=\omega_{*e}'/\omega_A$, and $\varepsilon_\eta=\eta c^2/4\pi r_0^2\omega_A$.

The most interesting case is the solution of (10) for values of $\lambda_H'$ such that $(\varepsilon_\eta/\hat{\omega}_i)^{1/2}<\lambda_H'<\hat{\omega}$. In this regime, the dispersion relation (10) reduces to

$$[\hat{\omega}+i2\lambda_H\lambda_h/(\hat{\omega}_i-\hat{\omega})](\hat{\omega}-\hat{\omega}_i)=-\lambda_H^2-5i\varepsilon_\eta/2(\hat{\omega}-\hat{\omega}_e), \tag{11}$$

and this can be related to the model dispersion relations (2) and (3). The relevant growth rate for the fishbone root with $\hat{\omega}\simeq\hat{\omega}_i$ is

$$\gamma\simeq(\pi/\hat{s}_0)^2(r_0/R)(\omega_A^2/\varpi_{Dh})\lambda_H\beta_{p,h}-\tfrac{5}{2}\varepsilon_\eta\omega_A^3/[\omega_{di}(\omega_{di}-\omega_{*e}')]. \tag{12}$$

The lower-frequency mode is made unstable by the electrical resistivity and has the growth rate $\gamma\simeq\frac{5}{2}\varepsilon_\eta\omega_A^3/|\omega_{di}\omega_{*e}'|$. For both roots, the eigenfunction reduces to[4] $\xi_r(x)=(\xi_\infty/2)[1-(2/\pi)\arctan(x/\lambda_H')]$, where $x=(r-r_0)/r_0$. For typical fishbone experiments, $\lambda_H\sim(r_0/R)^2$, $\hat{\omega}_i\sim r_0/R\gtrsim\beta_{p,h}\sim10^{-1}$, and $\varepsilon_\eta\sim10^{-7}$. These estimates, together with $\omega_{di}\sim20$ kHz, yield a growth time $\gamma^{-1}\lesssim1$ msec for the fishbone mode, consistent with the time over which significant beam particle losses were observed in the poloidal-divertor-experiment (PDX) machine.[1]

Equation (12) sets a threshold for the instability of the fishbone that involves the relative values of the core-plasma $\beta_p$ (appearing in the definition of $\lambda_H$), $\beta_{p,h}$, and $\varepsilon_\eta$. We observe that when the ideal-MHD instability condition is only marginally satisfied $[|\lambda_H'|<(\varepsilon_\eta/\hat{\omega}_i)^{1/2}]$, the limiting form of the dispersion relation (12) is $\hat{\omega}(\hat{\omega}-\hat{\omega}_i)(\hat{\omega}-\hat{\omega}_e)=-i\varepsilon_\eta$; i.e., it is independent of $\lambda_H'$. In this regime, the fishbone root is stable, having an eigenfrequency $\hat{\omega}\simeq\hat{\omega}_i-i\varepsilon_\eta/2|\hat{\omega}_i\hat{\omega}_e|$. We also observe that, in the limit $\mathrm{Im}(\lambda_H')>(\varepsilon_\eta/\hat{\omega}_i)^{1/2}>\mathrm{Re}(\lambda_H')\to0$, the mode eigenfunction associated with the eigenvalue $\hat{\omega}\simeq\hat{\omega}_i$ becomes singular at the $r=r_0$ surface. Consequently, $\mathrm{Re}(\lambda_H')>0$ is a necessary condition for the instability of this mode.

The resonant scattering of beam particles produced by an excited mode is a relatively easy process to understand and describe.[8,11] This process tends to remove the resonating particles and the associated ion dissipation, so that the "excergy" in the plasma pressure gradient is no longer accessible, leading to saturation of the fishbone mode. Another possibility that we consider is that the mode damping due to resistivity is responsible for the decay phase of the fishbone. Then, the following nonlinear model for the entire cycle, analogous to the one presented by Chen, White, and Rosenbluth,[12] but consistent with our linear instability process, can be constructed. We consider, for simplicity, the normalized mode amplitude $A=|\tilde{B}_\theta|/B_\theta$ and the beam density $n_h$ as the two quantities that vary significantly with time during a fishbone cycle. Considering values of $\lambda_H>(\varepsilon_\eta/\hat{\omega}_i)^{1/2}$, we have [see Eq. (12)]

$$\partial A/\partial t=-\gamma_\eta A+\gamma_{MHD}(n_h/n_0)A, \tag{13}$$

where $\gamma_\eta=\frac{5}{2}\varepsilon_\eta\omega_A^3/[\omega_{di}(\omega_{di}-\omega_{*e}')]$, $\gamma_{MHD}=\lambda_H\omega_A$, and

$$n_0=(\hat{s}_0/\pi)^2(R/r)(\varpi_{Dh}/\omega_A)\langle n_h\rangle/\langle\beta_{p,h}\rangle,$$

with $\langle n_h\rangle$ and $\langle\beta_{p,h}\rangle$ time-average values of $n_h$ and $\beta_{p,h}$ during a cycle. The beam particle density is taken to vary as

$$\partial n_h/\partial t=S_h-\gamma_L n_{crit}A^2, \tag{14}$$

where $S_h$ is the beam source term, $n_{crit}=n_0(\gamma_\eta/\gamma_{MHD})\sim n_h$, and $\gamma_L A^2\sim10^3$ sec$^{-1}$ is a measure of the hot-particle loss rate. A simple analysis of Eqs. (13) and (14) shows that a cyclic solution exists with period $t_{fb}\sim2\Delta\hat{n}_h/\gamma_\eta\Gamma^2$ when $(\Delta\hat{n}_h/\Gamma)^2>1$, where $\hat{n}_h=(n_h-n_{crit})/n_{crit}$, $\Delta\hat{n}_h=\max\{\hat{n}_h\}$, and $\Gamma^2=S_h/\gamma_\eta n_{crit}$. Then, for typical PDX parameters, we find $t_{fb}\sim2$–$6$ msec. The level of fluctuation of the poloidal magnetic field is typi-

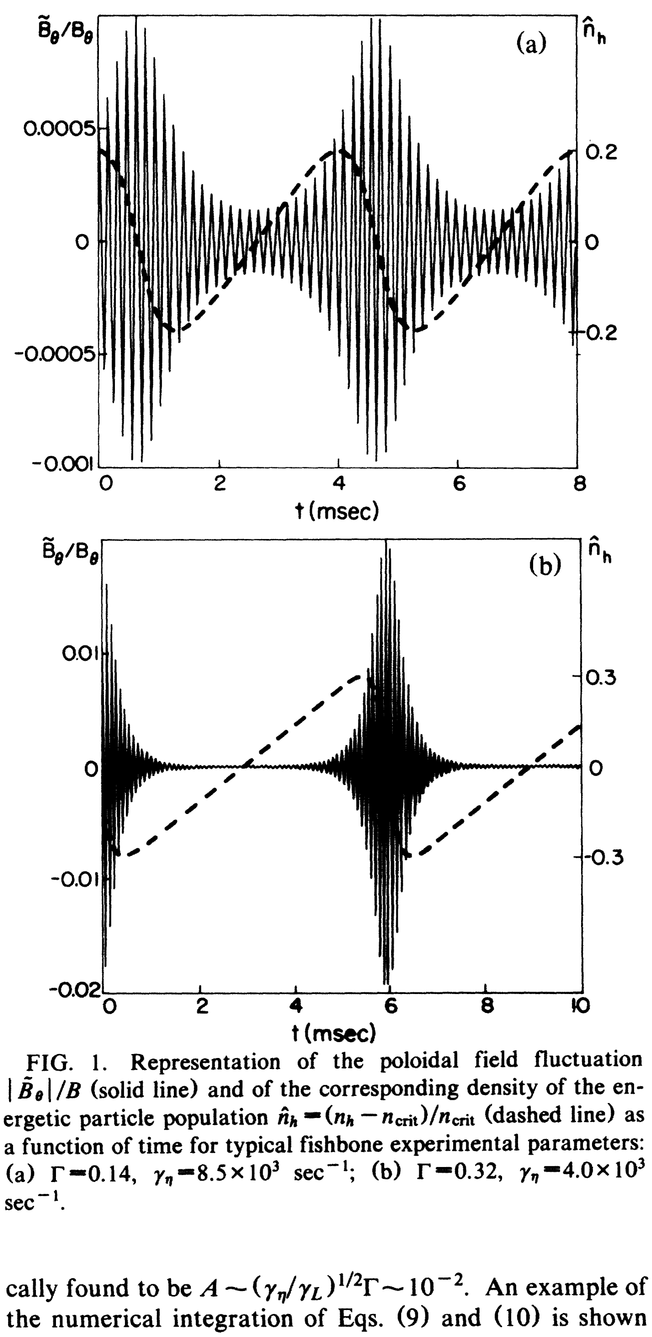


FIG. 1. Representation of the poloidal field fluctuation $|\tilde{B}_\theta|/B$ (solid line) and of the corresponding density of the energetic particle population $\hat{n}_h = (n_h - n_{crit})/n_{crit}$ (dashed line) as a function of time for typical fishbone experimental parameters: (a) $\Gamma = 0.14$, $\gamma_\eta = 8.5\times10^3$ sec$^{-1}$; (b) $\Gamma = 0.32$, $\gamma_\eta = 4.0\times10^3$ sec$^{-1}$.

cally found to be $A \sim (\gamma_\eta/\gamma_L)^{1/2}\Gamma \sim 10^{-2}$. An example of the numerical integration of Eqs. (9) and (10) is shown in Fig. 1. In the experiments, the type of fishbone shown in Fig. 1(b) was more frequently observed near the threshold of ideal marginal stability, where the fishbone activity is lower and the fraction of scattered beam particles ($\Delta\hat{n}_h$) is smaller. The excited mode, that has the structure of a kink, can be expected to produce a local flattening of the plasma pressure profile near the $r = r_0$ surface; the corresponding decrease of $\omega_{di}$ may explain the observed drop of the mode frequency (by as much as 30%) during a fishbone burst.

A different theoretical interpretation of the fishbone instability has been advanced and presented in Ref. 12. Considering the limit $\omega_{di} = 0$, the authors of Ref. 12 indicate having found a new $m^0 = 1$ mode with a frequency of oscillation related to $\bar{\omega}_{Dh}$. In this paper we have instead proposed that fishbone oscillations are related to an $m^0 = 1$ mode that is well known in the literature[4] but which necessitates a positive dissipation process to be driven unstable. In particular, the frequency of our mode is not anchored to $\bar{\omega}_{Dh}$ being finite. This conclusion is consistent with the most recent observations presented by Heidbrink *et al.*[13] and concerning experiments with beams injected tangentially to the magnetic field.

It is a pleasure to thank S. Migliuolo for his collaboration in the numerical analysis of the dispersion equation, and H. L. Berk, G. Bertin, L. Chen, F. Pegoraro, J. J. Ramos, M. N. Rosenbluth, and R. B. White for many illuminating discussions. This work has been supported in part by the U.S. Department of Energy and by the Comitato Ignitor of Torino, Italy.

[a] Also at Scuola Normale Superiore, Pisa, Italy.

[1] K. McGuire *et al.*, Phys. Rev. Lett. **50**, 891 (1983).

[2] D. O. Overskei *et al.*, in *Proceedings of the Fourth International Symposium on Heating in Toroidal Plasmas, Rome, 1984,* edited by H. Knoepfel and E. Sindoni (International School of Plasma Physics, Varenna, Italy, 1984).

[3] B. Coppi, contribution presented at the Varenna Workshop on Plasma Transport, Varenna, Italy, July 1982 (unpublished), and at the Princeton Plasma Physics Laboratory Workshop on Bean-Shaped and High-$\beta$ Tokamaks, Princeton, NJ, April 1983 (unpublished).

[4] G. Ara, B. Basu, B. Coppi, G. Laval, M. N. Rosenbluth, and B. V. Waddell, Ann. Phys. (N.Y.) **112**, 443 (1978).

[5] M. N. Bussac, R. Pellat, D. Edery, and J. L. Soulé, Phys. Rev. Lett. **35**, 1638 (1975).

[6] B. Coppi, M. N. Rosenbluth, and R. N. Sudan, Ann. Phys. (N.Y.) **55**, 230 (1969).

[7] B. Coppi and D. K. Bhadra, Phys. Fluids **18**, 692 (1975).

[8] R. B. White *et al.*, Phys. Fluids **26**, 2958 (1983).

[9] J. Crew and J. J. Ramos, Phys. Fluids **26**, 2621 (1983).

[10] M. Velli, F. Porcelli, and B. Coppi, Bull. Am. Phys. Soc. **29**, 1386 (1984).

[11] B. Coppi and A. Taroni, Plasma Phys. **16**, 161 (1974).

[12] L. Chen, R. B. White, and M. N. Rosenbluth, Phys. Rev. Lett. **52**, 1122 (1984).

[13] W. W. Heidbrink *et al.*, Phys. Rev. Lett. **57**, 835 (1986).