Late Phase Transitions and the Spontaneous Generation of Cosmological Density Perturbations

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Phase transitions occurring after photon decoupling are shown to result in significant density perturbations.

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With the emergence of unified models connecting high-energy particle theory and early-Universe evolution has come the notion that cosmological density perturbations were generated by physical processes at remote epochs.[1] However, increasingly accurate measurements of fluctuations in the cosmic microwave background[2] appear on the verge of ruling out truly primordial perturbations as the precursors of the known condensations in the present-day Universe. Moreover, recent systematic studies of the large-scale organization of galaxies[3] may require non-Gaussian initial fluctuations that are not symmetric about zero, although this is controversial.[4]

In view of these potential observational difficulties, we explore in this paper a speculative alternative to conventional theories of the origin of density perturbations. In order to circumvent microwave-background constraints, I assume that physical processes prior to photon decoupling (i.e., at red shifts $z \gtrsim 10^3$) produced at most negligible fluctuations. I then postulate a low-temperature ($T_c \lesssim 2700$ K or equivalently $T_c \lesssim 0.2$ eV), first-order phase transition in which the vacuum energy density drops from ρ_v to zero (or any other value $<< \rho_v$). On the assumption that $\rho_v \sim aT_c^4$ is small compared to the matter density ρ_0 at the onset of the phase transition, the possibility of an associated inflationary phase is neglected. The nucleation of bubbles of new, ordered phase, with the attendant relativistic bubble-wall expansion,[5] leads to small density perturbations substantially spherical in shape. The effective amplitude of the resulting fluctuations is $\delta\rho/\rho \lesssim \rho_v/\rho_0$, and the characteristic length scale associated with the perturbations is determined by the bubble nucleation rate.

At the outset it should be emphasized that there is at present no physical reason to suppose that such a phase transition ever occurred. The possibility of low-temperature phase transitions has been the subject of earlier speculation.[6] The most compelling argument in their favor, due to Primack and Sher,[6] is that a phase transition at $T_c \sim 10^{-1}$ eV would be about as far on a logarithmic scale from the electroweak transition ($T_{\rm EW} \sim 10^{11}$ eV) as is the hypothetical grand-unification-theoretic (GUT) transition ($T_{\rm GUT} \sim 10^{24}$ eV). I make no attempt here to construct a microphysical theory for such a transition, and concentrate on its macroscopic gravitational effects rather than its possibly observable but inherently model-dependent phenomenology.

To understand the perturbative effects of the phase transition, let us first consider the evolution of a single spherical bubble of disordered vacuum. Prior to the phase transition let us assume a flat ($k=0$) Robertson-Walker Universe, which may be the result of a GUT inflationary era.[7] Suppose that bubble nucleation occurs at the coordinate origin, $r=0$, at time t_0. Assume that t_0 is safely in the matter-dominated era, and thus ignore the (uniform) gravitational deceleration due to primordial radiation backgrounds. At $t < t_0$, points on any spherical shell of coordinate radius r are at a proper distance $d(t) = ra(t)$ from the origin, where, if $a(t_0) \equiv 1$,

$$\dot{a}^2(t) = \frac{8\pi G\rho_0}{3a(t)} + \frac{8\pi G\rho_v a^2(t)}{3}. \tag{1}$$

At $t > t_0$, the boundary of the spherical bubble of new phase expands outward at the speed of light,[5] reaching coordinate radius r at time $t_c(r) > t_0$, which may be determined from the relation ($c \equiv 1 \equiv \hbar$)

$$r = \int_{t_0}^{t_c(r)} dt'/a(t'). \tag{2}$$

At $t > t_c(r) > t_0$ the equation of motion for a spherical shell at radius r is $d(t) = ra(t;r)$ where instead of Eq. (1) we have

$$\dot{a}^2(t;r) = \frac{8\pi G\rho_0}{3a(t;r)} + \frac{8\pi G\rho_v a_c^2(r)}{3}, \tag{3}$$

with $a_c(r) \equiv a(t_c(r);r) = a(t_c(r))$. In writing Eq. (3) I have tacitly assumed that particle masses change only negligibly ($\Delta m/m \lesssim T_c/m << \rho_v a^3/\rho_0$) as a result of the phase transition. Equation (3) is the equation of motion for an unbound spherical shell with a total energy

$$E(r) = \tfrac{4}{3}\pi G\rho_v a_c^2(r) r^2. \tag{4}$$

Since $a_c(r)$ is an increasing function of r, $E(r)/r^2$ increases outwards. If we write $a(t;r) \equiv a(t)[1+\epsilon(r,t)]$, then Eqs. (1) and (3) imply

$$\epsilon(r,t) = \frac{\rho_v a_c^3(r)}{\rho_0}\left\{\frac{1}{5}\frac{a(t)}{a_c(r)} - \frac{1}{9}\left[\frac{a(t)}{a_c(r)}\right]^3 - \frac{4}{45}\left[\frac{a_c(r)}{a(t)}\right]^{3/2}\right\} \tag{5}$$

to lowest order in ρ_v/ρ_0 for $(\rho_v/\rho_0)a^3(t) \ll 1$ and $(\rho_v/\rho_0)a_c^2(r)a(t;r) \ll 1$.

The isolated stage in the evolution of individual density fluctuations ends when adjacent bubble walls collide. For the particular region considered in the last paragraph, let us suppose that this happens at a time $t_f = t_0(1+\delta_f)$ with $\delta_f \lesssim 1$. Then the total mass enclosed by the fluctuation region is

$$M_f \approx \tfrac{4}{3}\pi\rho_0\delta_f^3 t_0^3 \approx 3\times10^{17}\delta_f^3\left(\frac{1+z_0}{1000}\right)^{-3/2} h^{-1}M_\odot, \tag{6}$$

where $h = H_0/100$ km s^{-1} Mpc^{-1} and z_0 is the red shift at t_0. From Eq. (6) it is clear that δ_f must be small in order that the mass scale associated with the fluctuations not be too large. The required bubble nucleation rate is then $\Gamma_{\rm nuc} \approx (3/4\pi)(\delta_f t_0)^{-4}$, so that $T_c^4 \gg \Gamma_{\rm nuc} \gg [(\dot a/a)_{t=t_0}]^4$. The perturbation amplitude at t_f, like its scale, depends on δ_f. From Eq. (5) we find the fractional radial displacement

$$\epsilon(r,t_f) \approx -(2\rho_v/9\rho_0)(\delta_f - \delta_r)^2, \tag{7}$$

and the relative peculiar velocity

$$\frac{v(r,t_f)}{r(\dot a/a)_{t=t_f}} = \epsilon(r,t_f) + \frac{[\partial\epsilon(r,t)/\partial t]_{t=t_f}}{(\dot a/a)_{t=t_f}} \approx -(2\rho_v/3\rho_0)(\delta_f - \delta_r), \tag{8}$$

where $\delta_r \equiv t_c(r)/t_0 - 1 \approx r/t_0$ [see Eq. (2)].

The subsequent evolution of the fluctuation depends on the outcome of bubble-wall collisions.[8] One possibility is that colliding bubble walls disintegrate by emission of relativistic waves. I assume that the waves are at most very weakly coupled to the matter, so that they only influence the developing perturbation gravitationally. To get a crude idea of the effect of this "radiation," suppose that the waves uniformly and instantaneously fill the fluctuation region with density[5] ρ_v at $t = t_f$. The equation of motion for any spherical shell within the bubble is then

$$\ddot a(t;r) = -\frac{4\pi G\rho_0}{3a^2(t;r)} - \frac{8\pi G\rho_v a^4(t_f)a(t;r)}{3a^4(t)}, \tag{9}$$

where $a(t)$ satisfies Eq. (9) with $a(t;r) = a(t)$. Multiplying Eq. (9) by $\dot a(t;r)$ we find

$$\frac{d}{dt}\left[\frac{E(r)}{r^2}\right] = -\frac{8\pi G\rho_v a^4(t_f)a(t;r)\dot a(t;r)}{3a^4(t)}. \tag{10}$$

Thus the "radiation" tends to bind the spherical shells. Because the wave energy density drops rapidly, the right-hand side of Eq. (10) is only significant at early times, when $a(t;r) \approx a(t)$. Substituting $a(t)$ for $a(t;r)$ in Eq. (10) and then integrating to $t \gg t_f$ gives

$$E(r) \approx -\tfrac{4}{3}\pi G\rho_v[a^2(t_f) - a_c^2(r)]r^2 \approx -\tfrac{16}{9}\pi G\rho_v(\delta_f - \delta_r)r^2. \tag{11}$$

If one ignores shell crossing, Eq. (11) implies that a shell of radius r begins to recollapse at time

$$t_r \approx \left(\frac{3\pi}{32G\rho_0}\right)^{1/2}\left(\frac{3\rho_0}{4\rho_v(\delta_f - \delta_r)}\right)^{3/2}, \tag{12}$$

or, by use of the approximate relation $6\pi G\rho t^2 \approx 1$, at a red shift

$$1 + z_r \approx \frac{1}{\pi^{2/3}}\left(\frac{4}{3}\right)^{5/3}\frac{\rho_v(\delta_f - \delta_r)}{\rho_0}(1+z_0). \tag{13}$$

From Eq. (13) we see that only those shells satisfying the inequality

$$\frac{\rho_v(\delta_f - \delta_r)}{\rho_0} \gtrsim \pi^{2/3}(\tfrac{3}{4})^{5/3}(1+z_0)^{-1} \tag{14}$$

will have recollapsed by the present. Equation (14) requires a minimum characteristic amplitude $\rho_v\delta_f/\rho_0 \gtrsim (1+z_0)^{-1}$ for fluctuations ever to become nonlinear.

Equation (11) represents a shift of $E(r)/r^2$ so that it goes to zero on the (roughly spherical) fluctuation boundary. The resulting perturbation is bound even though $E(r)/r^2 > 0$ inside the bubble at $t < t_f$ since, in addition, $d[E(r)/r^2]/dr > 0$. Moreover, because $E(r)/r^2 = 0$ in a $k = 0$, $\rho^v = 0$ Robertson-Walker background, Eq. (11) should hold at least roughly whenever bubble-wall collisions result in a smooth redistribution of the swept-up energy in the form of particles or waves. As long as any products of bubble-wall collisions interact sufficiently weakly, there should be little reheating of the matter at $t \gtrsim t_f$ and little resultant perturbation of the microwave background.

Unbound perturbations can only result if the energy in colliding bubble walls remains localized at the fluctuation boundary at $t > t_f$. In this case one might expect some pileup of material at the cell boundary, since matter attempting to leave the fluctuation region experiences a small extra deceleration. This extra deceleration could be particularly important for shells crossing the boundary at relatively early times (i.e., $t \gtrsim t_f$ rather than $t >> t_f$), serving to compensate largely for their small initial $E(r)/r^2 > 0$. Once shells begin to accumulate at a cell boundary, a density ridge may result.[9] Large voids surrounded by galaxies would be the natural outcome in such a scenario.

Gravitational field variations associated with post-recombination phase transitions can generate significant density fluctuations without perturbing the microwave background. The effective amplitude of the perturbations just before bubble walls collide is

$$\Delta_{\rm eff} \sim \frac{\rho_v}{{}_c\rho_0}\delta_f \approx 0.02\left(\frac{hM_f}{3\times 10^{17}M_\odot}\right)^{1/3}\left(\frac{1+z_0}{1000}\right)^{3/2}, \qquad (15)$$

where we have used Eq. (6) and taken $\rho_v = aT_c^4$ with $T_c = 2.7(1+z_0)$ K. If $M_f \sim (10^{14}\text{-}10^{15})M_\odot$, which is characteristic of galaxy clusters,[10] then we must require $1+z_0 \approx 1000$ in order that nonlinear fluctuations may be built up by the present time. Fluctuations on larger mass scales, such as $M_f \sim 10^{16}M_\odot$ characteristic of the largest voids,[11] could allow slightly smaller values of $1+z_0$. It is conceivable that either clumps or voids can be produced in the transition and its aftermath. If approximately spherical overdense regions result then the centers of these overdense regions would follow a Poisson distribution, most likely resulting in a spongelike rather than cellular large-scale topology.[4] If unbound perturbations emerge from bubble-wall expansion and collision then the soap-bubble Universe favored by some could be naturally explained without explosions.[12]

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